Your Name
Mrs. T
1/25/2021
Notes

Lesson 4.5

Fundamental Theorem of Algebra and the Rational Root Theorem

and Synthetic Division

Day 6

Checking for Factors

Using Synthetic Division

$x^2 - 49 = 0$

$(x+7)(x-7) = 0$

$x+7=0 \quad x-7=0$

$x=-7 \quad x=7$

If a factor is a zero, then it is a solution to the function, it is an x intercept, so it has as a coordinate $\boxed{(x,0)}$

Is x - 4 a factor of $\rightarrow x = 4$

$f(x) = 9x^4 - 27x^3 - 20x^2 - 48x - 64$?

If so, write the coordinate

$$\begin{array}{r|rrrrr} 4 & 9x^4 & -27x^3 & -20x^2 & -48x & -64 \\ & \downarrow & 36 & 36 & 64 & 64 \\ \hline & 9x^3 & 9x^2 & 16x & 16 & \boxed{0} \end{array}$$

if remainder is 0, you found a factor/solution

$f(x) = (x-4)(9x^3 + 9x^2 + 16x + 16)$

(4, 0) is a root

Integer Root Theorem

Helps Factor

Using Synthetic Division

± counting numbers

The Integer Root Theorem is used to determine possible interger factors/roots of a polynomial

Divide the constant term by the leading coefficient of the polynomial. The positive and negative versions of every factor of that quotient are the possible solutions you should start checking for.

Leading Coefficient — # in front of highest degree term — term w/o a variable Constant

Factor: $2x^4 + 7x^3 - 39x^2 + 62x - 56$

$\frac{-56}{2} = -28 : \pm1, \pm2, \pm4, \pm7, \pm14, \pm28$

$+2 \,|\; 2x^4 \;\; 7x^3 \;\; -39x^2 \;\; 62x \;\; -56$

$\downarrow \;\; 4 \;\; 22 \;\; -34 \;\; 56$

x-2 is factor

$\boxed{x=2 \text{ is a solution}}$

$-7 \,|\; 2x^3 \;\; 11x^2 \;\; -17x \;\; 28 \;|\; 0$

$\downarrow \;\; -14 \;\; 21 \;\; -28$

x+7 is a factor

$\boxed{x=-7 \text{ is a solution}}$

$2 \;\; -3 \;\; 4 \;|\; 0$

$(x-2)(x+7)(2x^2-3x+4)$

Once you get down to a Quadratic: Factor or use Quadratic Formula

$x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$

$\boxed{x = \frac{3 \pm i\sqrt{23}}{4},\; x=-7,\; x=2}$

25. $1x^3 + 1x^2 - 17x + 15 = 0$

$\frac{15}{1} = 15 : \pm1, \pm3 \quad \pm5, \pm15$

$x = 1 \,|\; 1x^3 + 1x^2 - 17x + 15$

$\downarrow \;\; 1 \;\; 2 \;\; -15$

$(x-1)(1x^2 + 2x - 15) \;|\; 0$

$(x-1)(x+5)(x-3) = 0$

$x-1=0 \quad x+5=0 \quad x-3=0$

$\boxed{x=1 \quad x=-5 \quad x=3}$

27. $1x^3 - 10x^2 + 19x + 30 = 0$

$\frac{30}{1} = 30 \quad \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$$\boxed{x=-1} \;\; \begin{array}{|cccc} 1 & -10 & 19 & 30 \\ \downarrow & -1 & 11 & -30 \\ \hline 1 & -11 & 30 & \boxed{0} \end{array}$$

$(x+1)(1x^2 - 11x + 30) = 0$

$(x+1)(x-6)(x-5) = 0$

$$\boxed{x=-1,\ x=6,\ x=5}$$

30. $1x^3 - 16x^2 + 55x + 72 = 0$

$\frac{72}{1} = 72 \quad \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18, \pm 24, \pm 36, \pm 72$

$x = -1$

$+1 \quad +1$

$x + 1 = 0$

$$\begin{array}{|cccc} 1 & -16 & 55 & 72 \\ \downarrow & -1 & 17 & -72 \\ \hline 1 & -17 & 72 & \boxed{0} \end{array}$$

