

Complex Number Operations

Your Name

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Notes

Let's begin with Stations!

Adding
Subtracting
Imaginary
Numbers

Combine the Real parts together and
combine the Imaginary parts together

$$\begin{aligned} & \underline{(9 + 5i)} + \underline{(-5 - 7i)} \\ & \underline{9} + \underline{-5} + \underline{5i} + \underline{-7i} \\ & \boxed{4 - 2i} \end{aligned}$$

$$\begin{aligned} & \text{distribute subtraction} \\ & \underline{(3 - 8i)} - \underline{(2 + 4i)} \\ & \underline{3} - \underline{2} + \underline{-8i} - \underline{4i} \\ & 1 + -12i \\ & \boxed{1 - 12i} \end{aligned}$$

Adding
Subtracting
Imaginary
Numbers

Combine the Real parts together and
combine the Imaginary parts together

$$(a+bi) + (c+di)$$

$$= (a+c) + (b+d)i$$

distribute subtraction

$$(a+bi) - (c+di)$$

$$= (a-c) + (b-d)i$$

ex. $(6-5i) + (2+3i)$

$$6+2 + -5i+3i$$

$$8 - 2i$$

ex. $(6-5i) - (2+3i)$

$$6-5i-2-3i$$

$$6-2 + -5i-3i$$

$$4 - 8i$$

Multiplying
Imaginary
Numbers

Distribute and combine like terms
and apply and substitute $i^2 = -1$

$$i = \sqrt{-1}$$

$$i^2 = (\sqrt{-1})^2$$

$$i^2 = -1$$

A. $-2.5i(8 - 9i)$

~~$(3-2i)(2+2i)$~~

$$(1+3i)(2+5i)$$

$$2+5i+6i+15i^2$$

$$2+11i+15(-1)$$

$$2+11i-15$$

$$-13+11i$$

a. $\frac{2}{5}i(10 - \frac{5}{2}i)$

b. $(\frac{1}{2} + 2i)(\frac{1}{2} - 2i)$

Multiplying
Imaginary
Numbers

Distribute and combine like terms
and apply and substitute $i^2 = -1$

ex. $7i(-3 - 4i)$

$$-21i - 28i^2$$

$$-21i - 28(-1)$$

$$\boxed{28 - 21i}$$

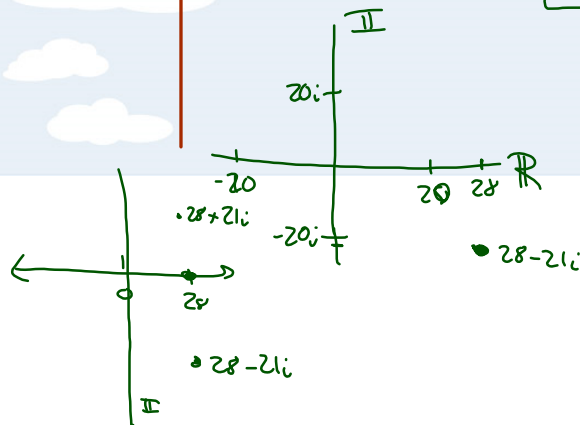
ex. $(6-5i)(2+3i)$

$$12 + 18i - 10i - 15i^2$$

$$12 + 18i - 10i - 15(-1)$$

$$12 + 15 + 18i - 10i$$

$$\boxed{27 + 8i}$$



Operating
on
Complex
Radicals

Express negative radicals
in terms of i first!

