Your Name
Mrs. Theo

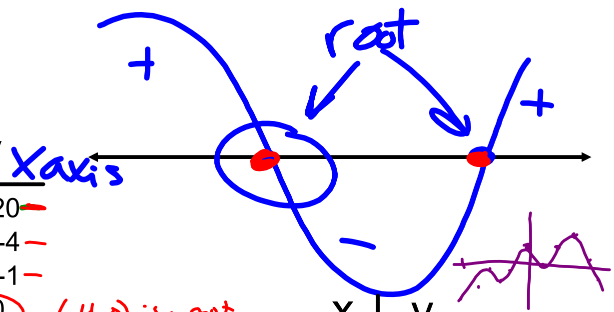
2/24/21
Notes

How to Graph Polynomials

Multiplicities

Intermediate Value Theorem
Location Principle

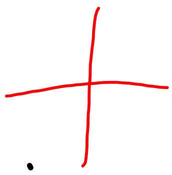
if there are positive and negative intervals on the graph, then there is a point where P(x) crosses x axis, basically there is a root where y = 0



x	y	
-10	-20	−
-8	-4	−
-6	-1	−
-4	0	(-4,0) is a root y=0
-2	1	+
0	7	+
2	8	+
4	7	+
6	2	+ A Root exists $x \in (6,8)$ $6 < x < 8$
8	-4	−
10	-13	−

[ ] =
( ) ≈

x	y	
-10	-30	Absolute min
-8	-14	local max $x \in (-8,-4)$
-6	-1	local min
-4	-6	A root exists
-2	-2	− $x \in (-2,0)$
0	5	+ y intercept $x=0$ local max
2	0	(2,0) is a root local min
4	5	
6	8	Absolute max
8	1	+ A Root exists
10	-3	− $x \in (8,10)$



Multiplicity

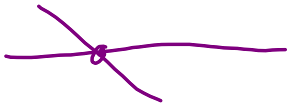
Solution
root
zero ← y = 0
x intercept

(x-c) is a factor more than once
the # of times it is a factor is its multiplicity m

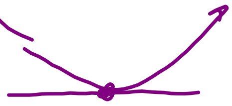
ex. $(x-4)^3$ 4 is a zero of P with multiplicity 3
$(x-4)(x-4)(x-4)$

ex. $(x+2)^4$ −2 is a zero w/ m = 4
$(x+2)(x+2)(x+2)(x+2) = 0$
$x + 2 = 0$
$-2 \quad -2 \qquad x = -2$

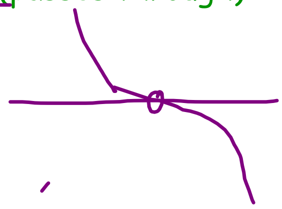
- If m is 1 it goes right through



- If m is even it changes direction (bounces)



- If m is odd it will keep its directions (passes through) but it will flatten at the zero



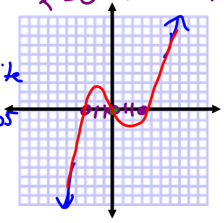
Sketching Polynomial Graphs

1) Plot x intercepts
2) Plot end behavior
3) Plot y intercept
4) Sketch passes and bounces

$f(x) = x^1(x-3)^1(x+3)^1$

Pass x = 0 Pass x = 3 Pass x = −3

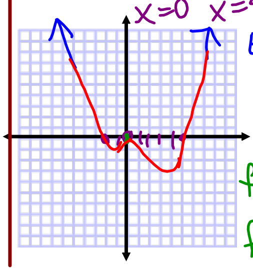
deg: x^3 3 opposite
Lead Coefficient: 1 pos



y int $f(0) = 0(0-3)(0+3)$
$= 0(-3)(3)$
$= 0$

$f(x) = 0.4x^2(x-5)^1(x+2)^1$

bounce even x = 0 pass x = 5 pass x = −2



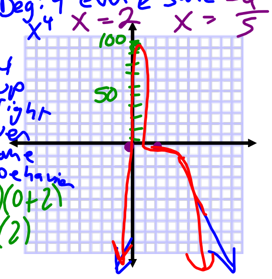
End behavior
Lead Coef: 0.4 positive up right
Degree: 4 even x^4 same behavior

