Chapter 9 conic sections

Your name

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Notes

Completing the Square

https://www.youtube.com/watch?v=YWY2RNzXko4

completing the square

Step 1:   Move C to the other side. Leave a space for the new C.

Step 2:   Divide the b value by 2

Step 3:   Square answer from step 2 and add to both sides
of the equation

New $c = \left(\frac{b}{2}\right)^2$

Step 4:   Factor the left side of the equation to $\left(x + \frac{b}{2}\right)^2$

Step 5:   Combine like terms on the right side of the equation

Step 6:   Solve the equation

Parabola

$$x^2 + 8x + 15 = 0$$

$$-15 \quad -15$$

$$x^2 + 8x + 16 = -15 + 16$$

$$\left(\frac{8}{2}\right)^2 = 16$$

$$(x+4)(x+4) = 1$$

$$(x+4)^2 = 1$$

$$\to f(x) = (x+4)^2 - 1$$

$$\text{vertex} : (-4, -1)$$

Completing the Square to: **Change into circle form.....**

Sketch the graph of $x^2 + y^2 + 6x - 4y - 12 = 0$.
$$+12 \quad +12$$

In order to sketch this graph, we will need the center and radius. We need to put this into standard form. In order to put this into standard form, we must complete the square TWICE.

Circle

$$1x^2 + 1y^2$$

$$x^2 + 6x + 9 + y^2 - 4y + 4 = 12 + 9 + 4$$

$$\left(\frac{6}{2}\right)^2 = (3)^2 = 9 \qquad \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

$$(x+3)(x+3) + (y-2)(y-2) = 25$$

$$(x+3)^2 + (y-2)^2 = 25 = r^2$$

$$C : (-3, 2) \qquad\qquad r = 5$$

You try……

Put into standard form: $x^2 + y^2 - 6x - y = -1/4$.

$$X^2 - 6x + 9 + y^2 - y + \frac{1}{4} = \frac{-1}{4} + 9 + \frac{1}{4}$$

$$(X-3)^2 + \left(y - \frac{1}{2}\right)^2 = 9$$

center: $\left(3, \frac{1}{2}\right)$ radius: 3

Completing the Square

Number in front

Step 1:   Move C to the other side.

Step 2:   Factor out lead coefficient, Leave a space for the new C.

Step 3:   Divide the new b value by 2

Step 4:   Square answer from step 2, add it inside the parentheses on one side and add its real value to other other side with the constant

New c $= \left(\frac{b}{2}\right)^2$ but on the other side $a \cdot \left(\frac{b}{2}\right)^2$

Step 5:   Factor the left side of the equation to

Step 6:   Combine like terms on the right side of the equation

Step 7:   Set equation equal to zero.
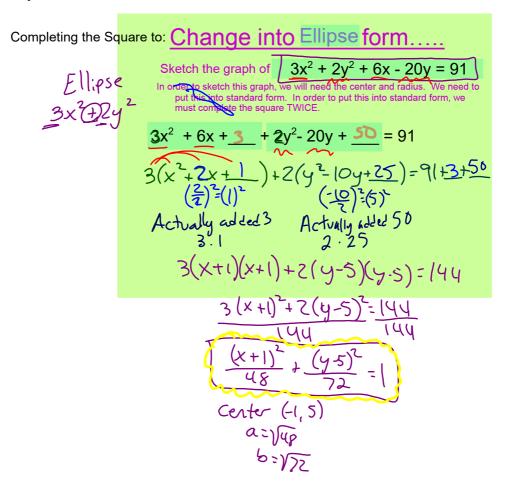
Parabola

$$2x^2 - 20x + 50 = 0$$

$$2(x^2 - 10x + 25) = -50 + 50$$

$$\left(\frac{-10}{2}\right)^2 = (-5)^2$$

Actually added 50

$2 \cdot 25$

$$2(x-5)(x-5) = 0$$

$$2(x-5)^2 = 0$$

Completing the Square to: **Change into Ellipse form.....**

Ellipse

$\underline{3x^2 \oplus 2y^2}$

Sketch the graph of $3x^2 + 2y^2 + 6x - 20y = 91$

In order to sketch this graph, we will need the center and radius. We need to put this into standard form. In order to put this into standard form, we must complete the square TWICE.

