Your Name

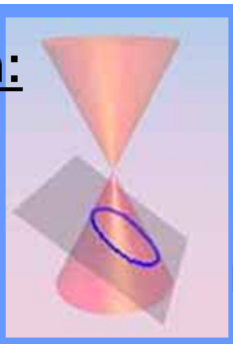
Mrs. Theo

5/11/2021

Ellipses

Notes

Formation:



Plane intersects the cone **at an angle with its base**

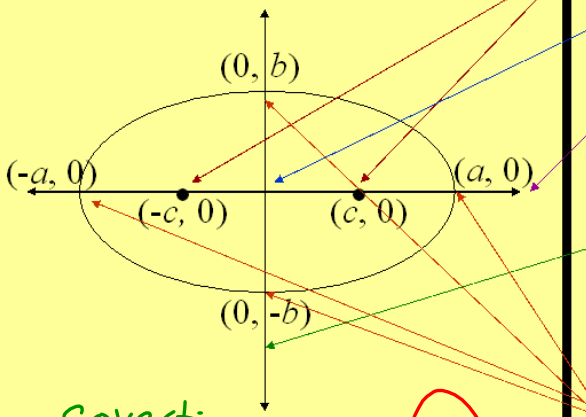
Ellipses

http://www.mathopenref.com/constellipse1.html

Definition: a collection of all points in a plane whose sum is a constant distance from two fixed points

a collection of all points in a plane the sum of whose distances from two fixed points is a constant

LABEL THIS PICTURE



Covertices are on minor axis

Foci

Foci

Center

Vertices are on major axis

DEFINE THESE TERMS

Foci – The two fixed points

Center – the midpoint of the line segment joining the foci

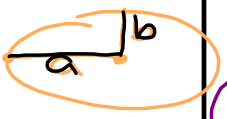
Major axis – line containing the foci (longer line)

Minor axis – line through center that is perpendicular to major axis (shorter line)

Vertices – where the ellipse and each axis intersect

Ellipses

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



(h,k) is the center of the circle

a is the length from the center to the side horizontally

b is the length from the center to the side vertically

Note: if a is bigger then it is a horizontal ellipse,

if b is bigger, then it is a vertical ellipse

**Notice the equations = 1. If they do not, divide by whatever it equals to get it to = 1.

For example: $\frac{4x^2}{36} + \frac{9y^2}{36} = \frac{36}{36}$

Divide everything by 36, but instead of using your calculator to divide it out – REDUCE!

$\frac{4x^2}{36} + \frac{9y^2}{36} = 1$

Center: (0,0)

$\frac{x^2}{9=a^2} + \frac{y^2}{4=b^2} = 1$

$a = 3$

$b = 2$

$\sqrt{9} = a$

Plug in $x = 2$

$\frac{2^2}{9} + \frac{y^2}{4} = 1$

$\frac{4}{9} + \frac{y^2}{4} = 1 = \frac{9}{9}$

$4 \cdot \frac{y^2}{4} = \frac{5}{9} \cdot 4$

$y^2 = \frac{20}{9}$

$y = \pm\sqrt{\frac{20}{9}}$

$y = \pm 1.49$

$(2, 1.49)$ $(2, -1.49)$

To Graph an Ellipse:

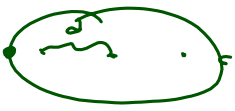
1. Plot the **center**
2. Find the **square root** of what is under **x** – move that many spaces **right and left**
3. Find the **square root** of what is under **y** – move that many spaces **up and down**
4. Connect in a **rounded** fashion using 4 more points found by plugging in x values
5. **Label** the important points – vertices and center.
6. To graph Foci Find the distance from the center on major axis by $c^2 = a^2 - b^2$

$c^2 = 3^2 - 2^2$

$c^2 = 9 - 4$

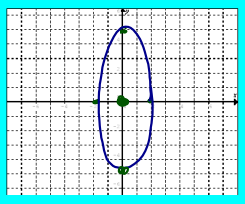
$c^2 = 5$

$c = \sqrt{5} = 2.23$



Examples: Graph. Label the center & vertices.

