Your Name

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11/19/2020

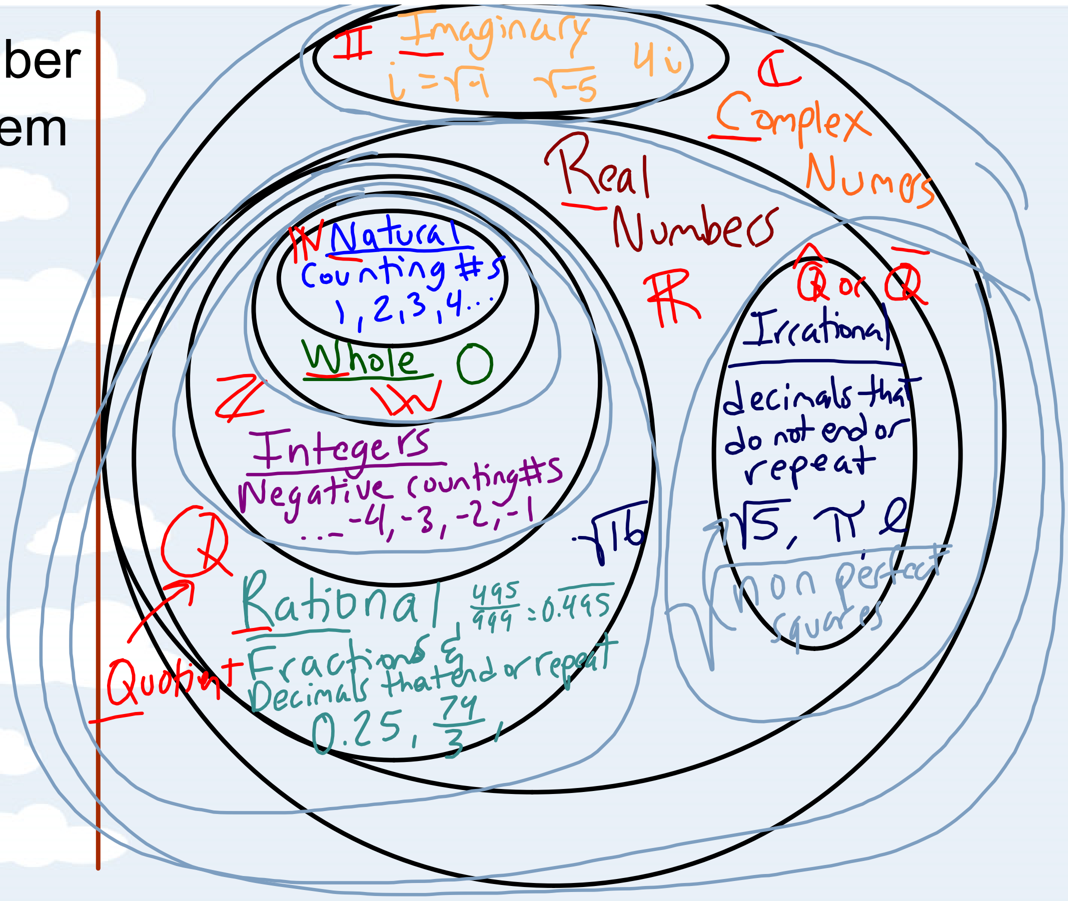
Notes

Complex Numbers

Objective: add, subtract, multiply and divide complex numbers to be able to find complex roots of polynomials

Life Lesson/Math Skill: To have total ability to find all possible zeros of any polynomial real and imaginary. Electricity and vectors uses complex numbers.

Number System



Number System

List all the number sets each of these are in:

(Hint: Simplify if you can, then find the smallest set it is included in and list all the sets that have that set inside them)

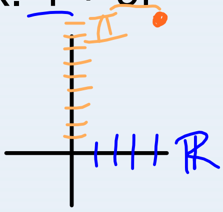
1. $\sqrt[3]{125}$ = 5 ℕ, 𝕎, ℤ, ℚ, ℝ, ℂ
2. √10 Q̂, ℝ, ℂ
3. √-16 = 4i 𝕀, ℂ
4. -3/2 ℚ, ℂ, ℝ

Complex Numbers:

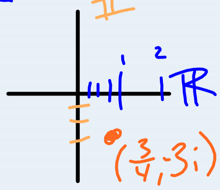
$a + bi$ where $i = \sqrt{-1}$

real part — imaginary part

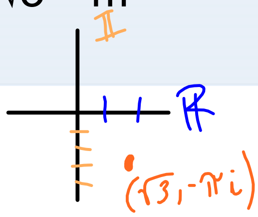
Ex. $4 + 9i$ $(4, 9i)$



$\frac{3}{4} - 3i$



$\sqrt{3} - \pi i$



$\sqrt{3}$

$\sqrt{1}$ $\sqrt{4}$

1 2

Imaginary Powers

$\sqrt{-1}^2 = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{-1 \cdot -1} = \sqrt{1} = 1$

$\boxed{i = \sqrt{-1}}$

$\boxed{i^2 = (\sqrt{-1})^2 = -1}$

$i^3 = i^2 \cdot i^1 = -1 \cdot i = \boxed{-i}$

$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = \boxed{1}$

$i^5 = i^4 \cdot i^1 = 1 \cdot i = i$

$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$

break up exponent into multiples of 4 and leftovers

$i^{18} = (i^4)^4 \cdot i^2 = (1)^4 \cdot -1$

$i^{16} \cdot i^2 = 1 \cdot -1$

$(i^6)^3 = (-1)^3$

$\boxed{= -1}$

$23/4 = 5.75$

$i^{23} = (i^4)^5 \cdot i^3 = 1 \cdot -i$

$i^{20} \cdot i^3 = \boxed{-i}$

Simplifying $\sqrt{\text{negative \#s}}$

$\sqrt{-36} = 6i$

Break it up into

$\sqrt{\text{real part}} \cdot \sqrt{-1}$

$\sqrt{36} \cdot \sqrt{-1}$

$6i$

has degree 2 needs $\pm$

$z^2 = -9$

$z = \pm\sqrt{-9}$

$z = \sqrt{-9}$ & $z = -\sqrt{-9}$

$z = \sqrt{9} \cdot \sqrt{-1}$ $z = -\sqrt{9} \cdot \sqrt{-1}$

$\boxed{z = 3i \text{ \& } z = -3i}$

$\sqrt{-32}$

$\sqrt{32 \cdot -1}$

$\sqrt{32} \cdot \sqrt{-1}$

$\sqrt{16} \cdot \sqrt{2} \cdot \sqrt{-1}$

$\boxed{4i\sqrt{2}}$

$-\sqrt{-75}$

$-\sqrt{25} \cdot \sqrt{3} \cdot \sqrt{-1}$

$\boxed{-5i\sqrt{3}}$

Imaginary Powers

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = i^2 \cdot i^1 = -1 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$

$i^5 = i^4 \cdot i^1 = 1 \cdot i = i$

$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$

break up exponent into multiples of 4 and leftovers

$i^{18} = (i^4)^4 \cdot i^2 = (1)^4 \cdot -1$

$i^{16} \cdot i^2 = 1 \cdot -1$

$(i^6)^3 = (-1)^3$

$= -1$

$i^{23} = i^{20} \cdot i^3 = -i$

$(i^4)^5 \cdot i^3$

$1 \cdot -i$

Simplifying $\sqrt{\text{negative \#s}}$

$\sqrt{-36} = 6i$

Break it up into $\sqrt{\text{real}} \cdot \sqrt{-1}$

$\sqrt{36 \cdot -1}$

$\sqrt{36} \cdot \sqrt{-1}$

$6i$

$z^2 = -9$

Two answers $\pm$

$z = \pm\sqrt{-9}$

$z = \sqrt{-9}$ & $z = -\sqrt{-9}$

$z = \sqrt{9} \cdot \sqrt{-1}$ $\quad z = -\sqrt{9} \cdot \sqrt{-1}$

$z = 3i$ $\quad z = -3i$

come in pairs

Break it up

Find biggest perfect square

$\sqrt{-32}$

$\sqrt{32} \cdot \sqrt{-1}$

real part

Imaginary part

$\sqrt{16} \cdot \sqrt{2} \cdot \sqrt{-1}$

$4i\sqrt{2}$

$\sqrt{-75}$

$\sqrt{75} \cdot \sqrt{-1}$

$\sqrt{25} \cdot \sqrt{3} \cdot \sqrt{-1}$

$5i\sqrt{3}$

Adding

Subtracting

Imaginary Numbers

Combine the Real parts together and combine the Imaginary parts together

Rules

$(a+bi) \boxed{+} (c+di)$

$=(a \boxed{+} c) + (b \boxed{+} d)i$

$(a+bi) \boxed{-} (c+di)$

$=(a \boxed{-} c) + (b \boxed{-} d)i$

distribute negative

ex. $(6-5i) + (2+3i)$

$(6+2) + (-5+3)i$

$8 - 2i$

ex. $(6-5i) - (2+3i)$

$(6-2) + (-5-3)i$

$4 - 8i$

Multiplying Imaginary Numbers

Distribute and combine like terms and remember $i^2 = -1$

Rule:

$$\boxed{(a+bi)\cdot(c+di)=(ac-bd) + (ad+bc)i}$$

$ac + adi + bci + bdi^2$

$bd(-1)$

$ac - bd + adi + bci$

ex. $(6-5i)(2+3i)$

$12 + 18i - 10i - 15i^2$

$-15(-1)$

$12 + 15 + 18i - 10i$

$\boxed{27 + 8i}$

Operating on Radicals

Express negative radicals in terms of i first!

$\sqrt{-6} - 3\sqrt{-25}$

$\sqrt{6}\cdot\sqrt{-1} - 3\cdot\sqrt{25}\cdot\sqrt{-1}$

$i\sqrt{6} - 3\cdot 5i$

$i\sqrt{6} - 15i$

$2.449i - 15i$

$\boxed{-12.551i}$

$(\sqrt{63} - \sqrt{-7})(7+\sqrt{-9})$

$\sqrt{9}\cdot\sqrt{7}$ $\quad \sqrt{7}\cdot\sqrt{-1}$ $\quad \sqrt{9}\cdot\sqrt{-1}$

$(3\sqrt{7} - i\sqrt{7})(7+3i)$

$21\sqrt{7} + 9i\sqrt{7} - 7i\sqrt{7} - 3i^2\sqrt{7}$

$-3(-1)\sqrt{7}$

$21\sqrt{7} + 3\sqrt{7} + 9i\sqrt{7} - 7i\sqrt{7}$

$\boxed{24\sqrt{7} + 2i\sqrt{7}}$

$\boxed{63.498 + 5.292i}$

$3\sqrt{-7}\,(5\sqrt{8} + i)$

$\sqrt{4}\cdot\sqrt{2}$

$3i\sqrt{7}\,(10\sqrt{2} + i)$

$30i\sqrt{14} + 3i^2\sqrt{7}$

$-3(-1)\sqrt{7}$

$\boxed{-3\sqrt{7} + 30i\sqrt{14}}$

$\boxed{-7.937 + 112.250i}$

Conjugates

Same terms but opposite operation signs. When multiplied the middle term will cancel

The conjugate would be...

$5x - 2$	$5x + 2$
$-5 + \sqrt{2}$	$-5 - \sqrt{2}$
$1 - 3\sqrt{5}$	$1 + 3\sqrt{5}$
$4i$	$-4i$
$13+11i$	$13 - 11i$
$3 - i\sqrt{2}$	$3 + i\sqrt{2}$

Dividing Imaginary Numbers

Mulitiply by the complex conjugate of the denominator

Multiply by 1

$$\frac{5+6i}{1-3i} \cdot \frac{1+3i}{1+3i}$$

$$\frac{5+15i+6i+18i^2}{1+3i-3i-9i^2} \quad 18\cdot(-1) \quad -9(-1)$$

$$\frac{5-18+15i+6i}{1+0+9}$$

$$\boxed{\frac{-13+21i}{10}}$$

$$\boxed{-1.3+2.1i}$$

$$\frac{13+11i}{4i} \cdot \frac{-4i}{-4i}$$

$$\frac{-52i-44i^2}{-16i^2} \quad -44\cdot -1 \quad -16\cdot -1$$

$$\boxed{\frac{44-52i}{16}}$$

$$\boxed{2.75-3.25i}$$

$$\frac{8}{3-i\sqrt{2}} \cdot \frac{3+i\sqrt{2}}{3+i\sqrt{2}}$$

$$\frac{24+8i\sqrt{2}}{9-i^2\sqrt{4}}$$

$$\frac{24+8i\sqrt{2}}{9+2}$$

$$\boxed{\frac{24+8i\sqrt{2}}{11}}$$

$$\boxed{2.182+1.029i}$$

Evaluation: 3-2-1

1. i^{33}

2. $(8-2i)(2+4i)$

3. $\dfrac{7}{-4-i\sqrt{3}}$

Practice:

pg. 108 # 5-12, 21-30, 37-44, 49-50