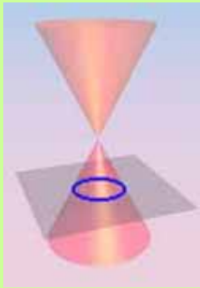
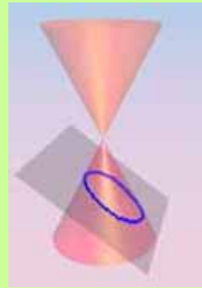


Conic Sections!!!



Circle

Plane intersects the cone **parallel to its base**



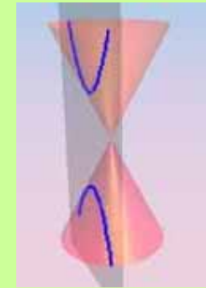
Ellipse

Plane intersects the cone **at an angle with its base**



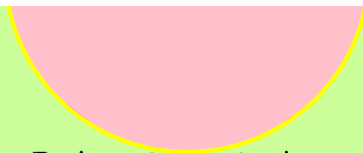
Parabola

Plane intersects the cone **parallel to its side**



Hyperbola

Plane intersects the cone **perpendicular to its base**

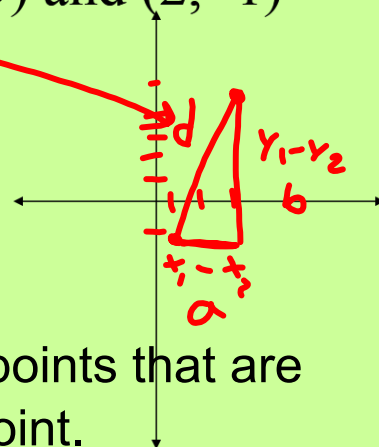
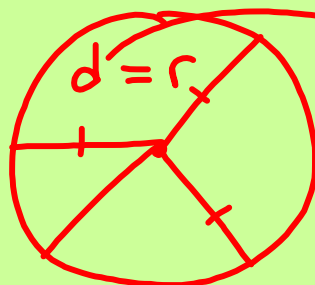


Circles

Each conic section has an equation that defines it. The circle's equation is derived from a familiar formula.....THE DISTANCE FORMULA!!

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance between (3, 5) and (2, -1)



A circle is created from all the points that are equal distant from the center point.

Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

(h, k) is the **center** of the circle

r is the **radius** (the distance from the center to any edge of the circle)

Notice: both the x and y are squared
and r

Examples:

