# Complex Numbers

Your Name

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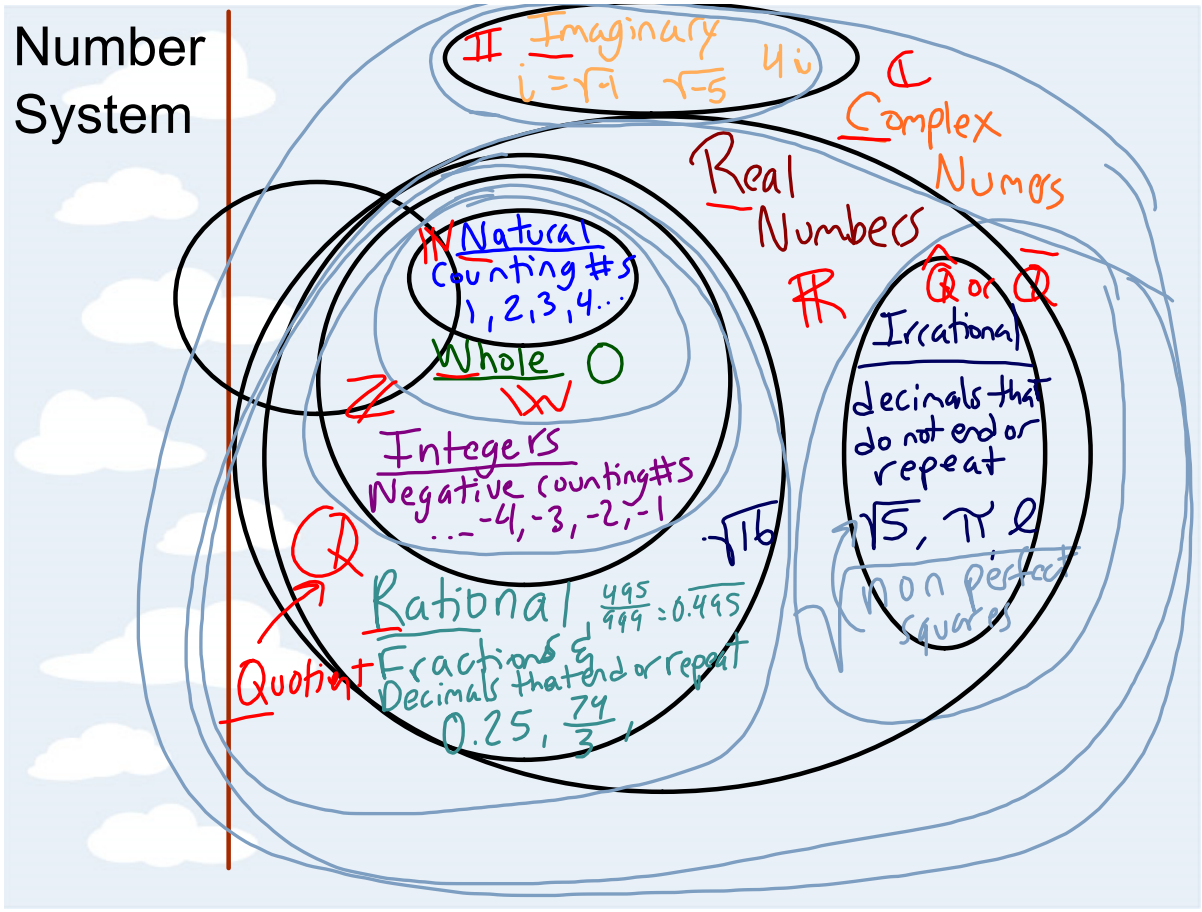
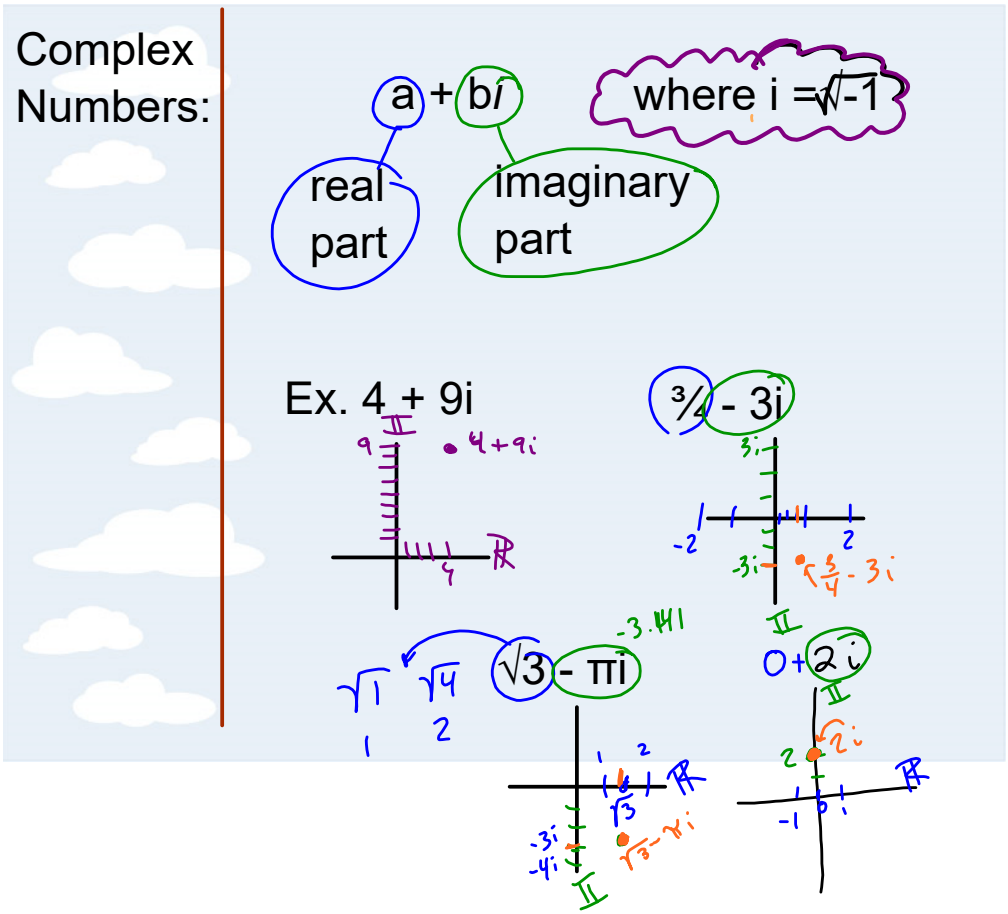
Notes