$f(0) = 0.4(0)^2(0-5)(0+2)$
$f(0) = 0.4(0)(-5)(2)$
$= 0$

$f(x) = -3(x-2)^3(5x+4)^1$

Pass & flatten x = 2 Pass $x = -\frac{4}{5}$

Lead: −15 neg
Deg: 4 even x^4 same

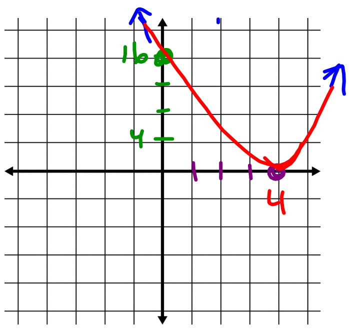


$f(0) = -3(0-2)^3(5(0)+4)$
$= -3(-2)^3(4)$
$= 96$

Homework Key

$y = (x-4)^2$

$f(0) = (0-4)^2 = 16$



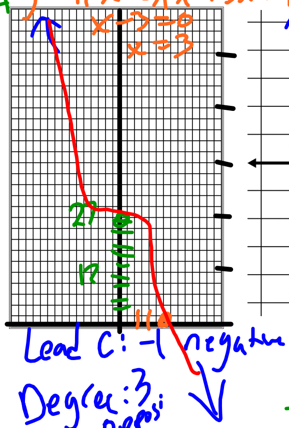
$y = -x^3+27$

$y = -1(x^3-27)$

$y = -1(x-3)(x^2+3x+9)$

$x-3=0$

$x=3$



Lead C: -1 negative

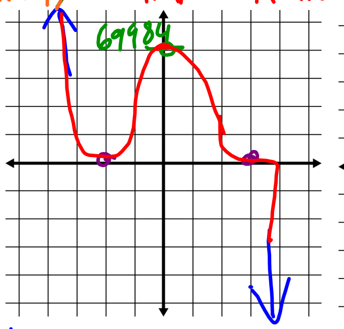
Degree: 3 opposi

$f(0) = -(0)^3+27 = 27$

x^{11}

$y = -2(x+2)^4(x-3)^7$

bounce flatter pass flatter

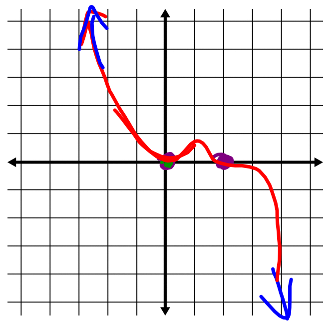


$f(0) = -2(0+2)^4(0-3)^7$

$= -2(2)^4(-3)^7$

$= 69984$

$y = -1/2x^2(x-2)^5$



$f(0) = -\frac{1}{2}(0)^2(0-2)^5$

$= 0$

Portfolio Work B Day

Polynomial Characteristics

$f(x) = -1/4x(x-2)^3 (x + 4)^2$

Plot x intercepts

Plot end behavior

Plot y intercept

Sketch passes and bounces

Label local max and local min

What intervals is this non-positive?

Assignment:

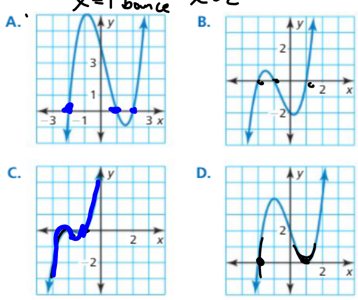
pg. 216 # 1,3-6, all

And Pick 3 from each section

7-16 , 23-30, and 31-36

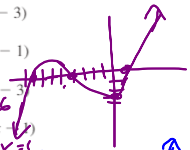
ANALYZING RELATIONSHIPS In Exercises 3–6, match the function with its graph.

