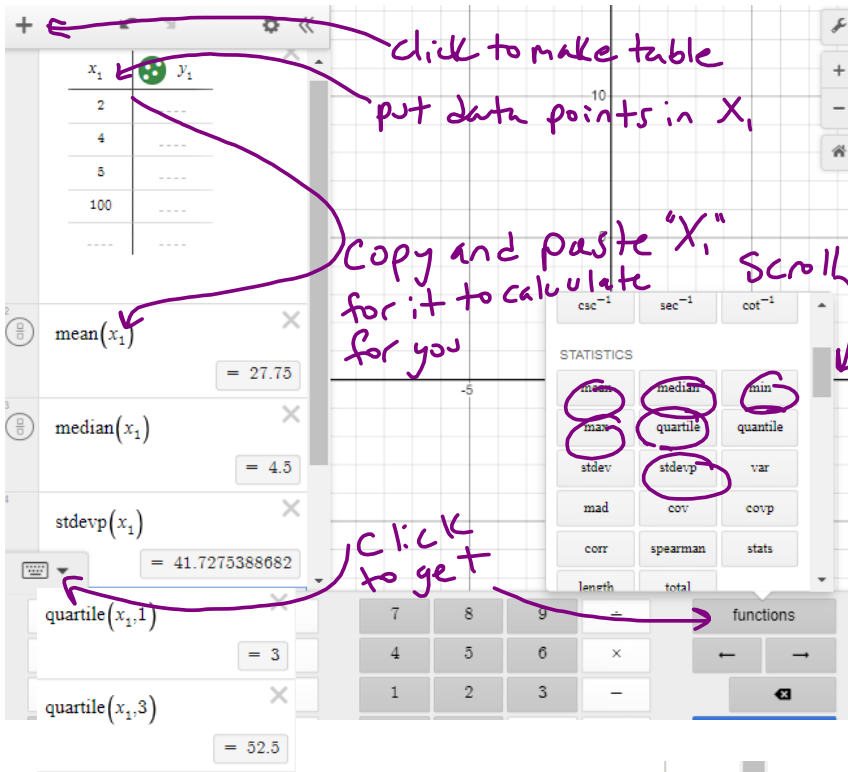


Data Distribution and Normal Curves

You can use Desmos Graphing Calculator at home! heres how



\bar{x} = mean

σ = Standard Deviation

-how far a data value is from the mean

5 Number summary

1. Minimum
2. Q1
3. Median/Q2
4. Q3
5. Maximum

On a calculator:

click Stat

Edit

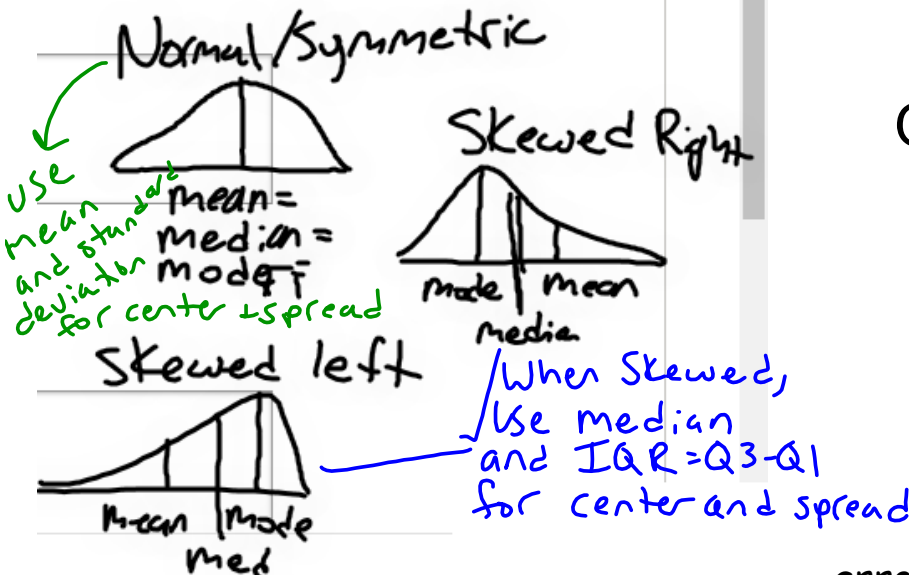
Enter Data

click Stat

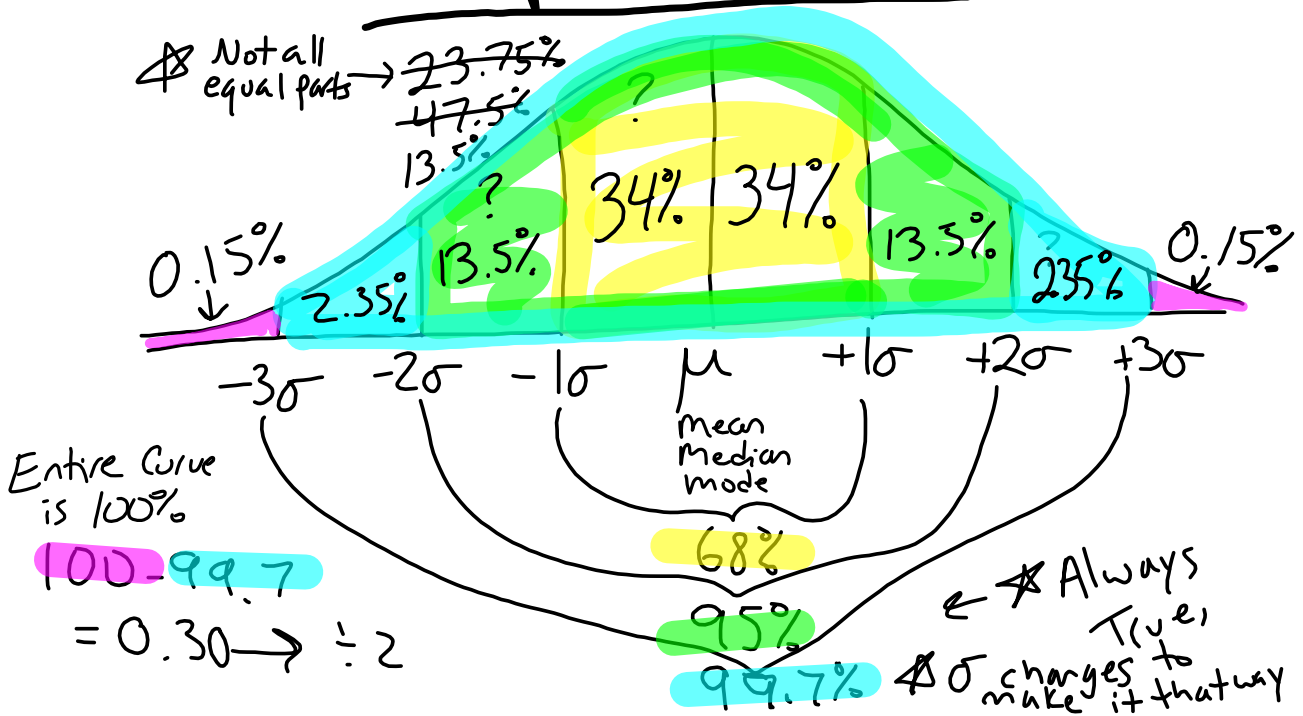
arrow to the right to Calc

1 Var-Stats

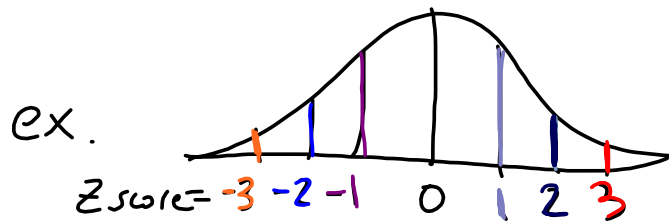
Enter



Empirical Rule



- σ standard deviation is the size of your intervals. You add + subtract it from the mean to get your values
- Z-score is the # of intervals away a value is from the mean



Normal Distribution Worksheet

Header: Key

Sec 11-4 Example 1 and Example 2

1. Generalizing: A normal distribution of scores has a standard deviation of 10. Find the z-scores corresponding to each of the following values:

a) A score that is 20 points above the mean.

