Your Name

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Notes

Solving Polynomial Equations

Applications

Graphing Rational Functions

Using Synthetic Division

Graph: ..../.... make sure to label the slant asymptote

$$f(x) = \frac{2x^3 + 11x^2 - 17x + 28}{x^2 + 5x - 14}$$

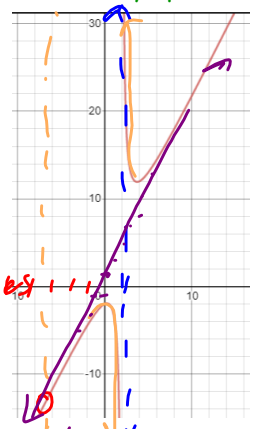
$$f(x) = \frac{(x+7)(2x^2 - 3x + 4)}{(x+7)(x-2)}$$

Canceled out factors make holes

Hole at x = -7

Vertical Asymptote at x=2

Slant Asymptote at y = $2x+1$

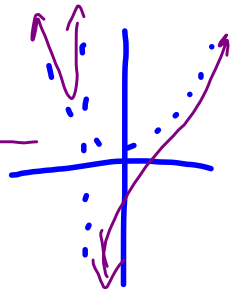


Divide leftover polynomials you may need long division

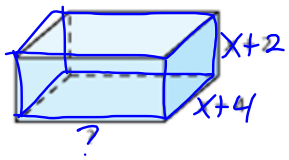
$2 \quad 2x^2 - 3x + 4$

$4 \quad 2$

$y = 2x + 1 \quad 6$



MATHEMATICAL CONNECTIONS The volume V of the rectangular prism is given by $V = 2x^3 + 17x^2 + 46x + 40$. Find an expression for the missing dimension.



$V = BH$ (Volume of Prisms = Area of Base × 3D height)

$V = l \cdot w \cdot h$

$V_{\square} = bhH$

$V_{\square} = \frac{1}{2}bhH$

$V_{\triangle} = \frac{1}{3}BH$ (Volume of Pyramid)

$V = 2x^3 + 17x^2 + 46x + 40$

$V = b \cdot h \cdot H$

$V = (2x+5)(x+4)(x+2)$

$x = -4$ | -4 | 2 | 17 | 46 | 40

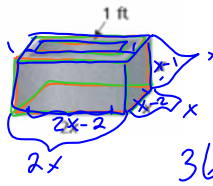
		-8	-36	-40
	2	9	10	0

$(x+4)(2x^2+9x+10)$ -2

		-4	-10
	2	5	0

$(x+4)(x+2)(2x+5)$

MODELING WITH MATHEMATICS You are designing a marble basin that will hold a fountain for a city park. The sides and bottom of the basin should be 1 foot thick. Its outer length should be twice its outer width and outer height. What should the outer dimensions of the basin be if it is to hold 36 cubic feet of water?



$V = b \cdot h \cdot H$

$36 = (2x-2)(x-2)(x-1)$

$36 = (2x-2)(x^2-3x+2)$

$36 = 2x^3 - 8x^2 + 10x - 4$

$-36 \quad -36$

$0 = 2x^3 - 8x^2 + 10x - 40$

$40: 1, 2, 4, 5, 8, 10, 20, 40$

$2: 1, 2$

$\pm 1, \pm\frac{1}{2}, \pm 2, \pm 4, \pm\frac{4}{2}$

$x = 4$ | 2 | -8 | 10 | -40

	8	0	40
$2x^2+0x+10$			0

$l = 2x = 2(4) = 8$

$w = x = 4$

$h = x = 4$

8 ft × 4 ft × 4 ft

$2x^2 + 10 = 0$

$2x^2 = -10$

$x^2 = -5$ (don't want imaginary #s for x)

