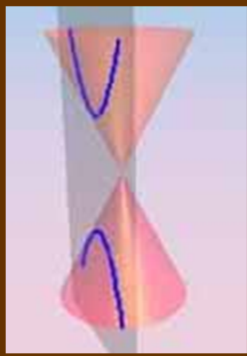
9.2 Hyperbolas

Formation: plane intersects the cone perpendicular to its base



https://www.youtube.com/watch?v=8nPMlW5NZSo

Definition: the collection of all points in a plane the difference of whose distances from two fixed points is a constant

LABEL THIS PICTURE:

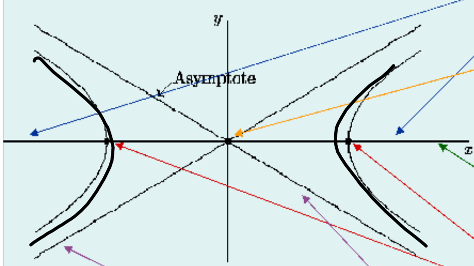
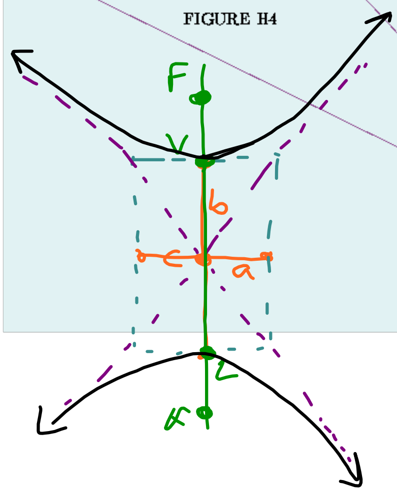


FIGURE H4



DEFINE THESE TERMS

Foci - the two fixed points

Center - the midpoint of the line segment joining the foci

Transverse axis - the line containing all important info

Vertices - two points of intersection of the hyperbola and transverse axis

Asymptotes - lines that guide the graph of the hyperbola (don't touch!)

Fundamental Rectangle - the box created by a and b whose corners determine the asymptotes

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Open left/right

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Open up/down

(h,k) is the center of the Hyperbola

a is the length from the center to the side horizontally of the fundamental rectangle

b is the length from the center to the side vertically of the fundamental rectangle

Transverse Axis: It is not about the size of a or b, but about which squared variable is positive!

Foci

To calculate the foci use: $\boxed{d^2 = a^2 + b^2}$

To Graph an Hyperbola:

1. Plot the center
2. Find the square root of what is under x – move that many spaces right and left
3. Find the square root of what is under y – move that many spaces up and down
4. Connect in a rectangle. This is called the fundamental rectangle.
5. Draw Asymptotes lines diagonally through the center and the Fun Rect corners.
6. Decide how the hyperbola opens (up/down or left/right) based on what is first and draw it, without crossing the using asymptotes. as guides
7. Finally, label the important info - vertex, center, and asymptotes.