$z = 2$

b) A score that is 10 points below the mean.

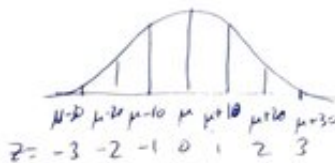
$z = -1$

c) A score that is 30 points above the mean.

$z = 3$

d) A score that is 30 points below the mean.

$z = -3$



2. The Wechsler Adult Intelligence Test Scale is composed of a number of subtests. On one subtest, the raw scores have a mean of 35 and a standard deviation of 6. Assuming these raw scores form a normal distribution:

a) What number represents the 84th percentile (what number separates the lower 84% of the distribution)?

$50\% + 34\% = 41$

b) What number represents the 97.5th percentile?

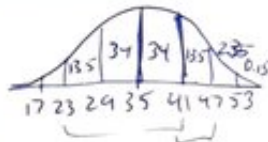
$50\% + 34\% + 13.5\% = 47$

c) What is the probability of getting a raw score between 23 and 41?

$13.5\% + 34\% + 34\% = 81.5\%$

d) What is the probability of getting a raw score between 41 and 47?

13.5%



3. Scores on the SAT form a normal distribution with $\mu = 500$ and $\sigma = 100$.

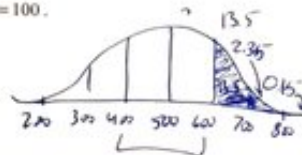
a) What is the minimum score necessary to be in the top 10% of the SAT distribution?

600

$13.5\% + 2.35\% + 0.15\% = 16\%$

b) Find the range of values that defines the middle 68% of the distribution of SAT scores.

400 to 600



4. A patient recently diagnosed with Alzheimer's disease takes a cognitive abilities test. The mean on this test is 52 and the standard deviation is 5.

a) The patient's score is a 47.

What is the patient's percentile rank? 16th percentile

$0.15\% + 2.35\% + 13.5\% = 16\%$

b) Another patient scored a 65.

Their score is between what percentiles?

They scored at the 97.5% to 99.85%

$50 + 34 + 13.5 = 97.5$

$50 + 34 + 13.5 + 2.35 = 99.85$



Standard Deviation Formula

$$S_x = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$

n = The number of data points
 X_i = Each of the values of the data
 \bar{X} = The mean of X_i

z = -3 -2 -1 1 2 3
Normal Distribution Curve

Standard Deviation

10 numbers

Data sample: 5, 5, 7, 11, 14, 14, 14, 18, 20, 22 Mean is 13.7

$$\text{Standard Deviation} = \sqrt{\frac{(5-13.7)^2 + (5-13.7)^2 + (7-13.7)^2 + (11-13.7)^2 + (14-13.7)^2 + (14-13.7)^2 + (14-13.7)^2 + (18-13.7)^2 + (20-13.7)^2 + (22-13.7)^2}{10-1}}$$

or

$$= \sqrt{\frac{2(5-13.7)^2 + 2(7-13.7)^2 + 3(11-13.7)^2 + 3(14-13.7)^2 + (18-13.7)^2 + (20-13.7)^2 + (22-13.7)^2}{10-1}}$$

$$= \sqrt{\frac{2(-8.7)^2 + 2(-6.7)^2 + (-2.7)^2 + 3(0.3)^2 + (4.3)^2 + (6.3)^2 + (8.3)^2}{9}} = \sqrt{\frac{375.79}{9}} =$$

$\sqrt{41.75444 \dots} = 6.462$ is standard deviation amount you add and subtract from the mean.