3. $f(x) = (x - 1)(x - 2)(x + 2)$
A $x=1$ $x=2$ $x=-2$
4. $h(x) = (x + 2)^2(x + 1)$
bounce $x=-2$ $x=1$ C
5. $g(x) = (x + 1)(x - 1)(x + 2)$ B
$x=-1$ $x=1$ $x=-2$
6. $f(x) = (x - 1)^2(x + 2)$
$x=1$ bounce $x=2$

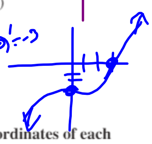


In Exercises 7–14, graph the function. *(See Example 1.)*

7. $f(x) = (x - 2)^2(x + 1)$
8. $f(x) = (x + 2)^2(x + 4)^2$
9. $h(x) = (x + 1)^2(x - 1)(x - 3)$
10. $g(x) = 4(x + 1)(x + 2)(x - 1)$
11. $h(x) = \frac{1}{3}(x - 5)(x + 2)(x - 3)$
$f(0) = -\frac{32}{12} = -2.6$
12. $g(x) = \frac{1}{12}(x + 4)(x + 8)(x - 1)$
$\frac{1}{12}x^3$ $x=-4$ $x=-8$ $x=1$



13. $h(x) = (x - 3)(x^2 + x + 1)$
$1x^3$ $x=3$ Imag $f(0)=-3$

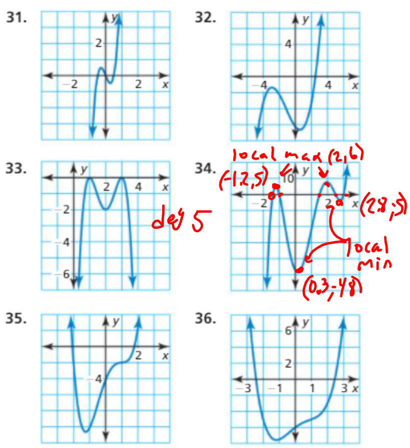


14. $f(x) = (x - 4)(2x^2 - 2x + 1)$

In Exercises 23–30, graph the function. Identify the x-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing. *(See Example 3.)*

23. $g(x) = 2x^3 + 8x^2 - 3$
24. $g(x) = -x^4 + 3x$
25. $h(x) = x^4 - 3x^2 + x$
26. $f(x) = x^5 - 4x^3 + x^2 + 2$
27. $f(x) = 0.5x^3 - 2x + 2.5$
28. $f(x) = 0.7x^4 - 3x^3 + 5x$
29. $h(x) = x^5 + 2x^2 - 17x - 4$
30. $g(x) = x^4 - 5x^3 + 2x^2 + x - 3$

In Exercises 31–36, estimate the coordinates of each turning point. State whether each corresponds to a local maximum or a local minimum. Then estimate the real zeros and find the least possible degree of the function.



Increasing min → max

$(x-0)(x-2)$

$x(x-2)$

$\cancel{\tfrac{4}{1}}(\cancel{\tfrac{1}{4}}x) = (0)\cdot\tfrac{4}{1}$

$x = 0$

$x \in (1.458, \infty)$ [redacted]

$\cup\ (-\infty, -1.627)$ [redacted]