$x = \pm i\sqrt{5}$

Factor and Solve: $2x^4 + x^3 + 8x^2 + 4x = 0$

$x(2x^3 + x^2 + 8x + 4) = 0$

4: 1, 2, 4

2: 1, 2

$\pm 1, \pm\frac{1}{2}, \pm 2, \pm 4$

$x(2x+1)(x-2i)(x+2i) = 0$

$x = 0 \quad x = -\frac{1}{2} \quad x = 2i \quad x = -2i$

Turn into a Factor

$x = -\frac{1}{2}$

$+\frac{1}{2} \quad +\frac{1}{2}$

$2\left(x + \frac{1}{2}\right) = 0$ 2

$(2x+1) = 0$

$x = -\frac{1}{2}$ | 2 1 8 4

↓ -1 0 -4

2 0 8 0

$2x^2 + 0x + 8 = 0$

$2x^2 + 8 = 0$

$2x^2 = -8$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$x = 2i$

$-2i \quad -2i$

$(x - 2i) = 0$

$x = -2i$

$+2i \quad +2i$

$(x + 2i) = 0$

WRITING EQUATIONS Write a polynomial function g of least degree that has rational coefficients, a leading coefficient of 1, and the zeros x=0, x = 7/3, x= -1

$x\left(x - \frac{7}{3}\right)(x + 1) = 0$

$x\left(x^2 + x - \frac{7}{3}x - \frac{7}{3}\right) = 0$

$x\left(x^2 - \frac{4}{3}x - \frac{7}{3}\right) = 0$

$x^3 - \frac{4}{3}x^2 - \frac{7}{3}x = 0$

WRITING EQUATIONS Write a polynomial function g of least degree that has rational coefficients, a leading coefficient of 1, and the zeros $-2+$ x= -3 - √5 and x=2 + i√3

Assume conjugate solutions

$x = -3 + \sqrt{5}$ $\quad x = 2 - i\sqrt{3}$

$+3 \quad +3 \quad -2 \quad -2$

$x + 3 = \sqrt{5} \quad x - 2 = -i\sqrt{3}$

$-\sqrt{5} \quad -\sqrt{5} \quad +i\sqrt{3} \quad +i\sqrt{3}$

$(x + 3 - \sqrt{5}) = 0 \quad (x - 2 + i\sqrt{3}) = 0$

Distribute conjugates together first

$0 = (x+3+\sqrt{5})(x+3-\sqrt{5})(x-2-i\sqrt{3})(x-2+i\sqrt{3})$

$x^2 + 3x - \sqrt{5}x + 3x + 9 - 3\sqrt{5} + \sqrt{5}x + 3\sqrt{5} - \sqrt{25}$

$x^2 - 2x + i\sqrt{3}x - 2x + 4 - 2i\sqrt{3} - i^2\sqrt{9}$ $(i^2 = -1)$

$+ 3$

$(x^2 + 6x + 4)(x^2 - 4x + 7)$

$x^4 - 4x^3 + 7x^2 + 6x^3 - 24x^2 + 42x + 4x^2 - 16x + 28$

$f(x) = x^4 + 2x^3 + 17x^2 + 26x + 28$

MODELING WITH MATHEMATICS During a 10-year period, the amount (in millions of dollars) of athletic equipment E sold domestically can be modeled by $E(t) = -20t^3 + 252t^2 - 280t + 21{,}614$, where t is in years.

a. Write a polynomial equation to find the year when about \$24,014,000,000 of athletic equipment is sold.

b. List the possible whole-number solutions of the equation in part (a). Consider the domain when making your list of possible solutions.

c. Use synthetic division to find when \$24,014,000,000 of athletic equipment is sold.

$E(t) = -20t^3+252t^2-280t+21614$

$24014 = -20t^3+252t^2-280t+21614$

$-24014 \qquad -24014$

$0 = -20t^3+252t^2-280t-2400$

$\frac{2400}{20} = \pm 5, \pm 10$

$5 \mid -20 \quad 252 \quad -280 \quad -2400$

$\downarrow \quad -100 \quad 760 \quad 2400$

$0 \mid -20 \quad 152 \quad 480 \mid 0$

$\downarrow \quad -200 \quad -480$

$-20 \quad -48 \mid 0$

$-20x - 48 = 0$

$\frac{-20x}{-20} = \frac{48}{-20}$

$x = -2.4$

negative years doesn't make sense

year

$x = 5$

year

$x = 10$

COMPARING METHODS You are taking a test where calculators are not permitted. One question asks you to evaluate $g(7)$ for the function $g(x) = x^3 - 7x^2 - 4x + 28$. You use the Factor Theorem and synthetic division and your friend uses direct substitution. Whose method do you prefer? Explain your reasoning.