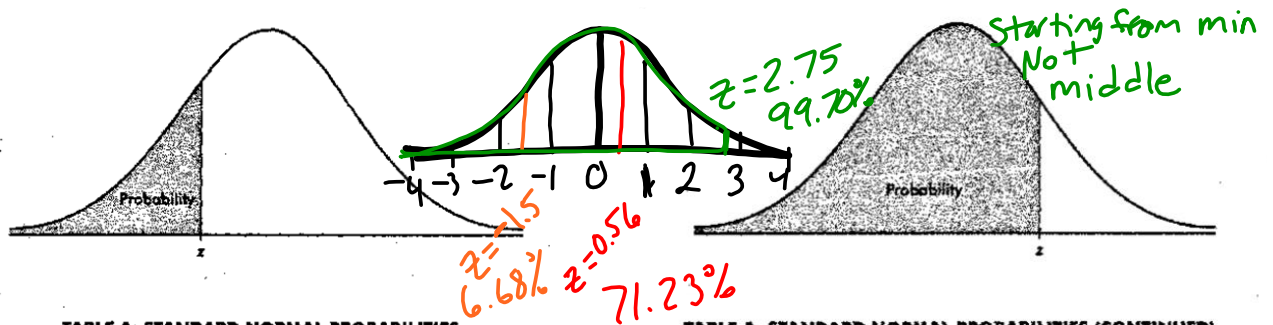


TABLE A: STANDARD NORMAL PROBABILITIES

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776

TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981

Z	A	Z	A	Z	A	Z	A	Z	A	Z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498
.11	.044	.67	.249	1.23	.391	1.79	.463	2.35	.491	2.91	.498
.12	.048	.68	.252	1.24	.393	1.80	.464	2.36	.491	2.92	.498
.13	.052	.69	.255	1.25	.394	1.81	.465	2.37	.491	2.93	.498
.14	.056	.70	.258	1.26	.396	1.82	.466	2.38	.491	2.94	.498
.15	.060	.71	.261	1.27	.398	1.83	.466	2.39	.492	2.95	.498
.16	.064	.72	.264	1.28	.400	1.84	.467	2.40	.492	2.96	.499
.17	.068	.73	.267	1.29	.402	1.85	.468	2.41	.492	2.97	.499
.18	.071	.74	.270	1.30	.403	1.86	.469	2.42	.492	2.98	.499
.19	.075	.75	.273	1.31	.405	1.87	.469	2.43	.493	2.99	.499
.20	.079	.76	.276	1.32	.407	1.88	.470	2.44	.493	3.00	.499
.21	.083	.77	.279	1.33	.408	1.89	.471	2.45	.493	3.01	.499
.22	.087	.78	.282	1.34	.410	1.90	.471	2.46	.493	3.02	.499
.23	.091	.79	.285	1.35	.412	1.91	.472	2.47	.493	3.03	.499
.24	.095	.80	.288	1.36	.413	1.92	.473	2.48	.493	3.04	.499
.25	.099	.81	.291	1.37	.415	1.93	.473	2.49	.494	3.05	.499
.26	.103	.82	.294	1.38	.416	1.94	.474	2.50	.494	3.06	.499
.27	.106	.83	.297	1.39	.418	1.95	.474	2.51	.494	3.07	.499
.28	.110	.84	.300	1.40	.419	1.96	.475	2.52	.494	3.08	.499
.29	.114	.85	.302	1.41	.421	1.97	.476	2.53	.494	3.09	.499
.30	.118	.86	.305	1.42	.422	1.98	.476	2.54	.495	3.10	.499
.31	.122	.87	.308	1.43	.424	1.99	.477	2.55	.495	3.11	.499
.32	.126	.88	.311	1.44	.425	2.00	.477	2.56	.495	3.12	.499
.33	.129	.89	.313	1.45	.427	2.01	.478	2.57	.495	3.13	.499
.34	.133	.90	.316	1.46	.428	2.02	.478	2.58	.495	3.14	.499
.35	.137	.91	.319	1.47	.429	2.03	.479	2.59	.495	3.15	.499
.36	.141	.92	.321	1.48	.431	2.04	.479	2.60	.495	3.16	.499
.37	.144	.93	.324	1.49	.432	2.05	.480	2.61	.496	3.17	.499
.38	.148	.94	.326	1.50	.433	2.06	.480	2.62	.496	3.18	.499
.39	.152	.95	.329	1.51	.435	2.07	.481	2.63	.496	3.19	.499
.40	.155	.96	.332	1.52	.436	2.08	.481	2.64	.496	3.20	.499
.41	.159	.97	.334	1.53	.437	2.09	.482	2.65	.496	3.21	.499
.42	.163	.98	.337	1.54	.438	2.10	.482	2.66	.496	3.22	.499
.43	.166	.99	.339	1.55	.439	2.11	.483	2.67	.496	3.23	.499
.44	.170	1.00	.341	1.56	.441	2.12	.483	2.68	.496	3.24	.499
.45	.174	1.01	.344	1.57	.442	2.13	.483	2.69	.496	3.25	.499
.46	.177	1.02	.346	1.58	.443	2.14	.484	2.70	.497	3.26	.499
.47	.181	1.03	.349	1.59	.444	2.15	.484	2.71	.497	3.27	.500
.48	.184	1.04	.351	1.60	.445	2.16	.485	2.72	.497	3.28	.500
.49	.188	1.05	.353	1.61	.446	2.17	.485	2.73	.497	3.29	.500
.50	.192	1.06	.355	1.62	.447	2.18	.485	2.74	.497	3.30	.500
.51	.195	1.07	.358	1.63	.449	2.19	.486	2.75	.497	3.31	.500
.52	.199	1.08	.360	1.64	.450	2.20	.486	2.76	.497	3.32	.500
.53	.202	1.09	.362	1.65	.451	2.21	.487	2.77	.497	3.33	.500
.54	.205	1.10	.364	1.66	.452	2.22	.487	2.78	.497		
.55	.209	1.11	.367	1.67	.453	2.23	.487	2.79	.497		