$(x+1)(1x^2 - 17x + 72)$

$(x+1)(x-9)(x-8) = 0$

$$\boxed{x=-1,\ x=9,\ x=8}$$

32. $3x^3 + x^2 - 38x + 24 = 0$

$\frac{24}{3} = 8 : \pm 1, \pm 2 \quad \pm 4, \pm 8$

$$\begin{array}{r|rrrr} -4 & 3 & 1 & -38 & 24 \\ & \downarrow & -12 & 44 & -24 \\ \hline & 3 & -11 & 6 & \boxed{0} \end{array}$$

$(x+4)(3x^2 - 11x + 6) = 0$

ac 18

$-9 \quad -2$

$+ \quad -11$

6

$(x+4)(3x^2 - 9x - 2x + 6)$

$(x+4)[3x(x-3) - 2(x-3)]$

$(x+4)(x-3)(3x-2) = 0$

$$\boxed{x = -4 \quad x = 3 \quad x = \frac{2}{3}}$$

Rational Root Theorem

Helps Factor

Using Synthetic Division

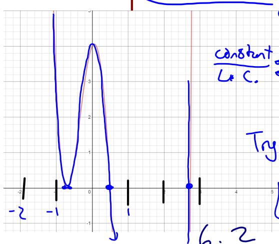
The Rational Root Theorem is used to determine possible rational factors/roots of a polynomial

fractions

Divide the constant term's factors by the leading coefficient of the polynomial's factors. The positive and negative versions of every factor and fraction of that quotient are the possible solutions you should start checking for.

Desmos.com

Use a graphing calculator to help pick out the rational factors



Leading coefficient

constant

e: $6x^4 - 11x^3 - 16x^2 + 2x + 4 = 0$

$\frac{\text{constant}}{\text{L.C.}} : \frac{4}{6} : \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$

$\pm 2, \pm\frac{2}{3}, \pm 4, \pm\frac{4}{3}$

Try: $-\frac{2}{3}$ and $\frac{1}{2}$

$6 \cdot \frac{2}{3}$

$$\begin{array}{r|rrrrr} \boxed{x = \frac{1}{2}} & 6 & -11 & -16 & 2 & 4 \\ & \downarrow & 3 & -4 & -10 & -4 \\ \hline \boxed{x = -\frac{2}{3}} & 6 & -8 & -20 & -8 & \boxed{0} \\ & \downarrow & -4 & 8 & 8 & \\ \hline & 6 & -12 & -12 & \boxed{0} & \end{array}$$

$3(x) = \left(-\frac{2}{3}\right)3$

$3x = -2$

$+2 \quad +2$

$3x + 2 = 0$

$x = \frac{1}{2}$

$2x = 1$

$2x - 1 = 0$

$6x^2 - 12x - 12 = 0$

$$\boxed{(3x+2)(2x-1)(6x^2 - 12x - 12) = 0}$$

a) b) c)

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(6)(-12)}}{2(6)}$

$$\boxed{x = \frac{12 \pm \sqrt{432}}{12}, \quad x = -\frac{2}{3}, \quad x = \frac{1}{2}}$$

Factor and Solve: $2x^3 + x^2 + 8x + 4=0$

$\frac{4}{2} = \pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{4}{1}$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 1 & 8 & 4 \\ & \downarrow & -1 & 0 & -4 \\ \hline & 2 & 0 & 8 & \boxed{0} \end{array}$$

$(2x+1)(2x^2+8)=0$

$\boxed{x = -\frac{1}{2}}$

$2x^2+8=0$

$\frac{2x^2}{2} = -\frac{8}{2}$

$x^2=-4$

$\boxed{x=\pm 2i}$

Lesson 4.5

Fundamental Theorem of Algebra and the Rational Root Theorem

and Synthetic Division

Day 6 pg. 194 # 25-32,

25. $x = -5, x = 1$, and $x = 3$

26. $x = -2, x = 1$, and $x = 3$

27. $x = -1, x = 5$, and $x = 6$

28. $x = -5, x = -2$, and $x = 3$

29. $x = -3, x = 4$, and $x = 5$

30. $x = -1, x = 8$, and $x = 9$

31. $x = -4, x = -0.5$, and $x = 6$

32. $x = -4, x = \frac{2}{3}$, and $x = 3$

Day 7

pg. 194 #33-40,odd only and 50,51

33. -5, 3, and 4

34. -4, -2, and 6

35. -5, -3, and -2

36. 1, 6, and 7

37. -4, 1.5, and 3

38. $\frac{4}{3}$, 2, and 5

39. 1, $\frac{-1+\sqrt{17}}{2} \approx 1.56$, and $\frac{-1-\sqrt{17}}{2} \approx -2.56$