1. $\frac{x^2}{4}+\frac{y^2}{25}=1$

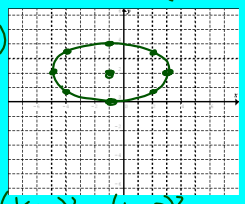


2. $\frac{(x+1)^2}{16}+\frac{(y-2)^2}{4}=\frac{16}{16}$

Center: (-1,2)

$a=4$

$b=2$



$\frac{(x+1)^2}{a^2=16}+\frac{(y-2)^2}{4=b^2}=1$

plug in $x=-4$

$\frac{(-4+1)^2}{16}+\frac{(y-2)^2}{4}=1$

$\frac{9}{16}+\frac{(y-2)^2}{4}=1=\frac{16}{16}$

$\frac{(y-2)^2}{4}=\frac{7}{16}\cdot\frac{4}{1}$

$(y-2)^2=\frac{7}{4}$

$y-2=\pm\sqrt{\frac{7}{4}}$

$y-2=1.322$ and $y-2=-1.322$

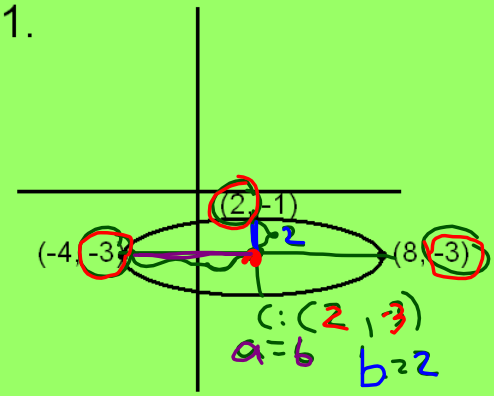
$+2\ +2$ $+2\ +2$

$y=3.322$ $y=0.677$

$(-4,3.322)$ $(-4,0.677)$

Examples: Find the equation for the given graphs.

1.

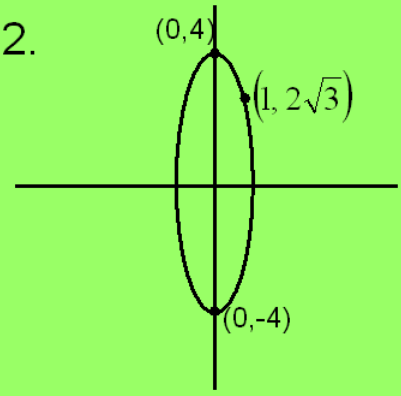


C: (2, -3)

$a=6$ $b=2$

$\frac{(x-2)^2}{36}+\frac{(y+3)^2}{4}=1$

2.



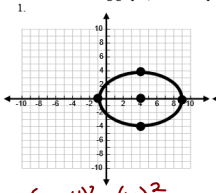
Example: Applications

The Colosseum in Rome seated 50,000 spectators around a central elliptical arena. The base of the Colosseum measured 615 feet long and 510 feet wide. Write an equation for the elliptical shape of the Colosseum.

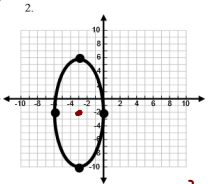
$$\frac{x^2}{615^2} + \frac{y^2}{510^2} = 1$$

Integrated Algebra 2 **Ellipse Worksheet** **Name** _____

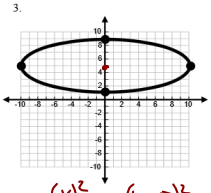
Given the following graphs, write the equation of the conic section.

1. 

$\frac{(x-4)^2}{25} + \frac{(y)^2}{9} = 1$

2. 

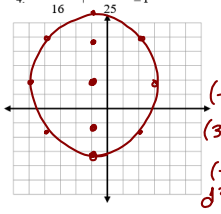
$\frac{(x+3)^2}{9} + \frac{(y+2)^2}{64} = 1$

3. 

$\frac{(x)^2}{100} + \frac{(y-5)^2}{16} = 1$

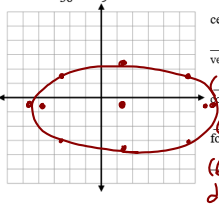
Graph each ellipse. Include the foci.

4. $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{25} = 1$



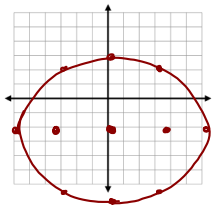
center: $(-1, 2)$
vertices: $b = 5$ $(-1, 7)$ & $(-1, -3)$
co-vertices: $a = 4$ $(3, 2)$ & $(-5, 2)$
foci: $d = 3$ $(-1, -1)$ & $(-1, 5)$
$d^2 = 5^2 - 4^2$
$d^2 = 25 - 16$
$d^2 = 9$

