

Solve the following system of equations.

1.
$$\begin{cases} A \ 2x + 3y - z = 9 \\ B \ -2x - y + 2z = 2 \\ C \ (x + y - 2z = 3) \cdot 2 \end{cases}$$

Step 1: Create 2 new equations of 2 Variables

~~A~~ $2x + 3y - z = 9$
~~B~~ $-2x - y + 2z = 2$
 D $2y + z = 11$
 $2(8) + z = 11$
 $16 + z = 11$
 $z = -5$
 E $y = 8$
 C $x + (8) - 2(-5) = 3$
 $x + 8 + 10 = 3$
 $x + 18 = 3$
 $x = -15$
 Solution: $(-15, 8, -5)$

Step 2: Solve the 2 Variable Equation for the two variables

2.
$$\begin{cases} A \ (4x - 2y - z = 5) \cdot -2 \\ B \ (x + 4y - z = -1) \cdot -1 \\ C \ (2x - 2y - 2z = -2) \end{cases}$$

~~A~~ $4x - 2y - z = 5$
~~B~~ $-x - 4y + z = 1$
 D $3x - 6y = 6$
~~D~~ $6x - 12y = 12$
~~E~~ $-4x + 2y = 12$
 $-10y = 0$
 $y = 0$
 E $-6x + 2(0) = -12$
 $-6x = -12$
 $x = 2$
 B $(2) + 4(0) - z = -1$
 $2 + 0 - z = -1$
 $-z = -3 \rightarrow z = 3$
 Solution: $(2, 0, 3)$

3.
$$\begin{cases} A \ -3x + 2y + 5z = -10 \\ B \ -x - 2y + 3z = 6 \\ C \ (2x - y - z = 8) \cdot -2 \end{cases}$$

~~A~~ $-3x + 2y + 5z = -10$
~~B~~ $-x - 2y + 3z = 6$
~~C~~ $-4x + 2y + z = -16$
 D $-4x + 8z = -4$
~~D~~ $4x - 40z = 20$
~~E~~ $-20x + 20z = -40$
 $-20z = -20$
 $z = 1$
 E $-5x + 5(1) = -10$
 $-5x + 5 = -10$
 $-5x = -15$
 $x = 3$

Step 3: plug in to one of the originals to find the third.

Solution: $(3, -3, 1)$
 $z = 1$
 $x = 3$
 C $2(3) - y - (1) = 8$
 $6 - y - 1 = 8$
 $5 - y = 8$
 $-y = 3$
 $y = -3$

~~A~~ $x + y + z = -2$
~~B~~ $3x - y - 4z = -25$
~~C~~ $-x - y + 9z = 52$
 D $10z = 50$
 $z = 5$
 E $4x - 3z = -27$
 $4x - 3(5) = -27$
 $4x - 15 = -27$
 $4x = -12$
 $x = -3$
 A $(-3) + y + (5) = -2$
 $y + 2 = -2$
 $y = -4$
 Solution: $(-3, -4, 5)$

5. Last year, a baseball team purchased new equipment. The equipment manager paid \$20 per bat and \$12 per glove and \$15 per ball, spending a total of \$646. The manager bought 40 pieces of equipment. They bought 7 more bats than balls. Write a system of equations and solve for the amount of bats, gloves, and balls that were bought.

more bats
 x is a letter

Determine Variables x : # bats y : # of gloves z : # of balls

Total Valued Equation: $20x + 12y + 15z = 646$
 A $20(z+7) + 12y + 15z = 646$
 B $(z+7) + y + z = 40$

Total Object Equation: $x + y + z = 40$
 B $20z + 140 + 12y + 15z = 646$
 C $7 + y + 2z = 40$

Relationship Equation: $x = z + 7$
 D $12y + 35z = 506$
 E $y + 2z = 33$

17 bats, 13 gloves, and 10 balls were bought.

~~E~~ $-12y - 24z = -396$
 $11z = 110$
 $z = 10$
 $x = (10) + 7$
 $x = 17$
 E $y + 2(10) = 33$
 $y + 20 = 33$
 $y = 13$

6. Andrea Liskow was the top scorer in a women's professional basketball league for the 2006 regular season, with a total of 822 points. The number of two-point baskets that Andrea made was 60 less than double the number of three-point baskets she made. The number of free throws (each worth one point) she made was 15 less than the number of two-point field goals she made. Find how many free throws, two-point baskets, and three-point baskets Andrea Liskow made during the 2006 regular season.

