

Your Name

Mrs. Theo

12/4/20

Notes

## Complex Zeros

## Sum and Difference of Cubes

## Day 4

Sum of Cubes

Form:  $x^3 + a^3$

Binomial of degree 3, addition

Factors into this

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$



a linear binomial and a quadratic  
 with 2 imaginary solutions which  
 can be found using the quadratic  
formula

Same sign

Opposite sign Always Positive

 $\sqrt[3]{1st\ term}$   $\sqrt[3]{2nd\ term}$ 

Product of 1st and 2nd terms



a)  $x^3 + 27$   
 $\sqrt[3]{27} = 3$   
 $(x+a)(x^2-ax+a^2)$   
 $(x+3)(x^2-3x+3^2)$   
 $(x+3)(x^2-3x+9)$

b)  $x^3 + 64 = 0 \Rightarrow \sqrt[3]{64} = 4$   
 $(x+4)(x^2-4x+16) = 0$   
 $(x+4)(x^2-4x+16) = 0$   
 $x+4=0 \Rightarrow x=-4$   
 $x^2-4x+16=0$   
 $a=1, b=-4, c=16$   
 Must use quadratic formula to solve for imaginary solutions  
 $X = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$   
 $x = \frac{4 \pm \sqrt{-48}}{2}$   
 $x = \frac{4 \pm i\sqrt{48}}{2} \Rightarrow x = \frac{4 \pm i4\sqrt{3}}{2}$   
 $x = 2 \pm 2i\sqrt{3}$   
 $x = -4, x = 2 + 2i\sqrt{3}, x = 2 - 2i\sqrt{3}$

c.  $216x^3 + 1 = 0$   
 Factored  
 $(6x+1)(36x^2-6x+1) = 0$   
 $m+a, m^2-am+a^2$   
 Side Note:  $\sqrt[3]{216x^3}$  and  $\sqrt[3]{1}$   
 $m=6x, a=1$   
 Solve for x  
 Separate and set = 0  
 $6x+1=0 \Rightarrow 36x^2-6x+1=0$   
 $a=36, b=-6, c=1$   
 $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(36)(1)}}{2(36)}$   
 $x = \frac{6 \pm \sqrt{36-144}}{72}$   
 split into + and -  
 $x = \frac{6 + i\sqrt{108}}{72} \Rightarrow x = \frac{6 + i3\sqrt{3}}{72}$   
 $x = \frac{6 - i\sqrt{108}}{72} \Rightarrow x = \frac{6 - i3\sqrt{3}}{72}$   
 $x = -\frac{1}{6}, x = \frac{1}{24} + \frac{i\sqrt{3}}{24}, \text{ and } x = \frac{1}{24} - \frac{i\sqrt{3}}{24}$

Difference of Cubes

Form:  $x^3 - a^3$

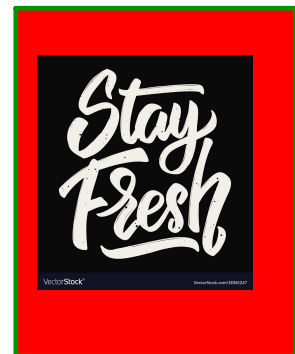
Binomial of degree 3, subtraction

Factors into this

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$   
 Same sign, opposite sign, always positive



a linear binomial and a quadratic with 2 imaginary solutions which can be found using the quadratic formula



a)  $x^3 - 8$   
 $\sqrt[3]{x^3} = x \quad \sqrt[3]{8} = 2$   
 $(x-a)(x^2 - ax + a^2)$   
 $(x-2)(x^2 - 2x + 2^2)$   
 $(x-2)(x^2 - 2x + 4)$

b)  $x^3 - 125 = 0$   
 $\sqrt[3]{x^3} = x \quad \sqrt[3]{125} = 5 = a$   
 $(x-a)(x^2 + ax + a^2)$   
 $(x-5)(x^2 + 5x + 25) = 0$   
 $x-5=0 \quad x^2 + 5x + 25 = 0$   
 $x=5 \quad x = \frac{-5 \pm \sqrt{5^2 - 4(1)(25)}}{2(1)}$   
 $x = \frac{-5 \pm \sqrt{-75}}{2}$   
 $x = \frac{-5 \pm 8.660i}{2}$   
 $x = \frac{-5}{2} \pm \frac{8.660}{2}i$   
 $x = -2.5 + 4.330i$   
 $x = -2.5 - 4.330i$

Extension

c)  $27x^3 - 8$   
 $\sqrt[3]{27x^3} = 3x \quad \sqrt[3]{8} = 2$   
 $(x-a)(x^2 + ax + a^2)$   
 $(3x-2)((3x)^2 + (2)(3x) + 2^2)$   
 $(3x-2)(9x^2 + 6x + 4)$