Objective: add, subtract, multiply and divide complex numbers to be able to find complex roots of polynomials

Life Lesson/Math Skill: To have total ability to find all possible zeros of any polynomial real and imaginary. Electricity and vectors uses complex numbers.


Number System
Imaginary
$i = \sqrt{-1}$ $\sqrt{-5}$ 4i
Complex Nums
Real Numbers
Natural
Counting #s
1, 2, 3, 4...
Whole
Integers
Negative counting #s
...-4, -3, -2, -1
Rational
Fractions & Decimals that end or repeat
0.25, 74/3, 495/999 = 0.495
Quotient
Irrational
decimals that do not end or repeat
√5, π, e
non perfect squares
√16


Complex Numbers:
a + bi
where $i = \sqrt{-1}$
real part
imaginary part
Ex. 4 + 9i
4 + 9i
¾ - 3i
$\sqrt{3} - \pi i$
-3.141
0 + 2i

Imaginary Powers

$i =$

$i^2 =$

$i^3 =$

$i^4 =$

$i^5 =$

$i^6 =$

Imaginary Powers

$i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = i^2 \cdot i^1 = (-1) \cdot i = -i$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$

$i^5 = i^4 \cdot i = (1)i = i$

$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$

$i^7 = -i$

$i^8 = 1$

it's a pattern

Simplifying Powers of i

break up exponent into multiples of 4 and leftovers

$i^9=$ $i^{18}=$ $i^{23}=$

$i^{43}=$ $i^{98}=$ $i^{52}=$

Simplifying Powers of i

break up exponent into multiples of 4 and leftovers

Use Fact: $i^4 = +1$

$i^9=$

$i^{18}=$

$i^{23}=$

$i^{43}=$

$i^{98}=$

$i^{52}=$

$i^{34}$ $i^{45}$ $i^{64}$

$i^{44} \cdot i^{1}$ $i^{56} \cdot i^{2}$ $(i^4)^{16}$

$(i^4)^{24} \cdot i^2$

## Simplifying $\sqrt{\text{negative \#s}}$

1
4
9
16
25
36
49
64
81
100

Can get a decimal

Break it up into $\sqrt{\text{real part}} \cdot \sqrt{-1}$ remember $i = \sqrt{-1}$