Determine Variables x: # Free throws y: # of 2pt baskets z: # of 3pt baskets

Total Valued Equation: $x + 2y + 3z = 822$ A $\textcircled{A} x + 2(x+15) + 3z = 822$ $\textcircled{B} (x+15) = 2z - 60$

Total Object Equation: $y = 2z - 60$ B $x + 2z + 30 + 3z = 822$ $\textcircled{C} x - 2z = -45$

Relationship Equation: $x = y - 15$ C $\textcircled{D} 3x + 3z = 792$ $\textcircled{E} -3x + 6z = 135$

161 Free throws
 176 2pt baskets
 103 3pt baskets

$9z = 927$
 $z = 103$
 $\textcircled{E} x - 2(103) = -45$
 $x - 206 = -45$
 $x = 161$
 $\textcircled{C} y = x + 15$
 $y = 161 + 15$
 $y = 176$

7. Write the system of 3 variable equations for the matrix.

$$\begin{bmatrix} 2 & 5 & 0 & | & 13 \\ -3 & 1 & 2 & | & 6 \\ 4 & 0 & -3 & | & 5 \end{bmatrix}$$

$2x + 5y = 13$
 $-3x + y + 2z = 6$
 $4x - 3z = 5$

8. Write the system of 3 variable equations for the matrix.

$$\begin{bmatrix} 6 & -3 & 6 & | & 5 \\ 4 & 6 & -7 & | & 4 \\ -2 & 6 & 6 & | & 7 \end{bmatrix}$$

$6x - 3y + 6z = 5$
 $4x + 6y - 7z = 4$
 $-2x + 6y + 6z = 7$

Write the matrix for the system of equations and solve (remember $[A]^{-1}[B]$).

9. $\begin{cases} 3x + y = -4 \\ -2x + 4y = 7 \end{cases}$

$$\left[\begin{array}{cc|c} 3 & 1 & -4 \\ -2 & 4 & 7 \end{array} \right]$$

$$\left[\begin{array}{cc|c} 3 & 1 & -4 \\ -2 & 4 & 7 \end{array} \right]^{-1} \cdot \begin{bmatrix} -4 \\ 7 \end{bmatrix} =$$

$(-1.643, 0.929)$

10. $\begin{cases} 4x - y + 2z = 10 \\ 5x + 2y - 3z = 0 \\ x - 3y + z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 4 & -1 & 2 & 10 \\ 5 & 2 & -3 & 0 \\ 1 & -3 & 1 & 6 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 4 & -1 & 2 & 10 \\ 5 & 2 & -3 & 0 \\ 1 & -3 & 1 & 6 \end{array} \right]^{-1} \cdot \begin{bmatrix} 10 \\ 0 \\ 6 \end{bmatrix} =$$

$x = 1.407$
 $y = -0.963$
 $z = 1.704$

11. $\begin{cases} 3x - 2y + z = 6 \\ 4x - 6z = 6 \\ -3y - 4z = -10 \end{cases}$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 4 & 0 & -6 & 6 \\ 0 & -3 & -4 & -10 \end{array} \right]$$

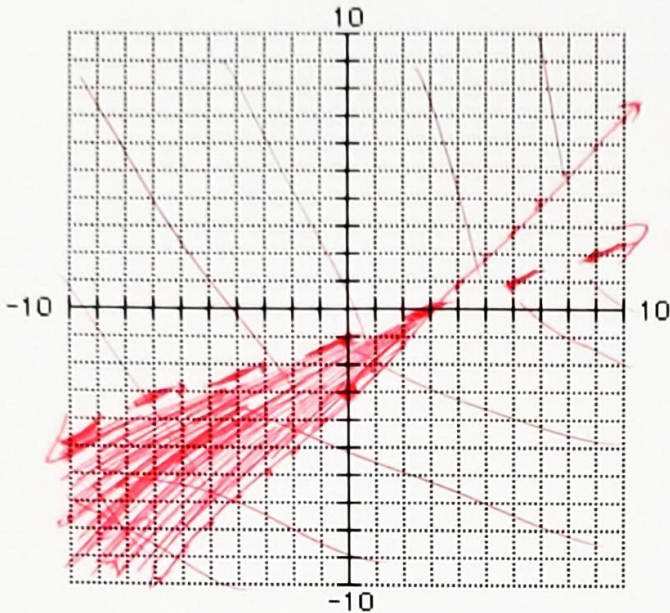
$$= \left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 4 & 0 & -6 & 6 \\ 0 & -3 & -4 & -10 \end{array} \right]^{-1} \cdot \begin{bmatrix} 6 \\ 6 \\ -10 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$(3, 2, 1)$

