Name____ Solving Exponential Equations Guided Notes and Practice

1. Solve the following equation using each of the following methods $\boxed{16 = 2^{5x-6}}$ exponent, base

Method 1: Definition of logarithm

Turn into log ☆

What is the base? 2

What is the exponent? 5x-6

What is left? 16

$$\log_{\boxed{2}}(\boxed{16}) = \boxed{5x-6}$$

base, exponent

$$4 = 5x - 6$$

$$+6 \qquad +6$$

$$\frac{10}{5} = \frac{5x}{5}$$

$$x = 2$$

Method 2: Logarithm as an inverse (same base)

Undo base ☆

take specific log on both sides

$$\log_2(16) = \log_2(\cancel{2}^{5x-6})$$

$$\frac{\log(16)}{\log(2)} = 5x - 6$$

$$4 = 5x - 6$$

$$x = 2$$

Method 3: One to One Property of exponential expressions (this method only applies in certain cases)

if the same base on both sides, then the exponents are equal

$$2^{\_\_} = 2^{\_\_}$$

$$2^4 = 2^{5x-6}$$

$$4 = 5x - 6$$

$$x = 2$$

Method 4: Graph of a system of an exponential function and a horizontal line (this only yields an approximate answer in some cases)

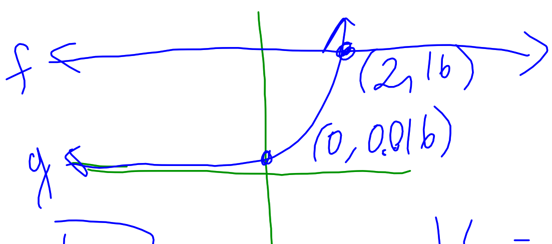
Original left side of equation

$f(x) = 16$ horizontal line

Original right side of equation

$g(x) = 2^{5x-6}$

Intersection is Solution



Method 5: Common logarithm to eliminate any base

Take log on both sides and exponent comes in front

$$\log(16) = \log(2^{5x-6})$$

☆ Power Rule

$$\frac{\log(16)}{\log(2)} = \frac{(5x-6)\cdot\log(2)}{\cancel{\log(2)}}$$

☆ divide

$$\frac{\log(16)}{\log(2)} = 5x - 6$$

Put in calculator

$$4 = 5x - 6$$

$$x = 2$$

~~Method 6: Properties of Exponents~~

Check of your exact answer____

$$5(2) - 6$$

$$16 = 2$$

$$16 = 2^{10-6}$$

$$16 = 2^4$$

$$16 = 16 \checkmark$$

2. Solve the following equation using each of the following methods $7 = 3^{2-4x}$

exponent

base

Method 1: Definition of logarithm

*Turn into Log
What is base? 3
What is exponent? 2-4x
What is left? 7

$\log_3(7) = 2-4x$ (exponent)

$\frac{\log(7)}{\log(3)} = 2-4x$

$1.771 = 2-4x$

$-2 \quad -2$

$\frac{-0.229}{-4} = \frac{-4x}{-4}$

$x = 0.057$

Method 2: Logarithm as an inverse (same base)

Undo base
Take specific log on both sides

$\cancel{\log_3}(7) = \cancel{\log_3}(\cancel{3}^{2-4x})$

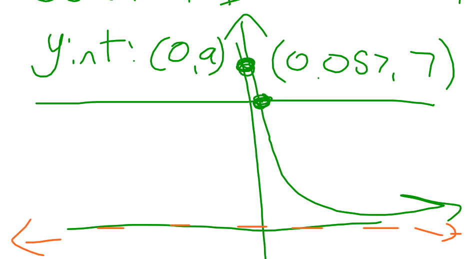
$1.771 = 2-4x$

$x = 0.057$

Method ~~3~~ 4: Graph of a system of an exponential function and a horizontal line (this only yields an approximate answer in some cases)

Original left side
$f(x) = 7$
Original right side
$g(x) = 3^{2-4x}$
Solution: Intersection

y-int: (0, 9) (0.057, 7)



horizontal Asymptote
$y = 0$
Imaginary line that the function approaches but never crosses

Method 5: Common logarithm to eliminate any base

Take log on both sides and the exponent comes infront

$\log(7) = \log(3^{2-4x})$

*Power Rule

$\frac{\log(7)}{\log(3)} = \frac{(2-4x)\cdot\cancel{\log(3)}}{\cancel{\log(3)}}$ *divide

$1.771 = 2-4x \rightarrow x = 0.057$

~~Method 6: Properties of Exponents~~

Check of your exact answer____

Why was the one to one property of exponents NOT an option in this problem?