1. Find an equation with center at $(-3, 2)$ having a radius of 5. $r=5$

$$(x - (-3))^2 + (y - (2))^2 = 5^2$$

$$(x + 3)^2 + (y - 2)^2 = 25$$

2. Give the equation of the circle with radius 3 whose center is at the origin. $(0, 0)$ $r=3$

$$(x - 0)^2 + (y - 0)^2 = 3^2$$

$$x^2 + y^2 = 9$$

Examples

$$x^2 + y^2 = 49$$

$$(x-0)^2 + (y-0)^2 = 7^2$$

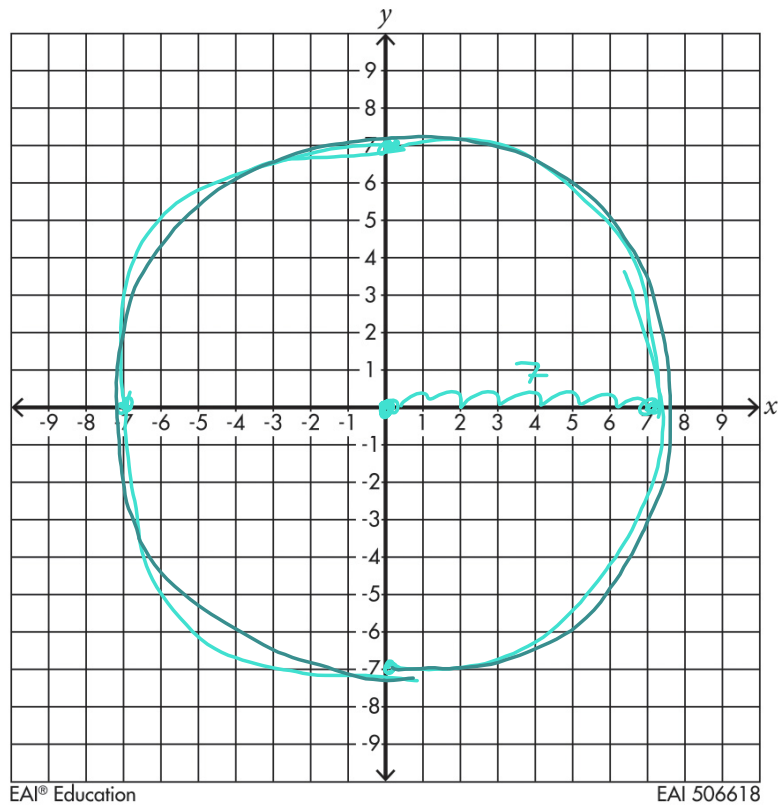
Center: $(0,0)$

Radius: 7

$$r^2 = 49$$

$$r = \sqrt{49}$$

$$r = 7$$



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Examples

$$(x-3)^2 + y^2 = 4$$

$$(x-3)^2 + (y-0)^2 = 2^2$$

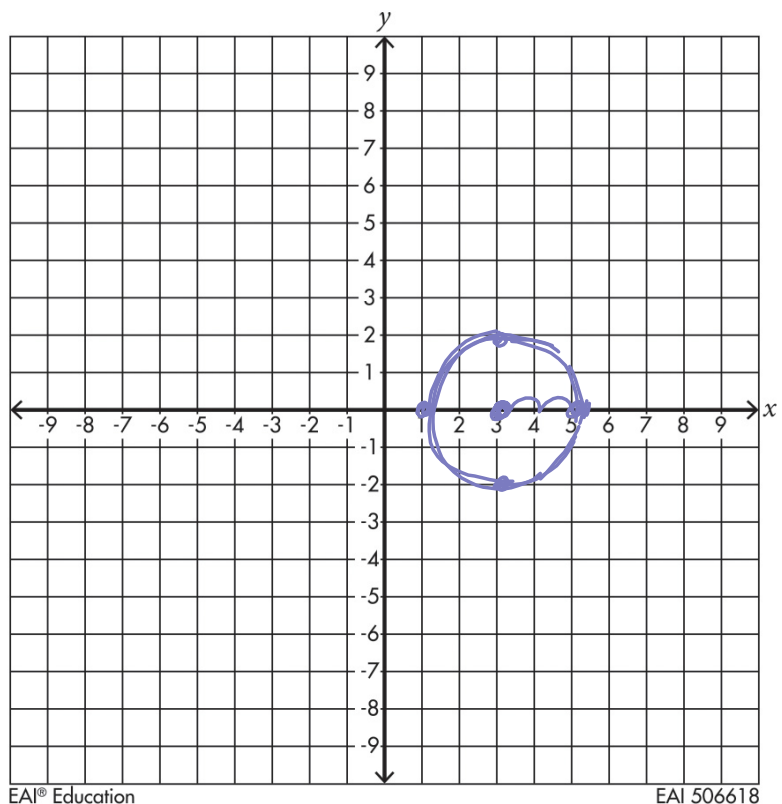
Center: $(3,0)$

Radius: 2

$$r^2 = 4$$

$$r = \sqrt{4}$$

$$r = 2$$



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Examples

$$x^2 + (y+2)^2 = 25$$

$$(x-0)^2 + (y-(-2))^2 = 5^2$$

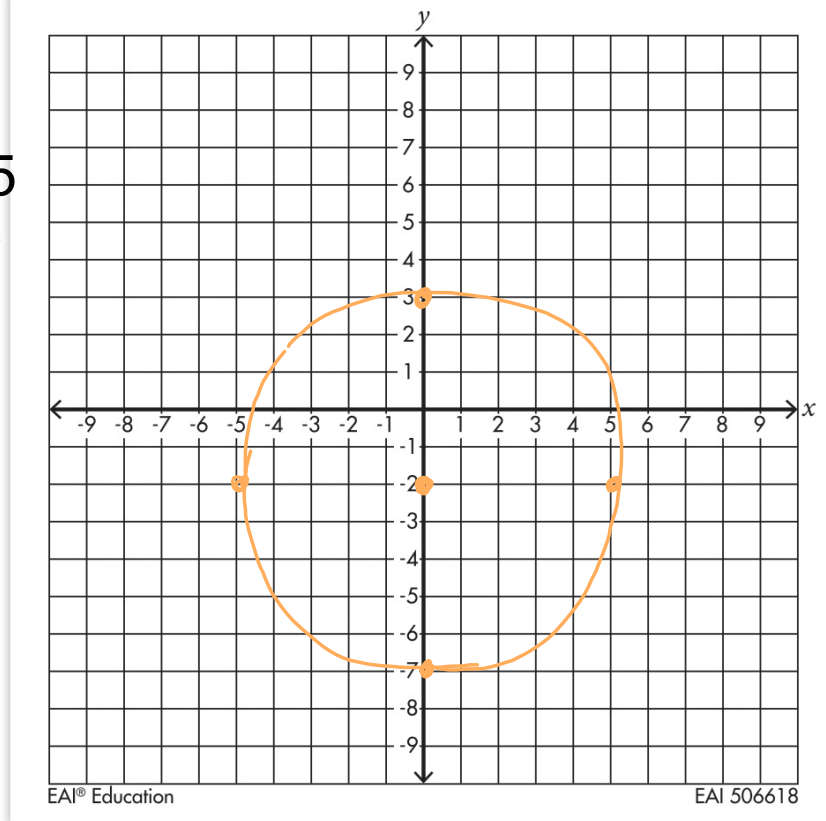
Center: $(0, -2)$

Radius: 5

$$r^2 = 25$$

$$r = \sqrt{25}$$

$$r = 5$$



Ellipses

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



(h,k) is the center of the ellipse