$\cup\ (0, 0.1687)$

Decreasing max → min

$x \in (-1.627, 0)$

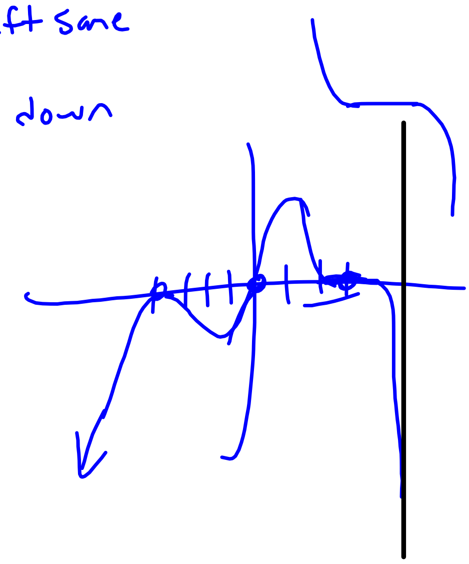
$(0.1687, 1.455)$

$\frac{1}{4}x(x-3)^3(x-2)^2$

$-\frac{1}{4}x^6$ → left same

→ right down

$f(0)$



1. turning

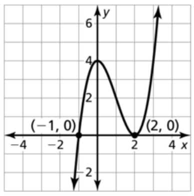
3. A

4. C

5. B

6. D

7.



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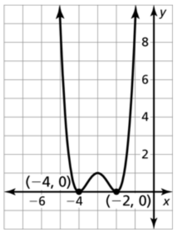

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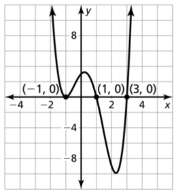

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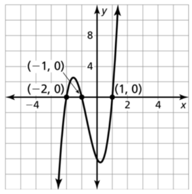

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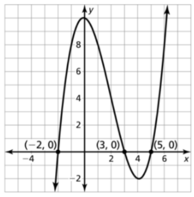
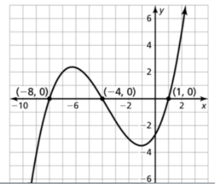
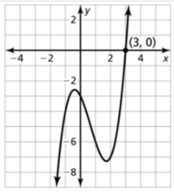


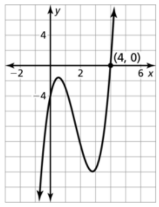
12.



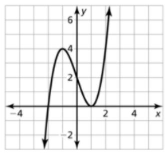
13.



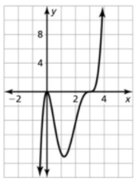
14.



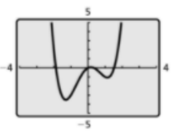
15. The x-intercepts should be -2 and 1.



16. Because 0 is a repeated zero with an even power, the graph should only touch the x-axis at 0, not cross it. Because 3 is a repeated zero with an odd power, the graph should cross the x-axis at 3.

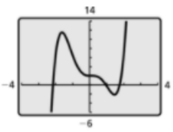


25.



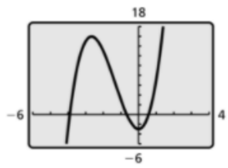
The x-intercepts of the graph are $x \approx -1.88$, $x = 0$, $x \approx 0.35$, and $x \approx 1.53$. The function has a local maximum at $(0.17, 0.08)$ and local minimums at $(-1.30, -3.51)$ and $(1.13, -1.07)$; The function is increasing when $-1.30 < x < 0.17$ and $x > 1.13$ and is decreasing when $x < -1.30$ and $0.17 < x < 1.13$.

26.



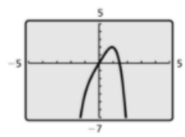
The x-intercepts of the graph are $x \approx -2.16$, $x = 1$, and $x \approx 1.75$. The function has a local maximum at $(-1.63, 10.47)$ and a local minimum at $(1.46, -1.68)$; The function is increasing when $x < -1.63$ and $x > 1.46$ and is decreasing when $-1.63 < x < 1.46$.

23.



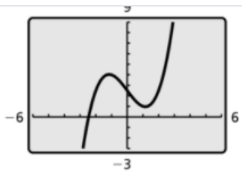
The x-intercepts of the graph are $x \approx -3.90$, $x \approx -0.67$, and $x \approx 0.57$. The function has a local maximum at $(-2.67, 15.96)$ and a local minimum at $(0, -3)$; The function is increasing when $x < -2.67$ and $x > 0$ and is decreasing when $-2.67 < x < 0$.