____

Use exact decimal

$7 = 3^{2-4(0.057...)}$

$7 = 7$ ✓

Use rounded decimal

$7 = 3^{2-4(0.057)}$

$7 = 7.005$

Solve the remaining equations using graphing and at least two of the algebraic methods previously mentioned

3. $5 = \left(\frac{1}{2}\right)^{2x-6}$

$$\log_{\frac{1}{2}} 5 = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{2x-6}$$

$$\frac{\log(5)}{\log(1/2)} = 2x - 6$$

x = 1.839

4. $16 = \cancel{4} \cdot 2^{x+4}$

$$\frac{16}{4} = \frac{\cancel{4}}{4}$$

Before you can solve, get base alone.

$4 = 2^{x+4}$ ← base; exponent

Method 1

$\log_{\boxed{2}}(4) = \boxed{x+4}$

base

exponent

$$\frac{\log 4}{\log 2} = x + 4$$

$$2 = x + 4$$

$$-4 \qquad -4$$

$$\boxed{-2 = x}\ \checkmark$$

Method 2

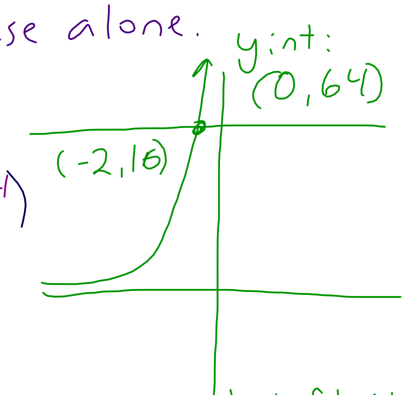
$$\log_2(4) = \cancel{\log_2}\left(\cancel{2}^{x+4}\right)$$

$$\frac{\log 4}{\log 2} = x + 4$$

$$2 = x + 4$$

$$-4 \qquad -4$$

$$\boxed{-2 = x}\ \checkmark$$



original leftside

$f(x) = 16$

original rightside

$g(x) = 4 \cdot 2^{x+4}$

5. $8 = 7 \cdot \left(\frac{2}{3}\right)^{6-2x}$

$$\frac{8}{7} = \frac{7}{7}$$

$$\frac{8}{7} = \left(\frac{2}{3}\right)^{6-2x}$$

log

$$\frac{\log(8/7)}{\log(2/3)} = 6 - 2x$$

x = -0.165

"e" is just a special mathematics number, it is approximately 2.718 , the inverse of e^x is $\boxed{\ln x}$ or $\boxed{\log_e x}$ (there are specific buttons on your calculator to deal with this special number and its inverse ln x) NOTE: ln x is called the NATURAL LOG

Solve the remaining equations using graphing and at least two of the algebraic methods previously mentioned

6. $7 = (e)^{2x+4}$

Method 1

$\log_{\boxed{e}}(7) = \boxed{2x+4}$ exponents

base

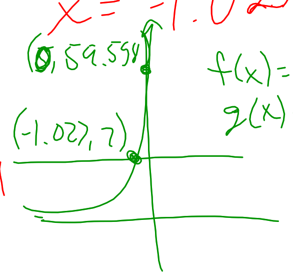
$\ln(7) = 2x+4$

$1.945 = 2x+4$

$-4 \quad -4$

$\frac{-2.054}{2} = \frac{2x}{2}$

$x = -1.027$



$f(x) = 7$

$g(x) = e^{(2x+4)}$

Method 2

$\ln(7) = \cancel{\ln}(e^{2x+4})$

$\ln(7) = 2x+4$

$1.945 = 2x+4$

$x = -1.027$

7. $(e)^{12} = (e)^{5x-7}$

one to one property

$12 = 5x-7$

x= 4.8

8. $12 = 3(e)^{\frac{1}{2}x+4}$

$\frac{12}{3} = \frac{3(e)^{\frac{1}{2}x+4}}{3}$

$4 = e^{\frac{1}{2}x+4}$ exponent, base

Method 1

$\log_{\boxed{e}}(4) = \boxed{\frac{1}{2}x+4}$ exponent

base

$\ln(4) = \frac{1}{2}x+4$

$1.386 = \frac{1}{2}x+4$

$-4 \quad -4$

$2(-2.614) = (\frac{1}{2}x)\frac{2}{1}$

$-5.227 = x$

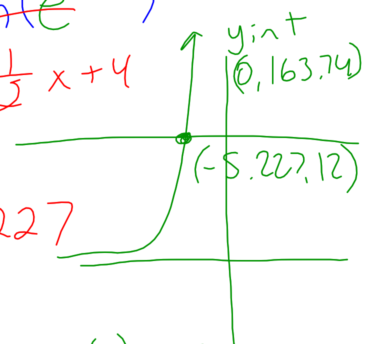
Method 2

$\ln(4) = \cancel{\ln}(e^{\frac{1}{2}x+4})$

$1.386 = \frac{1}{2}x+4$

. . .

$x = -5.227$



$f(x) = 12$

$g(x) = 3 \cdot e^{\frac{1}{2}x+4}$