a is the length from the center to the side horizontally

b is the length from the center to the side vertically

Note: if a is bigger then it is a horizontal ellipse,
if b is bigger, then it is a vertical ellipse

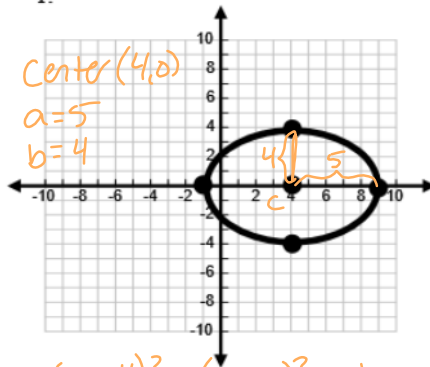
Foci

To calculate the foci use: $c^2 = a^2 - b^2$ or $b^2 - a^2$
depending on which is bigger a or b

Ellipse Examples:

Given the following graphs, write the equation of the conic section.

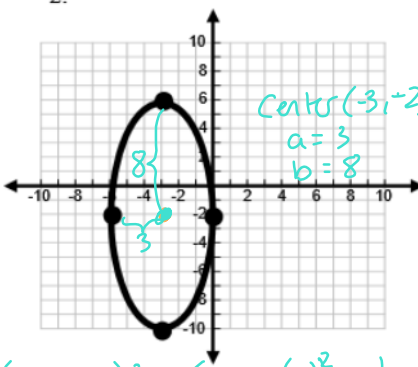
1.



$$\frac{(x-4)^2}{5^2} + \frac{(y-0)^2}{4^2} = 1$$

$$\frac{(x-4)^2}{25} + \frac{y^2}{16} = 1$$

2.



$$\frac{(x-(-3))^2}{3^2} + \frac{(y-(-2))^2}{8^2} = 1$$

$$\frac{(x+3)^2}{9} + \frac{(y+2)^2}{64} = 1$$

3. Find the equation of the ellipse with center at the origin having a horizontal distance of 8 and a vertical distance of 2.

Center: (0, 0)

a = 8

b = 2

$$\frac{(x-0)^2}{8^2} + \frac{(y-0)^2}{2^2} = 1$$

$$\frac{x^2}{64} + \frac{y^2}{4} = 1$$

Examples

$$\frac{x^2}{36} + \frac{y^2}{49} = 1$$

$a^2 \rightarrow$ $b^2 \rightarrow$

Center: (0, 0)