$64x^3 = 125$

$64x^3 - 125 = 0$   
 Difference of cubes Factored

$(4x - 5)(16x^2 + 20x + 25) = 0$   
 $\begin{matrix} m & - & a \\ 4 & - & 5 \\ \hline & & m^2 + ma + a^2 \\ & & 16x^2 + 20x + 25 \end{matrix}$

Solve for x

$4x - 5 = 0 \quad 16x^2 + 20x + 25 = 0$

$4x = 5$   
 $x = \frac{5}{4}$

$x = \frac{-20 \pm \sqrt{(20)^2 - 4(16)(25)}}{2(16)}$

$x = \frac{-20 \pm \sqrt{-1200}}{32}$

$x = \frac{-20 + i\sqrt{1200}}{32}$  and  $x = \frac{-20 - i\sqrt{1200}}{32}$

$x = \frac{5}{4}, x = -0.625 + 1.083i$   
 and  $x = -0.625 - 1.083i$

$0 = 216 + 8y^6$

$8y^6 + 216 = 0$

$\sqrt[3]{8y^6} = 2y^2 \quad \sqrt[3]{216} = 6$

$(2y^2 + 6)(4y^4 - 12y^2 + 36) = 0$

$(\sqrt{2}y + \sqrt{6}i)(\sqrt{2}y - \sqrt{6}i)$   
 $4y^4 - 12y^2 + 36 = 0$

$2y^2 + 6 = 0$

$2y^2 = -6$

$y^2 = -3$   
 $y = \pm i\sqrt{3}$

$y^2 = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(36)}}{2(4)}$

$y^2 = \frac{12 + i\sqrt{432}}{8} \quad y^2 = \frac{12 - i\sqrt{432}}{8}$

2 imag 2 imag

4 imaginary solutions

Factor And Solve

$$27x^3 - 8$$

$$x = -1/3 + 0.577i \quad x = -1/3 - 0.577i \quad x = 2/3$$

Factor and Solve :

$$64a^3 + 1$$

$$a = -1/4 \quad a = 0.125 + 0.217i \quad a = 0.125 - 0.217i$$

Factor and solve:

$$8y^3 - 27x^0$$

$$(2y - 3)(4y^2 + 6y + 9)$$

*1st term* *2nd Term squared* *1st + 2nd square* *AP*

$$2y - 3 = 0 \quad 4y^2 + 6y + 9 = 0$$

$$y = \frac{3}{2} \quad y = \frac{-6 \pm \sqrt{6^2 - 4(4)(9)}}{2(4)}$$

$$y = 1.5$$

$$y = \frac{-6 \pm \sqrt{108}}{8}$$

$$y = \frac{-6}{8} \pm \frac{\sqrt{108}}{8} i$$

$$y = -0.75 \pm \frac{10.39}{8} i$$

$$y = -0.75 + 1.299i$$

$$y = -0.75 - 1.299i$$

Factor And Solve

$$27x^3 - 8$$

Factor and Solve :
















$$64a^3 + 1$$

Factor and solve:

$$8y^3 - 27$$

### Polynomials - Factoring Sum and Differences of Two Cubes 2

Draw a line from the cube to its factors. The lines will go through a letter then a number. Write them at the bottom to find what happened when the students went to the Coca-Cola factory.

$8x^3 - 125y^3$		$(3x-4y)$		$(4x^2 + 6xy + 9y^2)$
$27x^3 - 125y^3$		$(2x-5y)$		$(9x^2 + 12xy + 16y^2)$
$27x^3 - 64y^3$		$(3x-5y)$		$(9x^2 - 12xy + 16y^2)$
$27x^3 - 8y^3$		$(2x-3y)$		$(4x^2 + 10xy + 25y^2)$
$8x^3 - 27y^3$		$(3x+4y)$		$(9x^2 + 15xy + 25y^2)$
$27x^3 + 64y^3$		$(3x-2y)$		$(16x^2 - 20xy + 25y^2)$
$8x^3 + 27y^3$		$(2x+3y)$		$(9x^2 + 6xy + 4y^2)$
$64x^3 + 125y^3$		$(4x+5y)$		$(4x^2 - 10xy + 25y^2)$
$27x^3 + 125y^3$		$(2x+5y)$		$(4x^2 - 6xy + 9y^2)$
$8x^3 + 125y^3$		$(4x+3y)$		$(16x^2 - 12xy + 9y^2)$
$64x^3 + 27y^3$		$(3x+5y)$		$(9x^2 - 15xy + 25y^2)$
$64x^3 - 125y^3$		$(4x-5y)$		$(9x^2 - 6xy + 4y^2)$
$27x^3 + 8y^3$		$(3x+2y)$		$(16x^2 + 20xy + 25y^2)$

9 6 1 10 1 8 3 12 3 7 5 7 11 2 13 4

### GridWords: Factoring #6 Sum / Difference of Two Cubes

Name: \_\_\_\_\_  
Date: \_\_\_\_\_ Class: \_\_\_\_\_

Factor each expression. Write the factors on the lines, then highlight the boxes in the grid containing each factor. When you are finished, write the letters that are formed by the highlighted boxes in order to create a word.