$\sqrt{-36}$

$\sqrt{36} \cdot \sqrt{-1}$

$6i$

$\sqrt{-32}$

$\sqrt{32} \cdot \sqrt{-1}$

$\sqrt{16} \cdot \sqrt{2} \cdot \sqrt{-1}$

$4 \cdot \sqrt{2} \cdot i$

$4i\sqrt{2}$

$\sqrt{-75}$

$\sqrt{75} \cdot \sqrt{-1}$

$-\sqrt{25} \cdot \sqrt{3} \cdot \sqrt{-1}$

$-5i\sqrt{3}$

## Simplifying $\sqrt{\text{negative \#s}}$

Break it up into $\sqrt{\text{real part}} \cdot \sqrt{-1}$

$\sqrt{-36}$

$\sqrt{36} \cdot \sqrt{-1}$

$6i$

$\sqrt{-32}$

$\sqrt{16} \cdot \sqrt{2} \cdot \sqrt{-1}$

$4 \cdot \sqrt{2} \cdot i$

$4i\sqrt{2}$

$-\sqrt{-75}$

$x^2 + 49 = 0$

$-49 \quad -49$

$\sqrt{x^2} = \sqrt{-49}$

$x = \pm 7i$

$a(x-h)^2 + k = 0$

55. $4(x+3)^2 + 52 = 0$

$-52 \quad -52$

$\frac{4(x+3)^2}{4} = \frac{-52}{4}$

$\sqrt{(x+3)^2} = \sqrt{-13}$

$x + 3 = \pm i\sqrt{13}$

$-3 \quad -3$

$x = -3 \pm i\sqrt{13}$

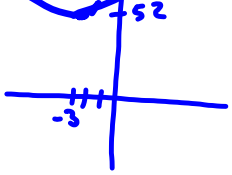
$x = -3 + i\sqrt{13}$ and $x = -3 - i\sqrt{13}$

real imaginary

$x = i\sqrt{13} - 3$ and $x = -i\sqrt{13} - 3$

V(h,k)

V(-3, 52)



## Homework: Simplifying Imaginary Roots

**In Exercises 5–12, find the square root of the number.** *(See Example 1.)*

5. $\sqrt{-36}$ — $\sqrt{36}\cdot\sqrt{-1}$ Separate; $6i$ Take radical
6. $\sqrt{-64}$ — $\sqrt{64}\cdot\sqrt{-1}$; $8i$
7. $\sqrt{-18}$ — $\sqrt{18}\cdot\sqrt{-1}$; $\sqrt{9}\cdot\sqrt{2}\cdot\sqrt{-1}$; $3i\sqrt{2}$
8. $\sqrt{-24}$

9. $2\sqrt{-16}$
10. $-3\sqrt{-49}$
11. $-4\sqrt{-32}$ — Separate $-4\cdot\sqrt{32}\cdot\sqrt{-1}$; Split $-4\cdot\sqrt{16}\cdot\sqrt{2}\cdot\sqrt{-1}$; Take Radical $-4\cdot 4\cdot i\cdot\sqrt{2}$; multiply $-16i\sqrt{2}$
12. $6\sqrt{-63}$

**In Exercises 49–54, solve the equation. Check your solution(s).** *(See Example 6.)*

49. $x^2+9=0$ — $-9\ -9$; $\sqrt{x^2}=\sqrt{-9}$; $x=\pm\sqrt{9}\cdot\sqrt{-1}$; $x=3i$ and $x=-3i$
50. $x^2+49=0$
51. $x^2-4=-11$ — $+4\ +4$; $\sqrt{x^2}=\sqrt{-7}$; $x=\pm\sqrt{7}\cdot\sqrt{-1}$; $x=i\sqrt{7}$ and $x=-i\sqrt{7}$
52. $x^2-9=-15$
53. $2x^2+6=-34$ — $-6\ -6$; $\frac{2x^2}{2}=\frac{-40}{2}$; $\sqrt{x^2}=\sqrt{-20}$; $x=\pm\sqrt{20}\cdot\sqrt{-1}$; $x=\pm\sqrt{4}\cdot\sqrt{5}\cdot\sqrt{-1}$; $x=\pm 2i\sqrt{5}$ and $x=-2i\sqrt{5}$
54. $x^2+7=-47$