40. 4, $-\frac{3}{2}$, and $-\frac{5}{2}$

41. $f(x) = x^3 - 7x^2 + 36$

42. $f(x) = x^3 + x^2 - 22x - 40$

43. $f(x) = x^3 - 10x - 12$

44. $f(x) = x^3 - 16x^2 + 77x - 116$

45. $f(x) = x^4 - 32x^2 + 24x$

46. $f(x) = x^4 + 5x^3 - 33x^2 - 85x$

50. a.
$$
\begin{aligned}
(x-3)(x-3)(x-3) &= 8 \\
(x-3)^3 &= 8 \\
x^3 - 9x^2 + 27x - 27 &= 8 \\
x^3 - 9x^2 + 27x - 35 &= 0
\end{aligned}
$$

b. The possible rational solutions are $\pm 1, \pm 5, \pm 7, \pm 35$.

c. $x = \frac{4 \pm 2\sqrt{3}i}{2} = 2 \pm i\sqrt{3}$

So, no other real solutions exist other than $x = 5$.

d. The dimensions of the cube are 2 centimeters × 2 centimeters × 2 centimeters.

51. The block is 3 meters high, 21 meters long, and 15 meters wide.

50.

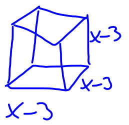
$V = l \cdot w \cdot h$

$V(x) = (x-3)(x-3)(x-3)$

$(x^2 - 3x - 3x + 9)(x-3)$

$(x^2 - 6x + 9)(x-3)$

$x^3 - 3x^2 - 6x^2 + 18x + 9x - 27$



$V = 8\ cm^3$

$V(x) = x^3 - 9x^2 + 27x - 27$

$8 = x^3 - 9x^2 + 27x - 27$

$-8 \qquad -8$

$\frac{35}{1}: \frac{\pm 1}{1}, \frac{\pm 5}{1}, \frac{\pm 7}{1}, \frac{\pm 35}{1}$ $\quad 0 = 1x^3 - 9x^2 + 27x - 35$

$(5, 0)$

$f(5) = 0$

$$
\begin{array}{c|cccc}
5 & 1 & -9 & 27 & -35 \\
 & \downarrow & 5 & -20 & 35 \\
\hline
 & 1 & -4 & 7 & \boxed{0}
\end{array}
$$

$(x-5)(1x^2 - 4x + 7)$

a b c

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$

$x = 5$, $x = \frac{4 \pm \sqrt{-12}}{2}$ Imaginary

real solution

$l = w = h = x - 3 = (5) - 3 = 2$

$2 \times 2 \times 2\ cm$

39. $4x^3 - 20x + 16 = 0$

$4x^3 + 0x^2 - 20x + 16$

$\frac{16}{4} : \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2, \pm 4, \pm\frac{8}{1}, \pm\frac{16}{1}, \pm$

$\boxed{x = 1}$

$x = 1$
$-1 \quad -1$
$x - 1 = 0$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -20 & 16 \\ & \downarrow & 4 & 4 & -16 \\ \hline & 4 & 4 & -16 & 0 \end{array}$$

$(x-1)(4x^2 + 4x - 16) = \boxed{0}$

a b c

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-16)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{272}}{8}$$

$$\boxed{x = 1, \quad x = -\frac{4}{8} + \frac{\sqrt{272}}{8}, \quad x = -\frac{4}{8} - \frac{\sqrt{272}}{8}}$$

51.

$V = l \cdot w \cdot h$

$V(x) = x \cdot (12x - 15)(12x - 21)$

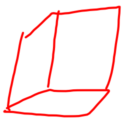
$V(x) = x(144x^2 - 252x - 180x + 315)$

$v(x) = 144x^3 - 432x^2 + 315x$

$945 = 144x^3 - 432x^2 + 315x$

$0 = 144x^3 - 432x^2 + 315x - 945$

$\frac{945}{144} : \pm 3$



$l = x$
$w = 12x - 15$
$h = 12x - 21$
$V = 945 \text{ cm}^3$

$$\begin{array}{r|rrrr} 3 & 144 & -432 & 315 & -945 \\ & \downarrow & 432 & 0 & 945 \\ \hline \end{array}$$