$g(7)$ means evaluate the function $g(x)$ when $x = 7$ and get the y coordinate $(x, g(x))$

$g(7) = (7)^3 - 7(7)^2 - 4(7) + 28$ use synthetic division

plug in calculator

or $7 \mid 1 \quad -7 \quad -4 \quad 28$

$\downarrow \quad 7 \quad 0 \quad -28$

$1 \quad 0 \quad -4 \mid 0$

$g(7) = 343 - 343 - 28 + 28$

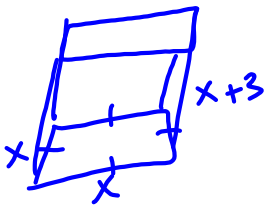
$g(7) = 0$

$g(7) = 0$

$(7, 0)$

Homework Key

1. x = 0, x = 3, and x = 6.5
2. 4cm, by 4cm, by 7cm
3. 4ft by 4ft by 8ft
4. $g(x) = x^4 - 2x^3 - 20x^2 + 39x - 21$
5. x = 2, b = 2ft, h = 2ft, and H = 3ft
6. x = 67.8, Thus in the year 1958
7. in the 3rd year and in the 9th year
8. x = 4.2577



$V = l \cdot w \cdot h$

$112 = x \cdot x \cdot (x+3)$

$112 = x^2(x+3)$

$112 = x^3 + 3x^2$

$-112 \quad -112$

$0 = 1x^3 + 3x^2 - 112$

$1^{x^3} \; 3^{x^2} \; 0^{x} \; -112$

$l = x = 4\,cm$

$w = x = 4\,cm$

$h = x+3 = 7\,cm$

112: 1, 2, 4

Desmos

4

$Y = 4$

$x = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$

$V = 2x^3 - 19x^2 + 39x$

$V = x(2x^2 - 19x + 39)$

$x = 0$

$x = 3$

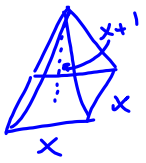
$\frac{39}{2}$

	2	-19	39
↓		6	-84
	2	-13	0

$2x + -13 = 0$

Solve for

$x = \frac{13}{2} = 6.5$



$V = \frac{1}{3}BH$

$V = \frac{1}{3}(b \cdot h)H$

$V = \frac{1}{3}(x)(x)(x+1)$

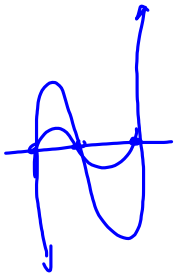
$y = \frac{1}{3}x^3 + \frac{1}{3}x^2$

$-4 \qquad -4$

$3(0 = \frac{1}{3}x^3 + \frac{1}{3}x^2 - 4)$

$0 = 1x^3 + 1x^2 - 12$

$1 \quad 1 \quad 0 \quad -12$



$1x^2 + 3x + 6$

$a \quad b \quad c$

Discriminant: $\sqrt{b^2 - 4ac}$

$(3)^2 - 4(1)(6)$

$9 - 24$

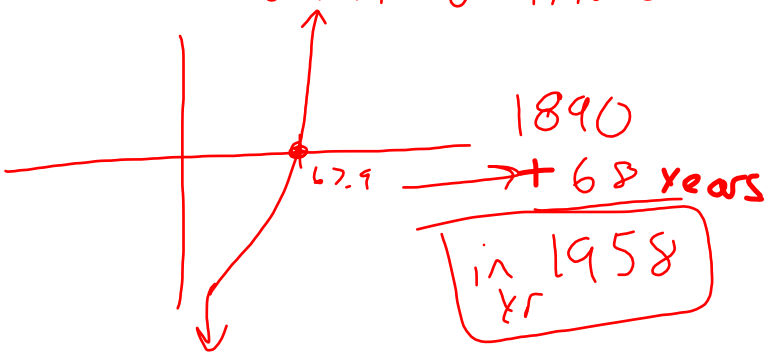
$P =$

$722 = .004t^3 - 0.24t^2 + 4.9t + 243$

$-722 \qquad\qquad -722$

$1000\left(0 = 0.004t^3 - 0.24t^2 + 4.9t - 479\right)$

$0 = 4t^3 - 240t^2 + 4900t - 479000$



in 1958 yr