Horizontal Distance: 6

Vertical Distance: 7

$$a^2 = 36$$

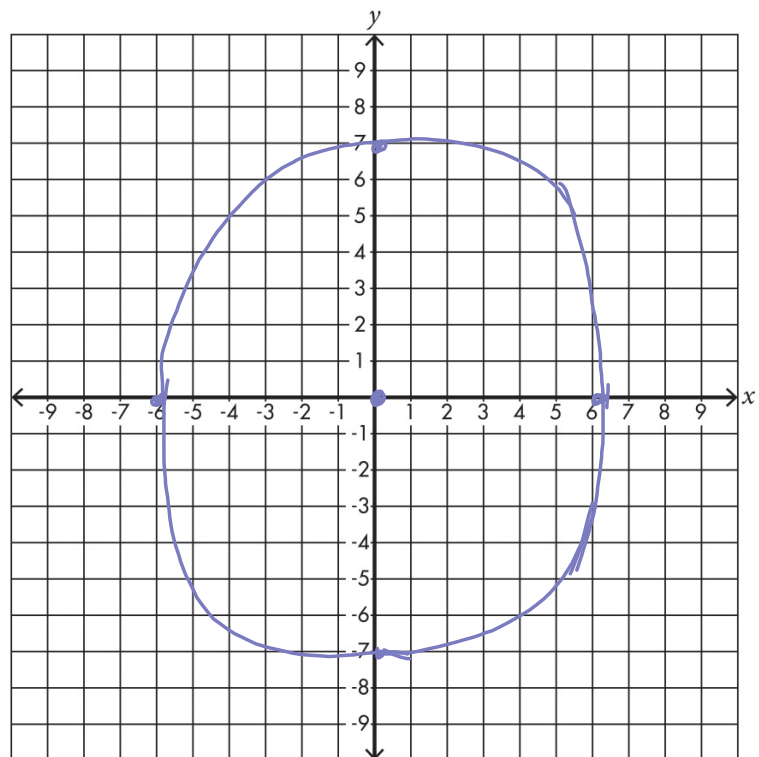
$$a = \sqrt{36}$$

$$a = 6$$

$$b^2 = 49$$

$$b = \sqrt{49}$$

$$b = 7$$

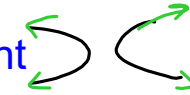


Hyperbola

horizontal when x is in front

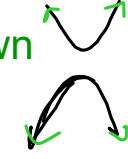
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Open left/right



$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Open up/down



Vertical when y is in front

(h,k) is the center of the Hyperbola

a is the length from the center to the side horizontally of the fundamental rectangle

b is the length from the center to the side vertically of the fundamental rectangle

Transverse Axis: It is not about the size of a or b, but about which squared variable is positive!

Foci

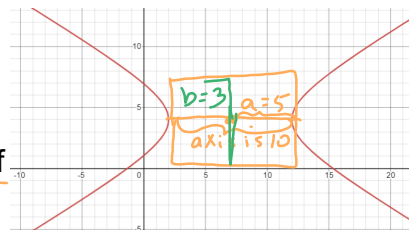
To calculate the foci use: $c^2 = a^2 + b^2$

Hyperbolas Examples:

1. Find the equation of the horizontal hyperbola with center at (7,4) having a horizontal axis of 10 and a vertical axis of 6.

$$\frac{(x-7)^2}{5^2} - \frac{(y-4)^2}{3^2} = 1$$

$$\frac{(x-7)^2}{25} - \frac{(y-4)^2}{9} = 1$$

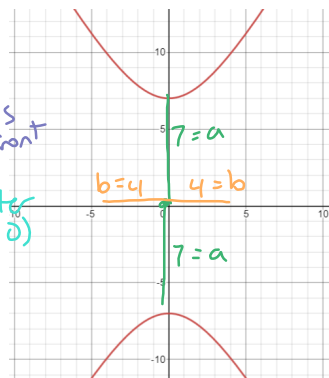


horiz. axis is all the way 'a' is half of it
Vertical Axis = 6 so half is 3

2. Find the equation of the vertical hyperbola with center at the origin having a vertical axis of 14 and a horizontal axis of 8.

$$\frac{(y-0)^2}{7^2} - \frac{(x-0)^2}{4^2} = 1$$

$$\frac{y^2}{49} - \frac{x^2}{16} = 1$$

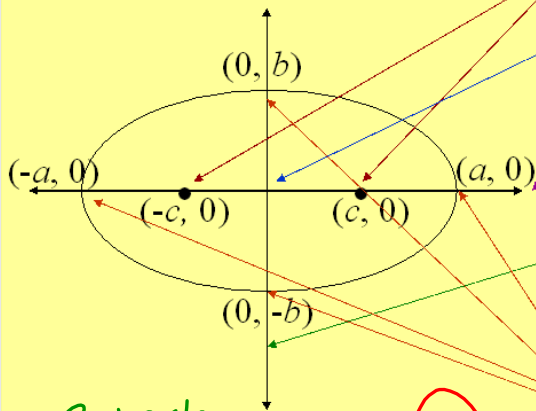


so y is in front

center is (0,0)