Graphing

$$\frac{x^2}{4} - \frac{y^2}{25} = 1$$

x is positive left + right

$a^2 = 4 \quad b^2 = 25$

Fun. Rectangle

Center: $(0,0)$

a = 2 b = 5

Asymptotes: $y = \pm\frac{5}{2}x$

Vertices: $(\pm 2, 0)$

Foci: $(\pm\sqrt{29}, 0)$

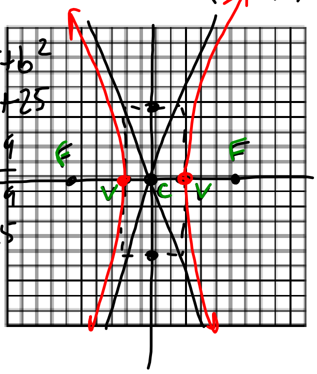
$d^2 = a^2 + b^2$

$d^2 = 4 + 25$

$d^2 = 29$

$d = \sqrt{29}$

≈ 5.5

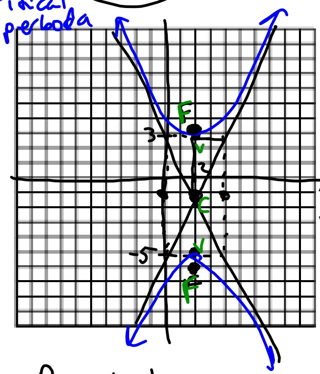


$$(y+1)^2 - 4(x-2)^2 = 16$$

$$\frac{(y+1)^2}{16} - \frac{4(x-2)^2}{16} = \frac{16}{16}$$

$$\frac{(y+1)^2}{b^2 = 16} - \frac{(x-2)^2}{a^2 = 4} = 1$$

y is positive Vertical hyperbola



Fun. Rectangle

Center: (h,k) $(2,-1)$

a = 2 b = 4

Asymptotes:

Vertices: $(2,3)$ $(2,-5)$

Foci: $(2, -1-\sqrt{20})$ $(2, -1+\sqrt{20})$

$d^2 = a^2 + b^2$

$d^2 = 4 + 16$

$d^2 = 20$

$d = \sqrt{20} \approx 4.5$

Point slope

$y - y_1 = m(x - x_1)$

Use center

$$\boxed{\begin{matrix} y+1 = 2(x-2) \\ y+1 = -2(x-2) \end{matrix}}$$

Graphing

$$\frac{x^2}{4} - \frac{(y+3)^2}{25} = 1$$

$a^2 = 4 \quad b^2 = 25$

Fun. Rectangle

Center: $(0,-3)$

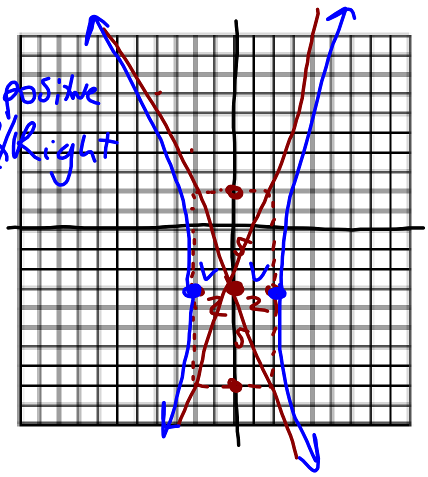
a = 2 b = 5

Asymptotes:

Vertices:

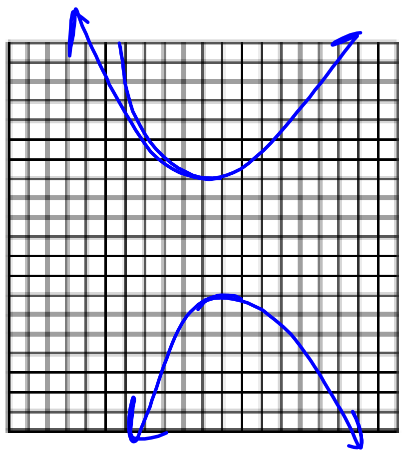
Foci:

x^2 is positive Left/Right



$$(y+1)^2 - 4(x-2)^2 = 16$$

y^2 is positive Up/down



Fun. Rectangle

Center:

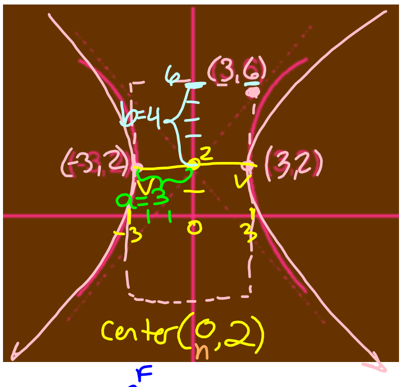
a = b =

Asymptotes:

Vertices:

Foci:

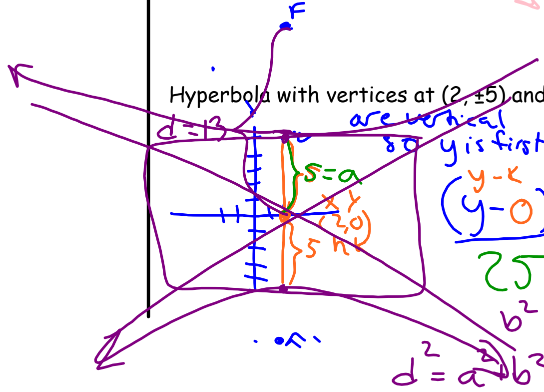
Write the Equation



is x first or y?
opens Left/Right

$$\frac{(x-0)^2}{9} - \frac{(y-2)^2}{16} = 1$$

Hyperbola with vertices at (2, ±5) and foci at (2, ±13).



are vertical so y is first

$$\frac{(y-0)^2}{25} - \frac{(x-2)^2}{144} = 1$$

$$d^2 = a^2 + b^2$$
$$13^2 = a^2 + 5^2$$
$$a^2 = 169 - 25$$
$$a^2 = 144$$

$$\frac{y^2}{25} - \frac{(x-2)^2}{144} = 1$$

https://www.youtube.com/watch?v=8nPMlW5NZSo

Name ______________________________ Hour______

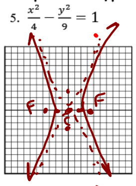
Honors Algebra II - Section 9.2 Wkst

Identify the following as ellipses or hyperbolas. Be careful!

1. $\frac{(y-3)^2}{8} - \frac{x^2}{16} = 1$ hyperbola
2. $x^2 + 12y^2 = 12$ ellipse
3. $8x^2 = 100 - 2y^2$ $+2y^2 \quad +2y^2$ $8x^2+2y^2=100$ Ellipse
4. $3x^2 = y^2 + 15$ $-y^2 \quad -y^2$ $3x^2-y^2=15$ Hyperbola

Graph each Hyperbola. Name then center, the vertices and the foci.

5. $\frac{x^2}{4} - \frac{y^2}{9} = 1$



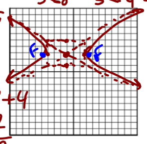
center $(0,0)$
vertices $(-2,0)\ (2,0)$
foci $(-\sqrt{13},0)\ (\sqrt{13},0)$
asymptotes $y=\pm\frac{3}{2}x$

$d^2=9+4$
$d^2=13$
$d=\sqrt{13}$

6. $4(x+1)^2 - 8(y-3)^2 = 32$

$\frac{(x+1)^2}{8} - \frac{(y-3)^2}{4} = 1$

$a=\sqrt{8}$
$b=2$
$d^2=8+4$
$d^2=12$
$d=\sqrt{12}$

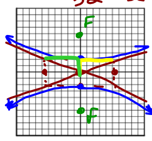


center $(-1,3)$
vertices $(-1\pm\sqrt{8},3)$
foci $(-1\pm\sqrt{12},3)$
asymptotes $y=\pm\frac{2}{3}(x+1)+3$

7. $8y^2 - x^2 = 32$

$\frac{y^2}{4} - \frac{x^2}{32} = 1$

$a=\sqrt{32}\approx 5.6$
$b=2$



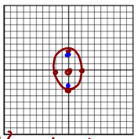
y^2 is pos. so Up/Dn

center $(0,0)$
vertices $(0,2)(0,-2)$
foci $(0,6)\ (0,-6)$
asymptotes $y=\pm\frac{2}{\sqrt{32}}x$

$d^2=a^2+b^2$
$d^2=32+4$
$d^2=36$
$d=6$

Graph each Conic Equation. Name the center, vertices, and asymptotes (EC. Foci).