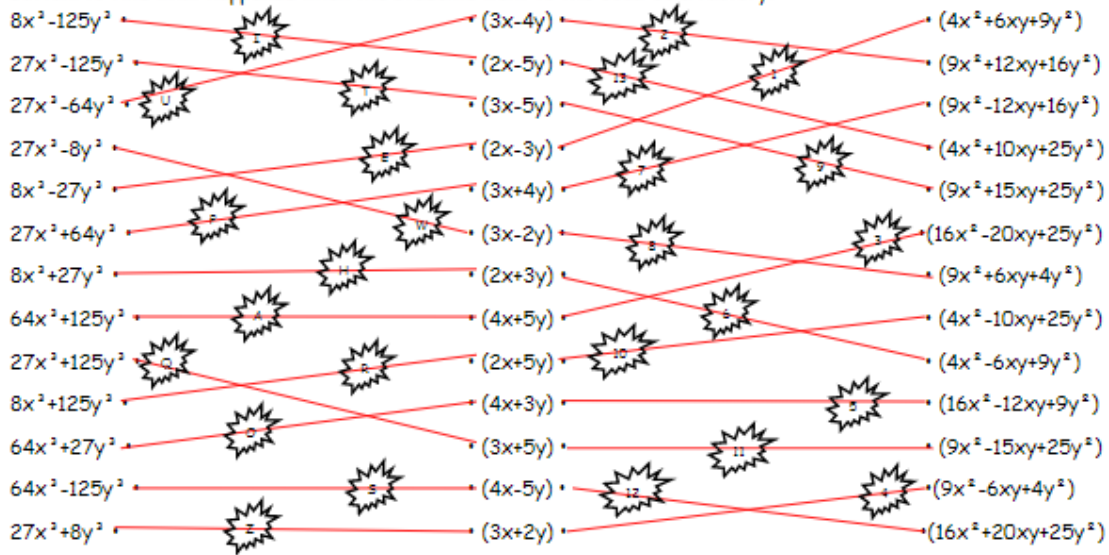
1 $8x^3 - 125$	_____	11 $x^{15} - 1$	_____
2 $a^3 b^6 + 8$	_____	12 $8b^3 + x^6$	_____
3 $27y^9 + 1$	_____	13 $y^6 b^3 + 1$	_____
4 $8a^3 - b^3$	_____	14 $64a^9 + 125$	_____
5 $x^6 - 8$	_____	15 $b^3 + x^3$	_____
6 $b^9 + x^6$	_____	16 $8y^6 - 27$	_____
7 $64a^3 - 1$	_____	17 $a^6 b^6 - 8$	_____
8 $8a^3 b^3 - x^6$	_____		
9 $x^3 + 125$	_____		
10 $27a^{12} + 1$	_____		

Write the GridWord here:

$7ab - x^2$	$b^2 - bx + x^2$	$7x - 5$	$a + 2$	$a^4 - a^2 b^2 - 2$	$b^3 - x^2$	$y^2 - 1$	$5x$	$x^{10} + x^5 + 1$	$b^3 + x^2$	$2a$	$a^4 b^2 + 4ab^2 + 8$	$x - y$	$4y^4 + 5y^2 + 8$	$ab + 4$
$3a^4 + 1$	$84a^2 + 4a - 1$	$b - x$	$5x^2$	$4x^2 - 8bx + 25$	$2a + b$	$27a^4 - 1$	$a$	$4a^2 + 2ab + b^2$	$3x^2 - 6$	$3$	$a^2 b^2 + 8$	$b^2 + 2$	$8y^6 - 3y^3 + 1$	$x + 1$
$x^2 - 2$	$x^2 - 5x + 25$	$y^2 + 1$	$bx + 1$	$8a^6 - 3a^4 + 1$	$4a + 1$	$4a^2 - a - 1$	$2$	$16a^6 - 28a^3 + 25$	$4a^3 + 5$	$b^4$	$a^2 b^4 - 4ab^2 + 8$	$4y^2 + 3$	$b^6 - b^3 x^2 + x^4$	$4a - 1$
$7b - x^2$	$b^6 - bx + 2$	$x + 5$	$yb + 1$	$a^2 b^2 - 2$	$2a - b$	$2y^2 - 3$	$bx$	$4a^2 b^2 + 2abx^2 + x^4$	$b^6 + x^4$	$8x$	$a^4 b^4 + 2a^2 b^2 + 4$	$b - 2x$	$x^2 - 25x - 25$	$x^2 - 3$
$b + x$	$x^4 + 7x^2 + 4$	$x^5 - 1$	$4x$	$8a^2 + 4a + 1$	$x^5 + 1$	$7b + x^2$	$y^4$	$y^4 b^2 - y^2 b + 1$	$3y^3 + 1$	$xy$	$4b^2 - 2bx^2 + x^4$	$ab^2 + 4$	$x^{10} - x^5 + 5$	$a - 5$

Polynomials - Factoring Sum and Differences of Two Cubes

Draw a line from the cube to its factors. The lines will go through a letter then a number. Write them at the bottom to find what happened when the students went to the Coca-Cola factory.



