

Self-Assessment:	Cannot complete	Attempted	Nearly There	Can explain to others
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Objective: Vocabulary Associated with Three Variable Systems

Resource Credit: Section 1.4
BIM Algebra 2 Text p29-36

Using Technology to Model Mathematics & Vocabulary

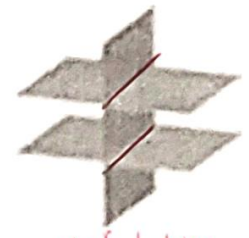
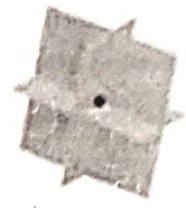
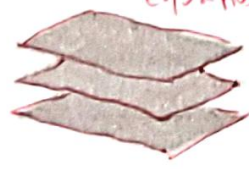
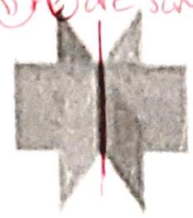
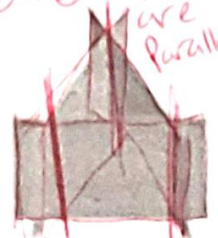
Unfortunately, sometimes the special cases of 3 x 3 systems of equations are the hardest to understand. If a system of equations does not have a single solution, then it either has NO solutions or it has infinitely many solutions. This is hard to determine which scenario is present using determinants or matrix inverse because the determinant of both types of systems is 0.

(D) + (E) Equations are Parallel

3 unique equations (D) + (E) are same

All 3 parallel equations

2 Parallel equations



No Solution System 1

Infinitely Many System 2 Solutions

No Solution System 3

1 solution System 4

No Solution System 5

Solutions to a 3 Variable System: Ordered Triples (x,y,z) points

Use the numbers under each representation of a three variable system to answer the questions below

- 4 Which of the systems has exactly one solution?
- 2 Which of the systems has infinitely many solutions?
- 1, 3, 5 Which of the systems has no solutions?

that work for all 3 equations

Directions: Match the systems A, B, C and D to the numbered system examples above and give an ALGEBRAIC reason for the selection of system

System A

System B

System C

System D

System E

$$\begin{aligned} x - 2y + 3z &= 2 \\ 2x - 4y + 6z &= 48 \\ -5x + 10y - 15z &= -50 \end{aligned}$$

$$\begin{aligned} x - 2y + 3z &= 2 \\ 2x - 4y + 6z &= 7 \\ -5x - 10y - 15z &= -50 \end{aligned}$$

$$\begin{aligned} -4x + 3y + 7z &= 25 \\ 2x - y + 6z &= 17 \\ -8x - 5y + 3z &= -5 \end{aligned}$$

$$\begin{aligned} 5x + 5y + 5z &= -20 \\ 4x + 3y + 3z &= -6 \\ -4x + 3y + 3z &= 9 \end{aligned}$$

$$\begin{aligned} x + 2y - 7z &= -4 \\ 2x + y + z &= 13 \\ 3x + 9y - 36z &= -33 \end{aligned}$$

not =
-5
Parallel

4. System A is an example like System 3
Algebraically how do you know this classification?

All Parallel equations, $A \cdot 2 = B + A \cdot -5 = C$ but constants are different

5. System B is an example like System 5
Algebraically how do you know this classification?

2 Parallel equations $A \cdot 2 = B$ but constant different
C is almost parallel to A but the z term is -15, instead of +15

6. System C is an example like System 4
Algebraically how do you know this classification?

None are parallel and (D) + (E) are not parallel or same

7. System D is an example like System 1
Algebraically how do you know this classification?

(D) + (E) are parallel

8. System E is an example like System 2
Algebraically how do you know this classification?

(D) + (E) are the same (D) \cdot -1 = (E)

$$\begin{aligned} (A) & -4x + 3y + 7z = 25 \\ (B) & +4x - 2y + 12z = 34 \\ (C) & y + 12z = 59 \\ (D) & 8x - 4y + 24z = 68 \\ (E) & -8x - 5y + 3z = -5 \end{aligned}$$

$$\begin{aligned} (A) & -2x - 4y + 14z = 8 \\ (B) & +2x + 4y + z = 13 \\ (C) & -2x + 15z = 91 \end{aligned}$$

$$\begin{aligned} (A) & -3x - 6y + 21z = 12 \\ (C) & 3x + 4y + 36z = -32 \\ (D) & 3y - 15z = -21 \end{aligned}$$

$$\begin{aligned} (A) & 80x + 20y + 20z = 80 \\ (C) & -20x + 15y + 15z = 135 \\ (D) & 4x + 3y + 3z = 9 \\ (E) & 4x + 3y + 3z = -6 \\ (F) & 35y + 35z = 55 \\ (G) & 6y + 6z = 3 \end{aligned}$$

Solve these systems of equations. Show all work or thinking. If there is no solution say so, if there are infinitely many solutions say so and write as an ordered triple in terms of y.