5. $\frac{(x-1)^2}{36} + \frac{y^2}{9} = 1$



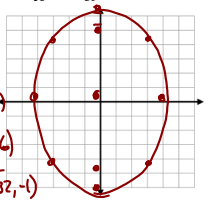
center: $(1, 0)$
vertices: $a = 6$ $(7, 0)$ & $(-5, 0)$
co-vertices: $b = 3$ $(1, 3)$ & $(1, -3)$
foci: $d = 5.196$ $(6.196, 0)$ & $(-4.196, 0)$
$d^2 = 6^2 - 3^2$
$d^2 = 36 - 9$
$d^2 = 27$

6. $\frac{x^2}{36} + \frac{(y+1)^2}{25} = 1$



center: $(0, -1)$
vertices: $a = 6$ $(6, -1)$ & $(-6, -1)$
co-vertices: $b = 5$ $(0, 4)$ & $(0, -6)$
foci: $d = 3.32$ $(3.32, -1)$ & $(-3.32, -1)$
$d^2 = 6^2 - 5^2$
$d^2 = 36 - 25$
$d^2 = 11$

7. $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{39} = 1$



center: $(-1, 2)$
vertices: $b = 6.2$ $(-1, 8.2)$ & $(-1, -4.2)$
co-vertices: $a = 4$ $(3, 2)$ & $(-5, 2)$
foci: $d = 4.79$ $(-1, 6.79)$ & $(-1, -2.79)$
$d^2 = 6.2^2 - 4^2$
$d^2 = 39 - 16$
$d^2 = 23$

Write the standard equation for each ellipse. Write the coordinates of the vertices, co-vertices and foci.

8. $25x^2+9y^2=225$ Equation $\frac{x^2}{9}+\frac{y^2}{25}=1$

$\frac{25x^2}{225}+\frac{9y^2}{225}=\frac{225}{225}$ $c^2=25-9$ $c^2=16$ $c=4$

$\frac{x^2}{9}+\frac{y^2}{25}=1$

vertices: (0, 5) (0, -5) co-vertices: (-3, 0) (3, 0) foci: (0, 4) (0, -4)

9. $49x^2+y^2=49$ Equation $\frac{x^2}{1}+\frac{y^2}{49}=1$

$\frac{49x^2}{49}+\frac{y^2}{49}=\frac{49}{49}$ $c^2=49-1$ $c^2=48$ $c=6.93$

$\frac{x^2}{1}+\frac{y^2}{49}=1$

vertices: (0, 7) (0, -7) co-vertices: (1, 0) (-1, 0) foci: (0, 6.93) (0, -6.93)

10. $4x^2+y^2-8x+4y=8$ Equation $\frac{(x-1)^2}{4}+\frac{(y+2)^2}{16}=1$

$4x^2-8x+y^2+4y=8$

$4(x^2-2x+1)+(y^2+4y+4)=8+4+4$

$\frac{4(x-1)^2}{16}+\frac{(y+2)^2}{16}=\frac{16}{16}$

vertices: _____ co-vertices: _____ foci: _____

11. $x^2+4y^2-18x-8y=-81$ Equation $\frac{(x-9)^2}{4}+\frac{(y-1)^2}{1}=1$

$x^2-18x+4y^2-8y=-81$

$(x^2-18x+81)+4(y^2-2y+1)=-81+81+4$

$(x-9)^2+4(y-1)^2=4$

vertices: _____ co-vertices: _____ foci: _____

12. $9x^2+4y^2-144x-8y=-544$ Equation $\frac{(x-18)^2}{264}+\frac{(y-1)^2}{594}=1$

$9x^2-144x+4y^2-8y=-544$

$9(x^2-36x+324)+4(y^2-2y+1)=-544+2916+4$

$9(x-18)^2+4(y-1)^2=2376$

vertices: _____ co-vertices: _____ foci: _____

Write the standard equation for each ellipse with the given characteristics.

13. Vertices (0, 5), (0, -5), Co-vertices (3, 0), (-3, 0)

$b=5$ $a=3$ center (0,0)

$\frac{x^2}{9}+\frac{y^2}{25}=1$

14. Vertices (7, 4), (-1, 4), Co-vertices (3, 6), (3, 2)

$a=\frac{8}{2}=4$ $b=\frac{4}{2}=2$ center (3, 4)

$\frac{(x-3)^2}{16}+\frac{(y-4)^2}{4}=1$

15. Vertices (-5, 0), (-5, -8), Foci (-5, -2), (-5, -6)

$b=\frac{8}{2}=4$ $d=\frac{4}{2}=2$ C: (-5, -4)

$\frac{(x+5)^2}{12}+\frac{(y+4)^2}{16}=1$

$d^2=b^2-a^2$

$2^2=4^2-a^2$

$a^2=16-4$

$a=\sqrt{12}\approx 3.5$