$z = 0.56 \rightarrow 71.2\%$
 add .212 to 0.5
 $0.5 + 0.212 = 0.712$

$z = -1.50 \rightarrow 6.7\%$
 \rightarrow subtract .433
 from .50
 $.50 - .433 = 0.067$

$z = 2.75 \rightarrow 99.7\%$
 add 0.497 to 0.5
 $= 0.997$

Normal Distribution Worksheet

Header: Key

Sec 11-4 Example 3 and 4

5. IQ scores have a mean of 100 and a standard deviation of 16. Albert Einstein reportedly had an IQ of 160.

$\mu = 100$ $\sigma = 16$

- a. What is the difference between Einstein's IQ and the mean?
 b. How many standard deviations is that?

60 pts



over 3 standard deviations away

- c. Convert Einstein's IQ score to a z score.

$z = \frac{160 - 100}{16} = 3.75$

- d. If we consider "usual IQ scores to be those that convert z scores between -2 and 2, is Einstein's IQ usual or unusual?

really unusual, our z score chart writ go higher than 3.49

6. For a normal distribution, find the z-score that separates the distribution as follows:

- a) Separate the highest 30% from the rest of the distribution.

$100\% - 30\% = 70\%$ $70\% = 0.7 \rightarrow z = 0.525$

- b) Separate the lowest 40% from the rest of the distribution.

$40\% = 0.4 \rightarrow z = -2.55$

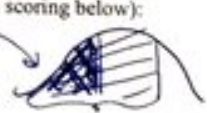
- c) Separate the highest 75% from the rest of the distribution.

$100\% - 75\% = 25\%$ $25\% = 0.25 \rightarrow z = -0.675$

7. For the z-scores below, find the percentile rank (percent of individuals scoring below):

a) $z = -0.47 \rightarrow 0.3142 = 31.42\%$

b) $z = 2.24 \rightarrow 0.9875 = 98.75\%$



8. For the numbers below, find the percent of cases falling above the z-score:

a) $z = 0.24 \rightarrow 0.5948 = 59.48\%$ below so $100\% - 59.48\% = 40.52\%$ above

b) $z = -2.07 \rightarrow 0.0192 = 1.92\%$ below so $100\% - 1.92\% = 98.08\%$ above



9. For the numbers below, find the area between the mean and the z-score:

a) $z = 1.17 \rightarrow 0.8790 = 87.90\% \rightarrow 87.9\% - 50\% = 37.9\%$

b) $z = -1.37 \rightarrow 0.0853 = 8.53\% \rightarrow 50\% - 8.53\% = 41.47\%$



10. A fifth grader takes a standardized achievement test (mean = 125, standard deviation = 15) and scores a 148. What is the child's percentile rank?

$X = 148$ $\bar{X} = 125$ $\sigma = 15$ $z = \frac{148 - 125}{15} = 1.53 \rightarrow 0.9370 = 93.7\%$ percentile