Graph and find the solution to the system of inequalities

12. $y \geq x - 3$

$y < \frac{1}{3}x - 1$



Two Solution points include: $(0, -3)$ $(-5, -5)$

Is $(-3, -3)$ a solution? Explain Is $(3, 0)$ a solution? Explain
 Prove it by plugging the points into the equations to check for true statements

Yes in shaded region
 $-3 \geq -3 - 3$
 $-3 \geq -6$ ✓ true
 (bigger b/c closer to 0)
 $-3 < \frac{1}{3}(-3) - 1$
 $-3 < -1 - 1$
 $-3 < -2$ ✓ true

No on a dashed line
 $0 \geq 3 - 3$
 $0 \geq 0$ ✓ True is =
 $0 < \frac{1}{3}(3) - 1$
 $0 < 1 - 1$
 $0 < 0$ X False not less than

13. which region/s are solutions to the above system graphed below? Give two points in the solution set, if there are more than one region include one point from each region. Determine if the given point is a solution and explain your thinking.

$y < -|x - 5| + 2$
 $y \leq \frac{1}{2}x - 6$

Solution Region/s: 4

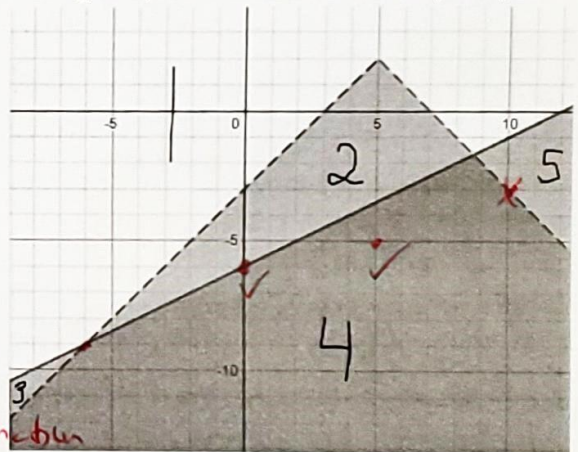
Two Solution points include: $(0, -6)$ $(5, -5)$

Is $(0, -6)$ a solution? Explain Is $(10, -3)$ a solution? Explain

Yes on a solid boundary line of solution region

No, on a dashed boundary line

Is $(-6, -9)$ a solution? Explain No
 works for linear function but not for Absolute Value function



Extension: Solve these Non Linear Systems using elimination or substitution on a separate sheet of paper.

14. $-x^2 + y = 12$
 $x^2 - 3x - y = 0$

$-3x = 12$
 $x = -4$
 $-(-4)^2 + y = 12$
 $-(16) + y = 12$
 $y = 28$
 $(-4, 28)$

15. $x - y = -2 \rightarrow x = y - 2$
 $x^2 - y^2 - 4y = 20$

$(y-2)^2 - y^2 - 4y = 20$
 $y^2 - 4y + 4 - y^2 - 4y = 20$
 $-8y + 4 = 20$
 $-8y = 16$
 $y = -2$
 $x = (-2) - 2$
 $x = -4$
 $(-4, -2)$

16. $y = x^2 + 2x$
 $y = 6 + x^2$

$6 + x^2 = x^2 + 2x$
 $-x^2 - x^2$
 $6 = 2x$
 $3 = x$
 $y = 6 + (3)^2$
 $y = 6 + 9$

- Can you graph a Linear Equation or Inequality by hand? Explain the steps.

Equation Graph y int b
move slope $\frac{rise}{run}$ from ynt to
connect points make
next points

Inequality Graph line points
Make dashed if $>$ or $<$
solid if \leq or \geq
shade side $y >$ above
 $y <$ below

- Do you understand how to perform a TEST point check of a system of inequality? Explain

plug point ~~into~~ for x and y into the inequalities
if you get a true statement for both
then the point is a solution

- Do you understand a solution can be a point on a solid line or in a shaded region only? Why is this?

if it is on a dashed line it ~~is~~ doesn't work
for that inequality. Solid lines mean the
 y can equal the expressions value so the points
on the line give a \geq or \leq it is true because it is =.

- Why is a solution to a system different that a solution to an equation?

↓
solution must work
for all equations/inequalities
in the system

↓
only works for
that equation

- How do you describe any of the regions created by one or more inequality functions/relations?

The overlapping shaded regions are the solution points
for the system, they are points that satisfy the
inequalities.