T H E R E W A S A P O P Q U I Z  
9 6 1 10 1 8 3 12 3 7 5 7 11 2 13 4

There was a pop quiz.

- 8.  $x^2 - 125$        $(2x - 5)(4x^2 + 10x + 25)$       11.  $x^{10} - 1$        $(x^5 - 1)(x^{10} + x^5 + 1)$
- 9.  $a^2b^4 + 64$        $(ab^2 + 4)(a^2b^4 - 4ab^2 + 16)$       12.  $8b^2 + x^6$        $(2b + x^2)(4b^2 - 2bx^2 + x^4)$
- 10.  $27y^3 - 1$        $(3y^3 + 1)(9y^6 - 3y^3 - 1)$       13.  $y^4b^2 - 1$        $(y^2b + 1)(y^2b^2 - y^2b - 1)$
- 11.  $8a^2 - b^2$        $(2a - b)(4a^2 + 2ab + b^2)$       14.  $64a^3 + 125$        $(4a^3 + 5)(16a^3 - 20a^2 + 25)$
- 12.  $x^6 - 8$        $(x^2 - 2)(x^4 + 2x^2 + 4)$       15.  $b^2 + x^3$        $(b + x)(b^2 - bx + x^2)$
- 13.  $b^3 + x^6$        $(b^3 + x^2)(b^3 - b^2x^2 + x^4)$       16.  $8y^4 - 27$        $(2y^2 - 3)(4y^4 + 6y^2 + 9)$
- 14.  $64a^2 - 1$        $(4a - 1)(16a^2 + 4a + 1)$       17.  $a^3b^4 - 8$        $(a^2b^2 - 2)(a^2b^4 + 2a^2b^2 + 4)$
- 15.  $8a^2b^2 - x^6$        $(2ab - x^2)(4a^2b^2 + 2abx^2 + x^4)$
- 16.  $x^2 - 125$        $(x - 5)(x^2 - 5x + 25)$
- 17.  $27a^{12} - 1$        $(3a^4 + 1)(9a^8 - 3a^4 + 1)$

Write the GridWord here:

SHELL

$2a - x^2$	$b^2 - bx + x^2$	$2x - 5$	$a + 2$	$a^4 + a^2b^2 - 2$	$b^2 - x^2$	$y^2 - 1$	$3x$	$x^{10} + x^5 + 1$	$b^2 + x^2$	$2a$	$a^4b^2 + 4ab^2 + 16$	$x - y$	$4y^4 + 6y^2 + 9$	$ab + 4$
$3a^4 + 1$	$64a^2 + 4a - 1$	$b - x$	$5x^2$	$4x^2 + 10x + 25$	$2a + b$	$27a^4 - 1$	$a$	$4a^2 + 2ab + b^2$	$3x^2 - 6$	$3$	$a^2b^2 + 8$	$b^2 + 2$	$8y^6 - 3y^3 + 1$	$x + 1$
$x^2 - 2$	$x^2 - 5x + 25$	$y^2b + 1$	$bx + 1$	$8a^3 - 3a^4 + 1$	$4a + 1$	$4a^2 - x - 1$	$2$	$16a^4 - 20a^2 + 25$	$4a^3 + 5$	$b^4$	$a^2b^4 + 4ab^2 + 16$	$4y^2 + 8$	$b^4 - b^2x^2 + x^4$	$4a - 1$
$2a - x^2$	$b^4 - bx + 2$	$x + 5$	$ya + 1$	$a^2b^2 - 2$	$2a - b$	$2y^2 - 3$	$bx$	$4a^2b^2 + 2abx^2 + x^4$	$b^4 - x^4$	$8x$	$a^4b^4 + 2a^2b^2 + 4$	$b - 2x$	$x^2 - 25x - 25$	$x^2 - 3$
$b + x$	$x^4 + 2x^2 + 4$	$x^2 - 1$	$4x$	$16a^2 + 4a + 1$	$x^2 + 1$	$2b - x^2$	$y^4$	$y^4b^2 - y^2b + 1$	$3y^2 + 1$	$xy$	$4b^2 - 2bx^2 - x^4$	$ab^2 + 4$	$x^{10} - x^5 + 5$	$a - 5$