LABEL THIS PICTURE

DEFINE THESE TERMS



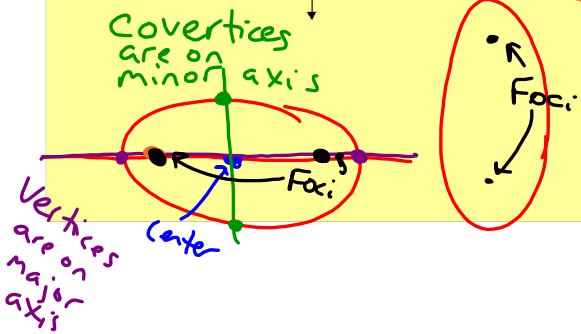
Foci – The two fixed points

Center – the midpoint of the line segment joining the foci

Major axis – line containing the foci (longer line)

Minor axis – line through center that is perpendicular to major axis (shorter line)

Vertices – where the ellipse and each axis intersect



LABEL THIS PICTURE:

DEFINE THESE TERMS

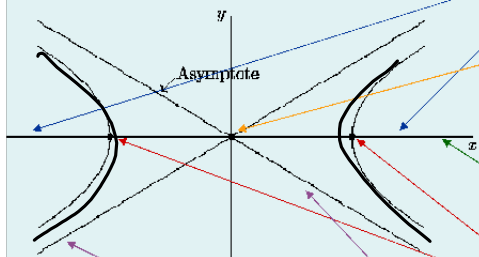


FIGURE H4

Foci - the two fixed points

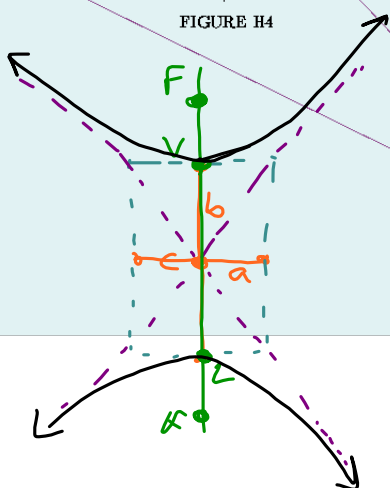
Center - the midpoint of the line segment joining the foci

Transverse axis - the line containing all important info

Vertices - two points of intersection of the hyperbola and transverse axis

Asymptotes - lines that guide the graph of the hyperbola (don't touch!)

Fundamental Rectangle - the box created by a and b whose corners determine the asymptotes



Name _____ Hour _____
Honors Algebra II - Section 9.2 Wkst

Identify the following as ellipses or hyperbolas. Be careful!

1. $\frac{(y-3)^2}{9} - \frac{x^2}{16} = 1$ hyperbola
 2. $x^2 + 12y^2 = 12$ ellipse
 3. $8x^2 = 100 - 2y^2$ ellipse
 4. $3x^2 = y^2 + 15$ hyperbola

Graph each Hyperbola. Name then center, the vertices and the foci.

5. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

 center (0,0)
 vertices (2,0), (-2,0)
 foci (5,0), (-5,0)
 asymptotes $y = \pm \frac{3}{2}x$
 $d^2 = 9 + 4 = 13$
 $d = \sqrt{13}$

6. $\frac{y(x+1)^2}{328} - \frac{x(y-3)^2}{32} = 32$

 center (-1,3)
 vertices (-1+sqrt(8), 3), (-1-sqrt(8), 3)
 foci (-1+sqrt(12), 3), (-1-sqrt(12), 3)
 asymptotes $y = \pm \frac{2}{3}(x+1) + 3$
 $d^2 = 8 + 4 = 12$
 $d = \sqrt{12}$

7. $8y^2 - x^2 = 32$

 center (0,0)
 vertices (0,2), (0,-2)
 foci (0,6), (0,-6)
 asymptotes $y = \pm \frac{2}{\sqrt{32}}x$
 $d^2 = 32 + 4 = 36$
 $d = 6$
 y^2 is pos. so Up/Down

Graph each Conic Equation. Name the center, vertices, and asymptotes (EC, Foci).

8. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ ellipse

 center (0,0)
 vertices (0,3), (0,-3)
 foci (0, sqrt(5)), (0, -sqrt(5))
 $d^2 = 9 - 4 = 5$
 $d = \sqrt{5}$

9. $y^2 - \frac{x^2}{16} = 1$ hyperbola

 center (0,0)
 vertices (0,1), (0,-1)
 foci (0, sqrt(17)), (0, -sqrt(17))
 asymptotes $y = \pm \frac{1}{4}x$
 $d^2 = 1 + 16 = 17$
 $d = \sqrt{17}$

10. $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{4} = 1$ circle

 center (2, -1)
 vertices (2,1), (2,-1), (4,-1), (0,-1)
 foci the center
 no asymptotes

Give the equation for each of the following shapes. (hint: first draw a sketch of the given info)

11. Hyperbola with x-intercepts at ± 3 and foci at (-5, 0) and (5, 0).

 Foci $(x-5)^2 - (y-0)^2 = 1$

12. Ellipse with a horizontal axis of 16, a vertical axis of 20, centered at (-1, 5).
 $(x-1)^2 + (y-5)^2 = 1$

13. Hyperbola with a horizontal axis of 16, a vertical axis of 20, centered at (-1, 5).
 doesn't say about direction... You choose!
 $\frac{(x+1)^2}{64} - \frac{(y-5)^2}{100} = 1$

14. Ellipse with center at origin, vertex at (0, 4) and co-vertex at (2, 0).

 $\frac{x^2}{4} + \frac{y^2}{16} = 1$

15. Hyperbola with vertices at (0, ± 3) and foci at (0, ± 6).

 center (0,0) vertical y^2 first
 $d^2 = a^2 + b^2$
 $b^2 = a^2 + 3^2$
 $6^2 = 3^2 + b^2$
 $36 = 9 + b^2$
 $b^2 = 27$
 $\frac{y^2}{9} - \frac{x^2}{27} = 1$

16. Hyperbola with center at (1, -2), focus at (4, -2), and vertex at (3, -2).

 horizontal x^2 first
 $d^2 = a^2 + b^2$
 $9 = a^2 + b^2$
 $3^2 = 1^2 + b^2$
 $9 = 1 + b^2$
 $b^2 = 8$
 $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{8} = 1$

17. An arch of a bridge has the shape of the top half of an ellipse. The arch is 40 ft wide and 12 ft high at the center. (A) Find the equation of the complete ellipse. (B) Find the height of the arch 10 ft from the center of the bottom.

A. $\frac{x^2}{20^2} + \frac{y^2}{12^2} = 1$ B. $\frac{(10)^2}{400} + \frac{y^2}{144} = 1$ $\sqrt{y^2} = \sqrt{144 - 144 \cdot \frac{1}{4}} = \sqrt{144 \cdot \frac{3}{4}} = \sqrt{3 \cdot 144} = 12 \cdot \sqrt{3} \approx 20.78$

18. The Elliptical chamber in the U.S. Capitol Building is 110 ft. long and 22 ft. wide. (A) Write the equation that models the shape of the room. (B) President John Quincy Adams discovered that he could overhear the conversations of opposing party leaders who were near the focus at the right side of the chamber if he situated his desk at the focus at the left side of the chamber. What were the coordinates of his desk's position?

A. $\frac{x^2}{256} + \frac{y^2}{3025} = 1$
 or $\frac{x^2}{3025} + \frac{y^2}{256} = 1$
 B. $d = 52.621$
 (0, 52.621) or (52.621, 0)
 or 2.379 ft from the vertex wall

Name _____ Hour _____ Date _____

Circles Practice

Find the Center and Radius for each.

1. $x^2 + y^2 = 49$

Center: (,)

Radius: r =

2. $x^2 + y^2 = 324$

3. $(x+2)^2 + (y-3)^2 = 183$

Center: (,)

Radius: r =

4. $(x+7)^2 + (y+8)^2 = 64$

5. $(x-10)^2 + (y+9)^2 = 36$

Center: (,)

Radius: r =

6. $(x+5)^2 + (y-10)^2 = 9$

Write the circle equation given the desired center and radius

7. (0,-2) and r = 11

8. (-14, 3) and r = 2

Challenge! For the following, You will need to complete the square first, if you know how to do it, go ahead and try, if not wait until we learn it in class, you will need to factor and complete the square for both the x's and also for the y's. Then Determine Center and radius.