9. A $5x + 5y + 5z = -20$ 4

B $4x + 3y + 3z = -6$

C $-4x + 3y + 3z = 9$ 15 Elimination

~~A~~ $4x + 3y + 3z = 6$

~~E~~ $-4x + 3y + 3z = 9$

~~D~~ $6y + 6z = 15$

~~A~~ $20x + 20y + 20z = -80$

~~C~~ $-20x + 15y + 15z = +45$

~~E~~ $35y + 35z = -35$

~~D~~ $y + z = 2.5$

~~E~~ $y + z = 1$

$0 = 3.5$
False

No Solution

System 1

~~E~~ $(y + z = -1) \cdot -1$
Parallel

~~E~~ $y + 11(-3) = -25$

~~E~~ $y + 33 = -25$
 $y = -58$

~~C~~ $x = -3(-3)$

~~C~~ $x = 9$

10. A $6x + 10y + 4z = 122$

B $-5x + y - 4z = -25$

C $-3z = x$ Substitution

~~A~~ $6(-3z) + 10y + 4z = 122$

~~A~~ $-18z + 10y + 4z = 122$

~~D~~ $10y - 14z = 122$

~~B~~ $-5(-3z) + y - 4z = -25$

~~B~~ $15z + y - 4z = -25$

~~E~~ $(y + 11z = -25) \cdot -10$

~~E~~ $-10y - 110z = +250$

~~F~~ $10y - 14z = 122$

~~D~~ $-124z = 372$

~~D~~ $z = -3$

System 4

$(9, 8, -3)$

1 solution

12. $3x + 3y + 3z = -12$

$2x + 3y + 5z = 9$

$-x - y - z = 4$

A and C are the same line $A \cdot -3 = C$

~~C~~ $-3x + 3y - 3z = 12$

~~A~~ $3x + 3y + 3z = -12$

$0 = 0$
True

11. $3x + 3y + 3z = -12$

$2x + 3y + 5z = 9$

$-3 \cdot (-x - y - z = 3)$ not $\cdot -3$

A and C are Parallel
B not parallel
No Solution
System 5

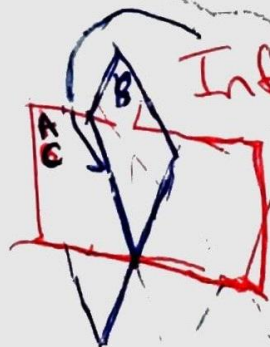
~~C~~ $-3x - 3y - 3z = 9$

~~A~~ $3x + 3y + 3z = -12$

$0 = -3$

False

No Solution



Infinite Solutions

7. Write the system of 3 variable equations for the matrix:

$$\begin{matrix} x & y & z & = \\ \text{1) } & & & \\ \text{2) } & & & \\ \text{3) } & & & \end{matrix} \left[\begin{array}{ccc|c} 2 & 5 & 0 & 13 \\ -3 & 1 & 2 & 6 \\ 4 & 0 & -3 & 5 \end{array} \right]$$

1) $2x + 5y = 13$
 2) $-3x + y + 2z = 6$
 3) $4x - 3z = 5$

8. Write the system of 3 variable equations for the matrix:

$$\left[\begin{array}{ccc|c} 6 & -3 & 6 & 5 \\ 4 & 6 & -7 & 4 \\ -2 & 6 & 6 & 7 \end{array} \right]$$

1) $6x - 3y + 6z = 5$
 2) $4x + 6y - 7z = 4$
 3) $-2x + 6y + 6z = 7$

Write the matrix for the system of equations and solve (remember $[A]^{-1}[B]$).

9. $\begin{cases} 3x + y = -4 \\ -2x + 4y = 7 \end{cases}$

$$\left[\begin{array}{cc|c} 3 & 1 & -4 \\ -2 & 4 & 7 \end{array} \right]$$

$x = -1.643$
 $y = 0.929$

10. $\begin{cases} 4x - y + 2z = 10 \\ 5x + 2y - 3z = 0 \\ x - 3y + z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 4 & -1 & 2 & 10 \\ 5 & 2 & -3 & 0 \\ 1 & -3 & 1 & 6 \end{array} \right]$$

$x = 1.407$
 $y = -0.963$
 $z = 1.704$

11. $\begin{cases} 3x - 2y + z = 6 \\ 4x - 6z = 6 \\ -3y - 4z = -10 \end{cases}$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 4 & 0 & -6 & 6 \\ 0 & -3 & -4 & -10 \end{array} \right]$$

$x = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

12. Last year, a baseball team purchased new equipment. The equipment manager paid \$20 per bat and \$12 per glove and \$15 per ball, spending a total of \$646. The manager bought 40 pieces of equipment. They bought 7 more bats than balls. Write a system of equations and solve for the amount of bats, gloves, and balls that were bought.