$(x-3)(144x^2 + 0x + 315) = \boxed{0}$

$\boxed{x = 3}$

$l = 3$
$w = 12(3) - 15$
$w = 21$
$h = 12(3) - 21$
$h = 15$

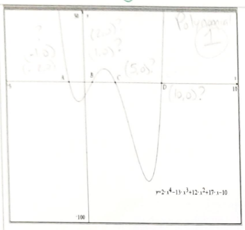
$144x^2 + 315 = 0$

$144x^2 = -315$

$\sqrt{x^2} = \sqrt{-\frac{315}{144}}$

$x = \pm i \frac{\sqrt{315}}{12}$

Name Key Rational Root Theorem Practice



$f(x) = 2x^4 - 13x^3 + 12x^2 + 17x - 10$

List the POSSIBLE INTEGER ROOTS
$\frac{10}{2}: \pm 1 \pm \frac{1}{2} \pm 2 \pm 5 \pm \frac{5}{2} \pm 10$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

5 | 2 −13 12 17 −10
 10 −15 −15 10
2 | 2 −3 −3 2 0
 4 2 −2
 2 1 −1 0

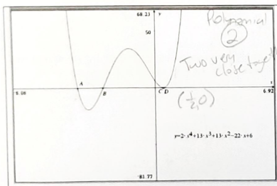
Are all the roots present? Yes

Are any of the roots irrational? No

Are any of the roots complex No

List A, B, C, and D as coordinates

A (-1,0) B (1/2,0)
C (2,0) D (5,0)



$f(x) = 2x^4 + 13x^3 + 13x^2 - 22x + 6$

List the POSSIBLE INTEGER ROOTS
$\frac{6}{2}: \pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

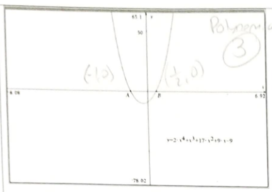
$\frac{1}{2}$ | 2 13 13 −22 6
 1 7 10 −6
−3 | 2 14 20 −12 0
 −6 −24 12
 2 8 −4 0

Are all the roots present? Yes

Are any of the roots irrational? Yes
$2x^2 + 8x - 4$
$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(-4)}}{2(2)}$
$x = \frac{-8 \pm \sqrt{96}}{4}$
$x = 0.449$
$x = -4.449$

Are any of the roots complex No

List A, B, C, and D as coordinates
$(\frac{1}{2}, 0)$
$(-3, 0)$



$f(x) = 2x^4 + x^3 + 17x^2 + 9x - 9$

List the POSSIBLE INTEGER ROOTS
$\frac{9}{2}: \pm 1 \pm \frac{1}{2} \pm 3 \pm \frac{3}{2} \pm 9 \pm \frac{9}{2}$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

−1 | 2 1 17 9 −9
 −2 1 −18 9
$\frac{1}{2}$ | 2 −1 18 −9 0
 1 0 9
 2 0 18 0

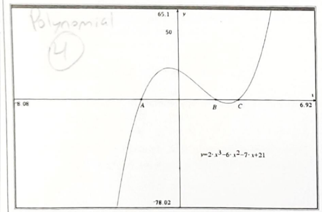
Are all the roots present? No

Are any of the roots irrational? No
$2x^2 + 18 = 0$
$2x^2 = -18$
$\sqrt{x^2} = \sqrt{-9}$
$x = \pm 3i$

Are any of the roots complex Yes $x = 3i$, $x = -3i$

List A and B as coordinates

A (-1,0) B (1/2,0)



$f(x) = 2x^3 - 6x^2 - 7x + 21$

List the POSSIBLE INTEGER ROOTS
$\frac{21}{2}: \pm 1 \pm \frac{1}{2} \pm 3 \pm \frac{3}{2} \pm 7 \pm \frac{7}{2} \pm 21 \pm \frac{21}{2}$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

3 | 2 −6 −7 21
 6 0 −21
 2 0 −7 0

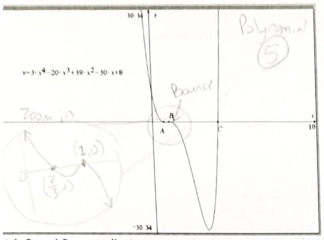
Are all the roots present? Yes
$2x^2 - 7 = 0$
$2x^2 = 7$
$\sqrt{x^2} = \sqrt{\frac{7}{2}}$
$x = \pm\sqrt{\frac{7}{2}}$