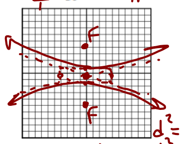
8. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ ellipse



center $(0,0)$
vertices $(0,3)\ (0,-3)$
foci $(0,\sqrt{5})\ (0,-\sqrt{5})$
~~asymptotes~~

$d^2=b^2-a^2$
$d^2=9-4$
$d^2=5$
$d=\sqrt{5}$

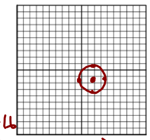
9. $y^2 - \frac{x^2}{16} = 1$ hyperbola



center $(0,0)$
vertices $(0,1)\ (0,-1)$
foci $(0,\sqrt{17})\ (0,-\sqrt{17})$
asymptotes $y=\pm\frac{1}{4}x$

$d^2=1+16$
$d^2=17$
$d=\sqrt{17}$

10. $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{4} = 1$ circle

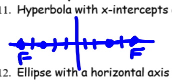


center $(2,-1)$
vertices $(0,-1)\ (2,1)\ (4,-1)\ (2,-3)$
foci the center
~~asymptotes~~

Give the equation for each of the following shapes. (hint: first draw a sketch of the given info)

11. Hyperbola with x-intercepts at ±3 and [illegible] at (-5, 0) and (5, 0).

Foci



$\frac{(x-\ )^2}{\ } - \frac{(y-\ )^2}{\ } = 1$

12. Ellipse with a horizontal axis of 16, a vertical axis of 20, centered at (-1, 5).

$\frac{(x-\ )^2}{\ } + \frac{(y-\ )^2}{\ } = 1$

13. Hyperbola with a horizontal axis of 16, a vertical axis of 20, centered at (-1, 5).

20 16

$a=8$ $b=10$

doesnt say about direction... You choose!

$\frac{(x+1)^2}{64} - \frac{(y-5)^2}{100} = 1$ or $\frac{(y-5)^2}{100} - \frac{(x+1)^2}{64} = 1$

14. Ellipse with center at origin, vertex at (0, 4) and co-vertex at (2,0)

$b=4$ $a=2$ $C(0,0)$

$\frac{x^2}{4} + \frac{y^2}{16} = 1$

15. Hyperbola with vertices at (0, ±3) and foci at (0, ±6).

$b=3$

center $(0,0)$

vertical y^2 first

$d^2=a^2+b^2$
$6^2=a^2+3^2$
$a^2=36-9=27$

$\frac{y^2}{9} - \frac{x^2}{27} = 1$

16. Hyperbola with center at (1, -2), focus at (4, -2), and vertex at (3, -2).

$b=2$
$d=3$

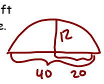
c v F

horizontal x^2 first

$d^2=a^2+b^2$
$9=a^2+4$
$a^2=5$

$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{5} = 1$

17. An arch of a bridge has the shape of the top half of an ellipse. The arch is 40 ft wide and 12 ft high at the center. (A) Find the equation of the complete ellipse. (B) Find the height of the arch 10 ft from the center of the bottom.

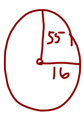


A. $\frac{x^2}{20^2} + \frac{y^2}{12^2} = 1$

B. $\frac{(10)^2}{400} + \frac{y^2}{144} = 1$ $\sqrt{y^2}=\sqrt{\frac{3}{4}\cdot 144}$

$y=10.392$ ft high

18. The **Elliptical** chamber in the U.S. Capitol Building is 110 ft. long and 32 ft. wide. (A) Write the equation that models the shape of the room. (B) President John Quincy Adams discovered that he could overhear the conversations of opposing party leaders who were near the focus at the right side of the chamber if he situated his desk at the focus at the left side of the chamber. What were the coordinates of his desk's position?



$d^2=3025-256$
$d^2=2769$
$d=52.621$

A. $\frac{x^2}{256} + \frac{y^2}{3025} = 1$

or $\frac{x^2}{3025} + \frac{y^2}{256} = 1$

B. $d=52.621$

$(0,52.621)$ or $(52.621,0)$

or 2.379 ft from the vertex wall