Determine Variables x : # of bats y : # of gloves z : # of balls

Total Valued Equation: $20x + 12y + 15z = 646$

Total Object Equation: $x + y + z = 40$

Relationship Equation: $x = z + 7$

17 bats, 13 gloves, and 10 balls were bought

$\textcircled{A} 20(z+7) + 12y + 15z = 646$
 $20z + 140 + 12y + 15z = 646$
 $35z + 12y = 506$
 $\textcircled{E} 12y - 24z = -396$
 $11z = 110$
 $z = 10$
 $x = z + 7$
 $x = 17$

$\textcircled{B} (z+7) + y + z = 40$
 $7 + y + 2z = 40$
 $\textcircled{E} (y + 2z = 33)$
 $-12 \cdot$
 $\textcircled{E} y + 2(10) = 33$
 $y + 20 = 33$
 $y = 13$

13. Andrea Liskow was the top scorer in a women's professional basketball league for the 2006 regular season, with a total of 822 points. The number of two-point baskets that Andrea made was 60 less than double the number of three-point baskets she made. The number of free throws (each worth one point) she made was 15 less than the number of two-point field goals she made. Find how many free throws, two-point baskets, and three-point baskets Andrea Liskow made during the 2006 regular season.

Determine Variables x: # of Free throws y: # of 2pt baskets z: # of 3pt baskets

Total Valued Equation: $x + 2y + 3z = 822$ (A) (A) $x + 2(y+15) + 3z = 822$ (B) $(x+15) = 2z - 60$

Total Object Equation: $y = 2z - 60$ (B) $x + 2x + 30 + 3z = 822$ (C) $(x - 2z = -45)$

Relationship Equation: $x = y - 15$ (C) (D) $3x + 3z = 792$ (E) $-3x + 6z = 13$

Andrea Liskow
161 Free throws
176 2 pt baskets
103 3 pt baskets

$y = x + 15$

$\frac{9z}{4} = \frac{927}{4}$
 $z = 103$

(E) $x - 2(103) = -45$
 $x - 206 = -45$
 $x = 161$

(C) $y = 161 + 15$
 $y = 176$

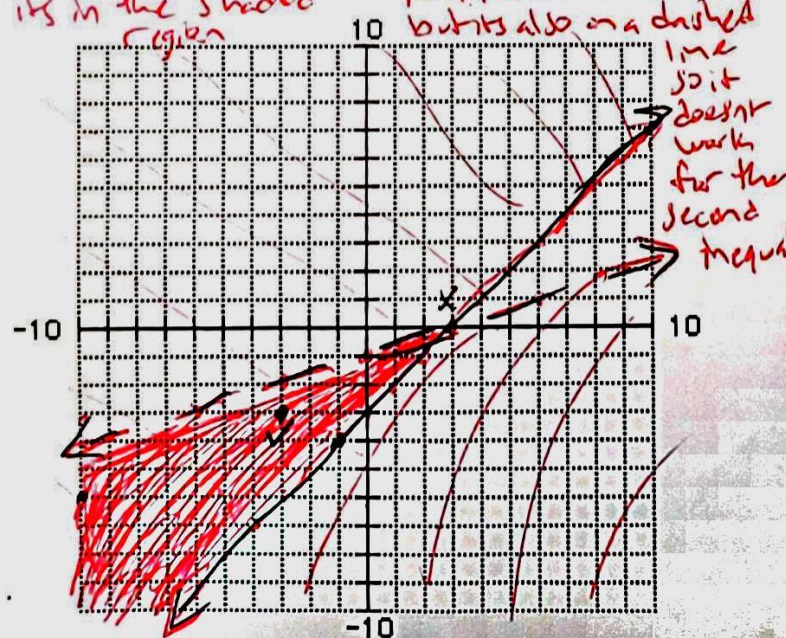
14. Graph and find the solution to the system of inequalities
 $y \geq x - 3$

slope $m=1$ $b=-3$ (0, -3)
shade above solid line

$y < \frac{1}{3}x - 1$ slope $m = \frac{1}{3}$ $b = -1$ (0, -1)
shade below dashed line

Two Solution points include: (-1, -4) (-10, -4) (1, -1)
Prove it by plugging the points into the equations to check (1, -2)
for true statements $(-2) \geq (1) - 3$ $(-2) < \frac{1}{3}(1) - 1$
 $-2 \geq -2$ $-2 < 0.33 - 1$
 $-2 < -0.66$

Is (-3, -3) a solution? Explain Yes, it's in the shaded region
Is (3, 0) a solution? Explain No, it's on one solid line but it's also on a dashed line so it doesn't work for the second inequality



15. Which region/s are solutions to the above system graphed below? Give two points in the solution set there are more than one region include one point from each region. Determine if the given point is a solution and explain your thinking

$y < -|x - 5| + 2$ Solution Region/s: 4
 $y \leq \frac{1}{2}x - 6$

Two Solution points include: (1, -6) (-5, -4)

Is (0, -6) a solution? Yes on boundary + on solid line
Is (10, -3) a solution? No, on boundary, but line is dashed
Is (-6, -9) a solution? No, intersection with dashed line
(0, 0) No, not included region 5
(5, -1) Not in overlapping shaded regions