Are any of the roots irrational? Yes $x = 1.871$, $x = -1.871$

Are any of the roots complex No

List A, B, and C as coordinates

A (-1.871,0) B (1.871,0)
C (3,0)
$(1.871, 0)$ or $(-1.871, 0)$



$f(x) = 3x^4 - 20x^3 + 39x^2 - 30x + 8$

List the POSSIBLE INTEGER ROOTS
$\frac{8}{3}: \pm 1 \pm \frac{1}{3} \pm 2 \pm \frac{2}{3} \pm 4 \pm \frac{4}{3} \pm 8 \pm \frac{8}{3}$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

1 | 3 −20 39 −30 8
 3 −17 22 −8
4 | 3 −17 22 −8 0
 12 −20 8
 3 −5 2 0

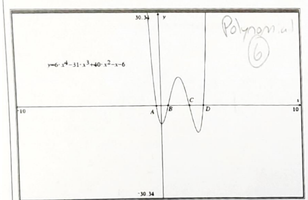
Are all the roots present? Yes

Are any of the roots irrational? No
$3x^2 - 5x + 2 = 0$
$(3x - 2)(x - 1) = 0$
$3x - 2 = 0$ $x - 1 = 0$
$x = \frac{2}{3}$ $x = 1$

Are any of the roots complex No

List A, B, and C as coordinates

A (2/3,0) B (1,0) Twice
C (4,0)



$f(x) = 6x^4 - 31x^3 + 40x^2 - x - 6$

List the POSSIBLE INTEGER ROOTS
$\frac{6}{6}: \pm 1 \pm \frac{1}{2} \pm \frac{1}{3} \pm \frac{1}{6} \pm 2 \pm \frac{2}{3} \pm 3 \pm \frac{3}{2} \pm 6$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

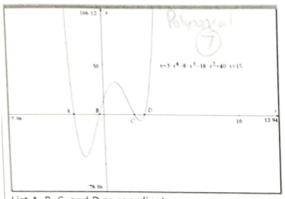
3 | 6 −31 40 −1 −6
 18 −39 3 6
2 | 6 −13 1 2 0
 12 −2 −2
 6 −1 −1 0

Are all the roots present? Yes

Are any of the roots irrational? No
$6x^2 - x - 1 = 0$
$(3x + 1)(2x - 1) = 0$

Are any of the roots complex No
$3x + 1 = 0$ $2x - 1 = 0$
$x = -\frac{1}{3}$ $x = \frac{1}{2}$

List A, B, C, and D as coordinates
D (3,0)
C (2,0)
A $(-\frac{1}{3}, 0)$
B $(\frac{1}{2}, 0)$



$f(x) = 3x^4 - 8x^3 - 18x^2 + 40x + 15$

List the POSSIBLE INTEGER ROOTS
$\frac{15}{3}: \pm 1 \pm \frac{1}{3} \pm 3 \pm 5 \pm \frac{5}{3} \pm 15$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

3 | 3 −8 −18 40 15
 9 3 −45 −15
$-\frac{1}{3}$ | 3 1 −15 −5 0
 −1 0 5
 3 0 −15 0

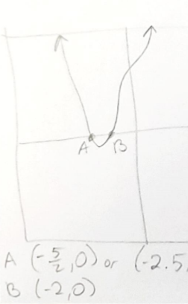
Are all the roots present?

Are any of the roots irrational? $x = \sqrt{5}$ $x = -\sqrt{5}$
$x = 2.236$ $x = -2.236$
$3x^2 - 15 = 0$
$3x^2 = 15$
$x = \pm\sqrt{5}$

Are any of the roots complex No

List A, B, C, and D as coordinates

A (-2.236,0) B (-1/3,0)
C (2.236,0) D (3,0)



List A, B, C, and D as coordinates

$f(x) = 2x^4 + 9x^3 + 12x^2 + 9x + 10$

List the POSSIBLE INTEGER ROOTS
$\frac{10}{2}: \pm 1 \pm \frac{1}{2} \pm 2 \pm 5 \pm \frac{5}{2} \pm 10$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

−2 | 2 9 12 9 10
 −4 −10 −4 −10
$-\frac{5}{2}$ | 2 5 2 5 0
 −5 0 −5
 2 0 2 0

Are all the roots present? No

Are any of the roots irrational? No
$2x^2 + 2 = 0$
$2x^2 = -2$
$\sqrt{x^2} = \sqrt{-1}$
$x = \pm i$

Are any of the roots complex Yes $x = i$, $x = -i$

A $(-\frac{5}{2}, 0)$ or $(-2.5, 0)$
B $(-2, 0)$