$(3+7i)(4+2i)$; $12+6i+28i+14i^2$; $14(-1)$; $12+6i+28i-14$; $-2+34i$

$9+5+5i+7i$; $4+2i$

## Homework: Simplifying Imaginary Roots

**In Exercises 5–12, find the square root of the number.** *(See Example 1.)*

5. $\sqrt{-36}$ — $\sqrt{36}\cdot\sqrt{-1}$; $6i$; $=\sqrt{9}\cdot\sqrt{2}\cdot\sqrt{-1}$; $3i\sqrt{2}$; $4.25i$
6. $\sqrt{-64}$ — $\sqrt{-1}\cdot 64$ Separate; $\sqrt{64}\cdot\sqrt{-1}$ Take radical; $8i$
7. $\sqrt{-18}$
8. $\sqrt{-24}$

9. $2\sqrt{-16}$
10. $-3\sqrt{-49}$ — $i^2=1$; $2^2=4$; $3^2=9$; $4^2=16$
11. $-4\sqrt{-32}$ — $-4\cdot\sqrt{32}\cdot\sqrt{-1}$ Separate; $-4\cdot\sqrt{4}\cdot\sqrt{8}\cdot\sqrt{-1}$; Split $-4\cdot\sqrt{4}\cdot\sqrt{4}\cdot\sqrt{2}\cdot\sqrt{-1}$; Take Radical $-4\cdot 2\cdot 2\cdot\sqrt{2}\cdot i$; multiply $-16i\sqrt{2}$
12. $6\sqrt{-63}$ — $6\cdot\sqrt{-1}\cdot 63$; $6\cdot\sqrt{63}\cdot\sqrt{-1}$; $6\cdot\sqrt{9}\cdot\sqrt{7}\cdot\sqrt{-1}$; $6\cdot 3\sqrt{7}\cdot i$; $18i\sqrt{7}$

**In Exercises 49–54, solve the equation. Check your solution(s).** *(See Example 6.)*

49. $x^2+9=0$
50. $x^2+49=0$
51. $x^2-4=-11$

52. $x^2-9=-15$
53. $2x^2+6=-34$
54. $x^2+7=-47$ — $-7\ -7$; $\sqrt{x^2}=\sqrt{-54}$; $x=\pm\sqrt{54}\cdot\sqrt{-1}$; $x=\pm\sqrt{9}\cdot\sqrt{6}\cdot\sqrt{-1}$; $x=\pm 3\sqrt{6}\cdot i$; $x=\pm 3i\sqrt{6}$; $x\approx\pm 7.35i$; $x=3i\sqrt{6}$ and $x=-3i\sqrt{6}$

# Homework: Simplifying Imaginary Roots

**In Exercises 5–12, find the square root of the number.** *(See Example 1.)*

5. $\sqrt{-36}$
$\sqrt{36}\cdot\sqrt{-1}$
$6i$

6. $\sqrt{-64}$
$8i$

7. $\sqrt{-18}$
$3i\sqrt{2}$

8. $\sqrt{-24}$
$\sqrt{24}\cdot\sqrt{-1}$
$\sqrt{4\cdot6}\cdot\sqrt{-1}$
$2i\sqrt{6}$

9. $\sqrt{-16}$
$2\sqrt{16}\cdot\sqrt{-1}$
$2\cdot4\cdot\sqrt{-1}$
$8i$

10. $-\sqrt{-49}$
$-3\cdot7i$
$-21i$

11. $-\sqrt{-32}$
$-16i\sqrt{2}$

12. $\sqrt{-63}$
$6\cdot\sqrt{9}\cdot\sqrt{7}\cdot\sqrt{-1}$
$18i\sqrt{7}$

**In Exercises 49–54, solve the equation. Check your solution(s).** *(See Example 6.)*

49. $x^2 - 9 = 0$
$-9\ \ -9$
$\sqrt{x^2} = \sqrt{-9}$
$x = \pm\sqrt{9}\cdot\sqrt{-1}$
$x = \pm 3i$
$x = 3i$ and $x = -3i$

50. $x^2 - 49 = 0$
$x = 7i$
and
$x = -7i$

51. $x^2 - 4 = -11$
$+4\ \ +4$
$x^2 = -7$
$x = \pm i\sqrt{7}$

52. $x^2 - 9 = -15$
$x^2 = -6$
$x = \pm i\sqrt{6}$

53. $2x^2 - 6 = -34$
$-6\ \ -6$
$\frac{2x^2}{2} = \frac{-40}{2}$
$x^2 = -20$
$x = \pm\sqrt{4\cdot5\cdot-1}$
$\boxed{x = \pm 2i\sqrt{5}}$

54. $x^2 + 7 = -47$
$-7\ \ -7$
$x^2 = -54$
$x = \pm\sqrt{9\cdot6\cdot-1}$
$x = 3i\sqrt{6}$
and $x = -3i\sqrt{6}$