24.



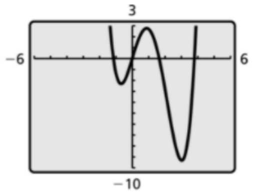
The x-intercepts of the graph are $x = 0$ and $x \approx 1.44$. The function has a local maximum at $(0.91, 2.04)$; The function is increasing when $x < 0.91$ and is decreasing when $x > 0.91$.

27.



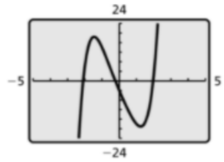
The x-intercept of the graph is $x \approx -2.46$. The function has a local maximum at $(-1.15, 4.04)$ and a local minimum at $(1.15, 0.96)$; The function is increasing when $x < -1.15$ and $x > 1.15$ and is decreasing when $-1.15 < x < 1.15$.

28.



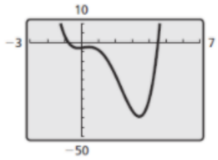
The x-intercepts of the graph are $x \approx -1.15$, $x = 0$, $x \approx 1.64$, and $x \approx 3.79$. The function has a local maximum at $(0.87, 2.78)$ and local minimums at $(-0.68, -2.31)$ and $(3.02, -9.30)$; The function is increasing when $-0.68 < x < 0.87$ and $x > 3.02$ and is decreasing when $x < -0.68$ and $0.87 < x < 3.02$.

29.



The x-intercepts of the graph are $x \approx -2.10$, $x \approx -0.23$, and $x \approx 1.97$. The function has a local maximum at $(-1.46, 18.45)$ and a local minimum at $(1.25, -19.07)$; The function is increasing when $x < -1.46$ and $x > 1.25$ and is decreasing when $-1.46 < x < 1.25$.

30.



The x-intercepts of the graph are $x \approx -0.77$ and $x \approx 4.54$. The function has a local maximum at $(0.47, -2.56)$ and local minimums at $(-0.16, -3.09)$ and $(3.44, -39.40)$; The function is increasing when $-0.16 < x < 0.47$ and $x > 3.44$ and is decreasing when $x < -0.16$ and $0.47 < x < 3.44$.

31. $(-0.29, 0.48)$ and $(0.29, -0.48)$; $(-0.29, 0.48)$ corresponds to a local maximum and $(0.29, -0.48)$ corresponds to a local minimum; The real zeros are -0.5, 0, and 0.5. The function is of at least degree 3.

32. $(-2.91, -1.36)$ and $(0.57, -6.63)$; $(-2.91, -1.36)$ corresponds to a local maximum and $(0.57, -6.63)$ corresponds to a local minimum; The real zero is 2.5. The function is of at least degree 3.

33. $(1, 0)$, $(3, 0)$, and $(2, -2)$; $(1, 0)$ and $(3, 0)$ correspond to local maximums, and $(2, -2)$ corresponds to a local minimum; The real zeros are 1 and 3. The function is of at least degree 4.

34. $(-1.22, 5.07)$, $(1.96, 7.71)$, $(0.15, -48.35)$, and $(2.79, -3.74)$; $(-1.22, 5.07)$ and $(1.96, 7.71)$ correspond to local maximums, and $(0.15, -48.35)$ and $(2.79, -3.74)$ correspond to local minimums; The real zeros are -1.4, -1, 1.5, 2.5, and 3. The function is of at least degree 5.

35. $(-1.25, -10.65)$; $(-1.25, -10.65)$ corresponds to a local minimum; The real zeros are -2.07 and 1.78. The function is of at least degree 4.

36. $(-1.18, -7.57)$; $(-1.18, -7.57)$ corresponds to a local minimum; The zeros are -2.45 and 2.45. The function is of at least degree 4.