

Division Algorithm

Factored Form 1:

$$f(x) = d(x) \cdot q(x) + r(x)$$

Divisor
 Divided by • Quotient ending factor + Remainder left over polynomial
 what you get after division

Divided Form 2:

$$\frac{f(x)}{d(x)} = \frac{d(x) q(x) + r(x)}{d(x)}$$

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

Synthetic Division Uses

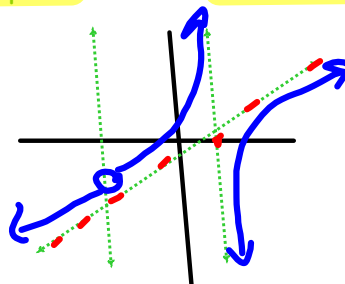
Can only be used when dividing by a linear binomial like:

$$x^1 - a \text{ or } x^1 + a$$

*(coefficient for x must be 1)

Uses:

1. To factor a polynomial into a linear binomial factor and a polynomial factor
2. To check if a number is a root/factor/solution for x, if the remainder is 0, then it is
3. To evaluate the polynomial function for a given x value, the remainder value is the y-value paired with the x factor you checked
4. To divide two functions and get the slant asymptote for the new rational function



What method would you use for each of the following?

- A. $(2x^3 - 4x + 5) \div (x^2 - 4)$ long
- B. $(3x^3 - 4x^2 + 5x) \div (x - 4)$ synthetic
- C. $(40x^{21} - 4x^{20} + 5) \div (3x + 7)$ long division
- D. $(40x^{21} - 4x^{20} + 5) \div (x^3 + 7x + 10)$ long

How to Synthetically Divide

1. Set it up:
 - A. Only write the coefficients of the top polynomial, making sure to put 0 as a place holder for missing terms.
 - B. You will be dividing by the zero/solution of the factor: if divisor is $x-3$ then $x = 3$, you will write 3 on the left
2. Synthetic Division to find the left over factor after "factoring out the linear factor":
 - A. Bring down the first constant to the third row
 - B. Multiply the divisor by the number in the third row
 - C. Write product in the second row of the next column
 - D. Add the first and second row, put sum in same column third row
 - E. Repeat steps #3,4,5 until finished
3. The bottom row:
 - A. The first numbers are the coefficients for the left over factor starting with one degree less than you started with
 - B. The last number if 0, means that the original linear binomial is a factor and a zero of the polynomial, if not, it is the remainder and is the fraction over the linear binomial that is added to the quotient

ex. $2x^3 - 7x^2 + 5 \div x-3$

$2x^3 - 7x^2 + 0x + 5$

$x-3=0$
 $+3 \quad +3$
 $x=3$

3	2	-7	0	5
		6	-3	-9
	2	-1	-3	-4

box off last number *this is the remainder

$2x^2 - 1x - 3 + \frac{-4}{x-3}$

$f(x) = 2x^2 - x - 3 = \frac{4}{x-3}$

$f(x) = (x-3)(2x^2 - x - 3) - 4$

Synthetic Division

$$(x^3 + 2x^2 - 3x + 1) \div (x + 2)$$

1. Set it up:
 - A. Only write the coefficients of the top polynomial, making sure to put 0 as a place holder for missing terms.
 - B. You will be dividing by the zero/solution of the factor: if divisor is $x-3$, then $x = 3$, you will write 3 on the left
2. Synthetic Division:
 - A. Bring down the first constant to the third row
 - B. Multiply the divisor by the number in the third row
 - C. Write product in the second row of the next column
 - D. Add the first and second row, put sum in same column third row
 - E. Repeat steps #3,4,5 until finished

$$\begin{array}{r|rrrr}
 -2 & 1 & 2 & -3 & 1 \\
 & & -2 & 0 & 6 \\
 \hline
 & 1 & 0 & -3 & 7
 \end{array}$$

$$\frac{f(x)}{d(x)} = 1x^2 + 0x - 3 + \frac{7}{x+2}$$

$$f(x) = (x+2)(x^2 - 3) + 7$$

Assignment

Textbook: pg. 177 #13-18

write $f(x)$ and $f(x)/d(x)$ form

13. $2x - 11 + \frac{62}{x + 5}$

14. $x^2 - 3x + 5 - \frac{9}{x + 3}$

15. $x + 3 + \frac{18}{x - 3}$

16. $3x^2 - 2x - 2 - \frac{4}{x - 1}$

17. $x^3 + x^2 - 2x + 1 - \frac{6}{x - 6}$

18. $x^3 - x^2 + 5x - 9 + \frac{10}{x + 5}$