$3x^2 + 6x + \underline{3} + 2y^2 - 20y + \underline{50} = 91$

$3(x^2 + 2x + \underline{1}) + 2(y^2 - 10y + \underline{25}) = 91 + \underline{3} + \underline{50}$

$\left(\frac{2}{2}\right)^2 = (1)^2 \qquad \left(\frac{-10}{2}\right)^2 = (5)^2$

Actually added 3   Actually added 50
$3 \cdot 1$ $2 \cdot 25$

$3(x+1)(x+1) + 2(y-5)(y-5) = 144$

$\dfrac{3(x+1)^2 + 2(y-5)^2}{144} = \dfrac{144}{144}$

$\dfrac{(x+1)^2}{48} + \dfrac{(y-5)^2}{72} = 1$

Center $(-1, 5)$

$a = \sqrt{48}$

$b = \sqrt{72}$

Completing the Square to: **Change into Hyperbola Form**

Sketch the graph of $4x^2 - 5y^2 - 8x + 20y = 416$

We will have to factor out the negative and the coefficient, switching the signs!

$4x^2 - 8x + \underline{4} - 5y^2 + 20y + \underline{-20} = 416$ ☆

$4(x^2 - 2x + \underline{1}) - 5(y^2 - 4y + \underline{4}) = 416 + \underline{4} + \underline{-20}$

$\left(\frac{-2}{2}\right)^2 = (-1)^2 \qquad \left(\frac{-4}{2}\right)^2 = (-2)^2$

$4(x-1)^2 - 5(y-2)^2 = 400$

$\dfrac{4(x-1)^2}{400} - \dfrac{5(y-2)^2}{400}$

$\dfrac{(x-1)^2}{100} - \dfrac{(y-2)^2}{80} = 1$

C: $(1, 2)$ Vertices: $(11, 2) (-9, 2)$

$a = 10$

$b = \sqrt{80} \approx 8.9$

9. $(-12,-5)$ r = 3   10. $(-13,-14)$ r = 1
11. $(3,16)$ r = 1   12. $(3,5)$ r = 11.4

For the following, You will need to complete the square first, if you know how to do it, go ahead and try, if not wait until we learn it in class. Then Determine Center and radius.

9. $x^2 + y^2 + 24x + 10y + 160 = 0$

$$x^2 + 24x + \underline{144} + y^2 + 10y + \underline{25} = -160 + \underline{144} + \underline{25}$$

$$(x+12)^2 + (y+5)^2 = 9$$

10. $364 + 28y + y^2 + x^2 = -26x$

11. $-6x = -x^2 + 32y - 264 - y^2$

12. $-6x + x^2 = 97 + 10y - y^2$

$-10y - y^2 \qquad -10y + y^2$

$$x^2 - 6x + \underline{9} + y^2 - 10y + \underline{25} = 97$$

$\left(\frac{-6}{2}\right)^2 = 9 \qquad \left(\frac{-10}{2}\right)^2 = 25 \qquad \begin{array}{l} +9 \\ +25 \end{array}$

$$(x-3)^2 + (y-5)^2 = 131$$

$$C : (3,5) \qquad r = \sqrt{131}$$

10. $4x^2 + y^2 - 8x + 4y = 8$

Equation _____

vertices:_____ co-vertices:_____ foci:_____

11. $x^2 + 4y^2 - 18x - 8y = -81$

Equation _____

vertices:_____ co-vertices:_____ foci:_____

12. $9x^2 + 4y^2 - 144x - 8y = -544$

Equation _____

vertices:_____ co-vertices:_____ foci:_____

10. $4x^2 - 8x + y^2 + 4y = 8$
$4(x^2 - 2x + \underline{1}) + (y^2 + 4y + \underline{4}) = 8 + \underline{4} + \underline{4}$
$\dfrac{4(x-1)^2}{16} + \dfrac{(y+2)^2}{16} = \dfrac{16}{16}$ $\dfrac{(x-1)^2}{4} + \dfrac{(y+2)^2}{16} = 1$

11. $x^2 - 18x + 4y^2 - 8y = -81$
$(x^2 - 18x + \underline{81}) + 4(y^2 - 2y + \underline{1}) = -81 + \underline{81} + \underline{4}$
$(x-9)^2 + 4(y-1)^2 = 4$ $\dfrac{(x-9)^2}{4} + \dfrac{(y-1)^2}{1} = 1$

12. $9x^2 - 144x + 4y^2 - 8y = 544$
$9(x^2 - 36x + \underline{324}) + 4(y^2 - 2y + \underline{1}) = -544 + \underline{2916} + \underline{4}$
$9(x-18)^2 + 4(y-1)^2 = 2376$ $\dfrac{(x-18)^2}{264} + \dfrac{(y-1)^2}{594} = 1$