9. $x^2 + y^2 + 24x + 10y + 160 = 0$

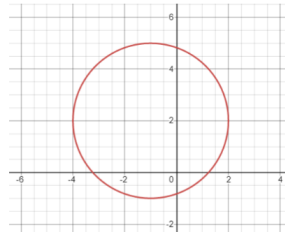
10. $364 + 28y + y^2 + x^2 = -26x$

11. $-6x = -x^2 + 32y - 264 - y^2$

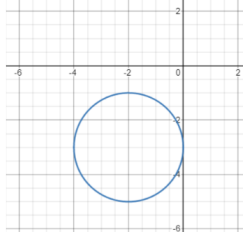
12. $-6x + x^2 = 97 + 10y - y^2$

From the Graphs determine the center and radius and write the Equation of each circle.

13. $(x \quad)^2 + (y \quad)^2 = \quad$

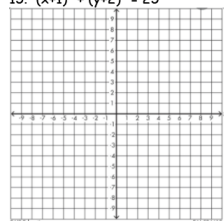


14. _____

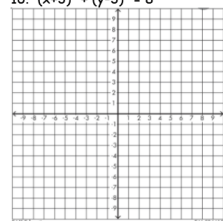


Graph the following circles Using the center and the radius up, down, left and right.

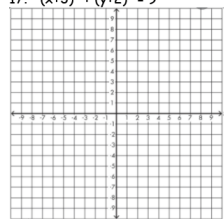
15. $(x+1)^2 + (y+2)^2 = 25$



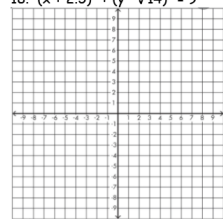
16. $(x+3)^2 + (y-3)^2 = 8$



17. $(x+3)^2 + (y+2)^2 = 9$



18. $(x + 2.5)^2 + (y - \sqrt{14})^2 = 9$



Answers

1. $(0,0) r = 7$

3. $(-2,3) r = 13.5$

5. $(-10,-9) r = 6$

7. $(0,-2) r = 11$

9. $(-13,-14) r = 1$

11. $(3,16) r = 1$

13. $(-1,2) r = 3$

15. $(-1,-2) r = 5$

17. $(-3,-2) r = 3$

2. $(0,0) r = 18$

4. $(-7,-8) r = 8$

6. $(-3,10) r = 3$

8. $(14,2) r = 2$

10. $(-12,-5) r = 3$

12. $(3,5) r = 11.4$

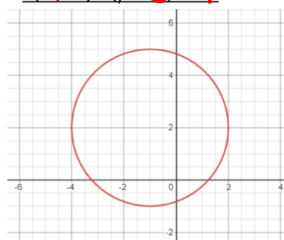
14. $(-2,-3) r = 2$

16. $(-3,3) r = 2.8$

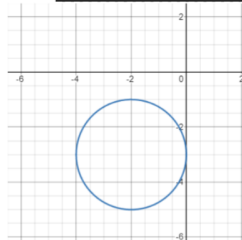
18. $(6, 5) r = 4$

From the Graphs determine the center and radius and write the Equation of each circle.

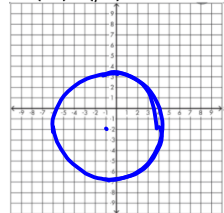
13. $(x+1)^2 + (y-2)^2 = 9$



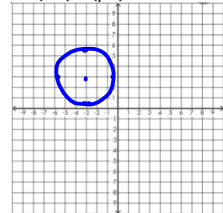
14. $(x+2)^2 + (y+3)^2 = 4$



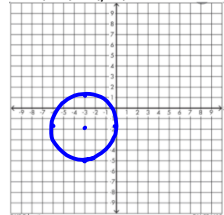
15. $(x+1)^2 + (y+2)^2 = 25$



16. $(x+3)^2 + (y-3)^2 = 8$



17. $(x+3)^2 + (y+2)^2 = 9$



18. $(x-6)^2 + (y-5)^2 = 16$

