

Objective: Solving Systems of Equations in Three Variables

<https://bit.ly/2NgTPxm>

- Set up a system of equations in three variables to model real-life behavior
- Solving system of three variable equations through elimination method

1. Uncle Scrooge claims he has a bag of 30 coins containing nickels, dimes and quarters. The total value of the coins is \$2.55. There are twice as many nickels as there are dimes. How many of each type of coin does he have?

Study

Define Variables:

n: # of nickels

d: # of dimes

q: # of quarters

Plan

The 3 Equations are:

A) $n + d + q = 30$

B) $0.05n + 0.10d + 0.25q = 2.55$

C) $n = 2d$
 $n = 2(9)$
 $n = 18$

Act
Solve

Step 1

(A) $(2d) + d + q = 30$

(D) $3d + q = 30$

(E) $q = 30 - 3d$

Step 2

(E) $0.20d + 0.25(30 - 3d) = 2.55$
 $0.20d + -0.75d + 7.5 = 2.55$
 $-0.55d + 7.5 = 2.55$

Step 3

(E) $q = -3(9) + 30$
 $q = -27 + 30$
 $q = 3$

Reflect

8 nickels
 4 dimes
 3 quarters

2. Cornstock sold a total of 440 tickets for \$3940. Each regular ticket cost is \$5, each premium ticket is \$15 and each elite ticket cost is \$25. The number of regular tickets was three times the number of premium and elite tickets combined. How many of each ticket were sold?

Study

Define Variables:

x: # of regular tickets sold

y: # of premium tickets sold

z: # of elite tickets sold

Plan

The 3 Equations are:

(A) $x + y + z = 440$

(B) $5x + 15y + 25z = 3940$

(C) $x = 3(y + z)$
 $x = 3y + 3z$

(D) $0 = -x + 3y + 3z$
 $0 = -5x + 15y + 15z$

Act
Solve

Step 1

(A) $x + y + z = 440$

(C) $-x + 3y + 3z = 0$

(D) $-4y + 4z = 440$

(E) $-40y - 40z = 4400$

(E) $30y + 40z = 3940$

Reflect

3. The sum of three integers is 40. Three times the smaller integer is equal to the sum of the others. Twice the larger is equal to 8 more than the sum of the others. Find the integers.

Study

Define Variables:

X: first smaller number

Y: second middle number

Z: third larger number

Plan

The 3 Equations are:

(A) $x + y + z = 40$

(B) $3x = y + z$

(C) $2z = (x + y) + 8$

(D) $-x - y + 2z = 8$

Act
Solve

Step 1

(B) $-3x + y + z = 0$

(C) $-x - y + 2z = 8$

(D) $-4x + 3z = 8$

Step 2

(E) $3z - 4z = 8$
 $-z = 8$
 $z = -8$

Pair up to cancel variable

(A) $x + y + z = 40$

(C) $-x - y + 2z = 8$

(E) $3z = 48$

Solve 2 variables system

(D) $-4x + 3(16) = 8$
 $-4x + 48 = 8$
 $-4x = -40$
 $x = 10$

Reflect

the three integers are: 10, 14, 16

Step 3: Plug into Original
 (A) $(10) + y + (16) = 40$
 $y + 26 = 40$
 $y = 14$

4. The sum of the angles A, B, and C of a triangle is 180° . Angle C is equal to the sum of the other two angles. Five times angle A is equal to the sum of angle C and B. Find the angles.

Study

Define Variables:

X: measure of Angle A

Y: measure of Angle B

Z: measure of Angle C

Plan

The 3 Equations are:

(A) 1) $x + y + z = 180$

(B) 2) $z = x + y$
 $-x - y + z = 0$

(C) 3) $5x = z + y$
 $5x - y - z = 0$

Act

Solve

Step 1

(A) $x + y + z = 180$
 (B) $-x - y + z = 0$

 $2z = 180$
 $\frac{2z}{2} = \frac{180}{2}$
 (D) $z = 90^\circ$

(A) $x + y + z = 180$
 (C) $5x - y - z = 0$

 $6x = 180$
 $\frac{6x}{6} = \frac{180}{6}$
 (E) $x = 30^\circ$

Step 3

(A) $(30) + y + (90) = 180$
 $y + 120 = 180$
 $y = 60^\circ$

Reflect

$m\angle A = 30^\circ$
 $m\angle B = 60^\circ$
 $m\angle C = 90^\circ$

5. A parabola passes through three points $(-2, 11)$, $(-1, 4)$, and $(1, 2)$. Use these points and $y = ax^2 + bx + c$ to construct a system of three linear equations in terms of a, b, and c and solve it.

Study

Define Variables:

a: the coefficient a for x^2

b: the coefficient b for x

c: the constant c term

Plan

The 3 Equations are:

1) $11 = a(-2)^2 + b(-2) + c$

(A) $11 = 4a - 2b + c$

2) $4 = a(-1)^2 + b(-1) + c$

(B) $4 = a - b + c$

(C) $2 = a(1)^2 + b(1) + c$

(D) $2 = a + b + c$

(E) $4 = 2a + 2b + 2c$

Act

Solve

Step 1

(A) $11 = 4a - 2b + c$
 (B) $4 = a - b + c$

 (C) $2 = a + b + c$
 (D) $(6 = 2a + 2c) \div 2$
 (E) $15 = 6a + 3c$

(D) $-18 = -6a - 6c$
 (E) $15 = 6a + 3c$

 $-3 = -3c$
 $\frac{-3}{-3} = \frac{-3c}{-3}$
 $1 = c$

Step 2

(E) $15 = 6a + 3(1)$
 $15 = 6a + 3$
 $-3 \quad -3$
 $12 = 6a$
 $\frac{12}{6} = \frac{6a}{6}$
 $2 = a$

Step 3

(C) $2 = (2) + b + (1)$
 $2 = 3 + b$
 $-3 \quad -3$
 $-1 = b$

Reflect

The Parabola that goes through these points is:

$y = 2x^2 - x + 1$

6. A parabola passes through three points $(-1, 7)$, $(1, -1)$ and $(2, -2)$. Use these points and $y = ax^2 + bx + c$ to construct a system of three linear equations in terms of a, b, and c and then solve the system.

Study

Define Variables:

a:

b:

c:

Plan

The 3 Equations are:

1)

2)

3)

Act

Reflect

The Parabola that goes through these points is:

2. Reg=330, Prem=46, Elite= 64
 4. A=30°, B=60°, C=90°
 6. a=1, b=-4, c=2 $y = x^2 - 4x + 2$