to combine
further
get decimal

$$\sqrt{-6} - 3\sqrt{-25}$$

$$\widehat{\sqrt{6}} \cdot \sqrt{-1} - 3\sqrt{25} \cdot \sqrt{-1}$$

$$i\sqrt{6} - 3 \cdot 5i$$

$$i\sqrt{6} - 15i$$

$$2.449i - 15i$$

$$\boxed{-12.551i}$$

$$3\sqrt{-7} (5\sqrt{8} + i)$$

$$3\sqrt{7} \cdot \sqrt{-1} (5\sqrt{4} \cdot \sqrt{2} + i)$$

$$\boxed{3i\sqrt{7}} (10\sqrt{2} + i)$$

$$30i\sqrt{14} + 3i^2\sqrt{7}$$

$$+ 3(-1)\sqrt{7}$$

$$\boxed{30i\sqrt{14} - 3\sqrt{7}}$$

or $-3\sqrt{7} + 30i\sqrt{14}$

$$\boxed{\text{or } -7.937 + 112.249i}$$

$$(3\sqrt{-12} + 9) - (4 - 8i)$$

$$\begin{array}{l} 3 \cdot \sqrt{12} \cdot \sqrt{-1} \\ 3 \cdot \sqrt{4} \cdot \sqrt{3} \cdot \sqrt{-1} \\ 3 \cdot 2i\sqrt{3} \end{array}$$

$$(6i\sqrt{3} + 9) - (4 - 8i)$$

Subtract Reals + Subtract imaginary

$$9 - 4 + 6i\sqrt{3} + 8i$$

$$\boxed{5 + 6i\sqrt{3} + 8i}$$

get decimal
to combine
further

$$5 + 10.392i + 8i$$

$$\boxed{5 + 18.392i}$$

Simplify
radicals

$$(\sqrt{63} - \sqrt{-7})(7 + \sqrt{-9})$$

$$(\sqrt{9} \cdot \sqrt{7} - \sqrt{7} \cdot \sqrt{-1})(7 + \sqrt{9} \cdot \sqrt{-1})$$

$$(3\sqrt{7} - i\sqrt{7})(7 + 3i)$$

$$21\sqrt{7} + \underline{9i\sqrt{7}} - \underline{7i\sqrt{7}} - 3i^2\sqrt{7}$$

$$-3(-1)\sqrt{7}$$

$$\underline{21\sqrt{7} + 2i\sqrt{7} + 3\sqrt{7}}$$

$$\boxed{24\sqrt{7} + 2i\sqrt{7}}$$

Conjugates

Same terms but opposite operation signs. When multiplied the middle term will cancel

The conjugate would be...

$$5x - 2$$

$$\boxed{5x} + \boxed{2}$$

$$-5 + \sqrt{2}$$

$$\boxed{-5} - \boxed{\sqrt{2}}$$

$$1 - 3\sqrt{5}$$

$$1 + 3\sqrt{5}$$

$$4i$$

$$0 - 4i$$

$$13 + 11i$$

$$13 - 11i$$

$$3 - i\sqrt{2}$$

$$3 + i\sqrt{2}$$

only signs in front of $\sqrt{\quad}$ or i will change (or second term)

Conjugates

Explore Adding

$$(\underline{2} + \underline{6i}) + (\underline{2} - \underline{6i})$$

$$2 + 2 + \underbrace{6i + -6i}$$

$$4 + 0$$

$$4$$

You get a real number

Shortcut: just add real parts

Explore Multiplying

$$(2 + 6i)(2 - 6i)$$

$$4 - 12i + 12i - 36i^2$$

$$4 - \underbrace{12i + 12i} + 36$$

$$4 + 36$$

$$40$$

$$2^2 + 6^2$$

$$4 + 36$$


$$40$$

square both terms always add

$$1. \begin{aligned} & (4a^2b - 3ab^2 + 2ab + 5) - (2a^2b + 3ab^2 - 7ab) \\ & + -2a^2b - 3ab^2 + 7ab \\ \hline & 2a^2b - 6ab^2 + 9ab + 5 \end{aligned}$$

$$4. (5x + 2y)(x^2 - xy + 3y^2)$$

$$\begin{aligned} & 5x^3 - 5x^2y + 15xy + 2x^2y - 2xy^2 + 6y^2 \\ & 5x^3 - 3x^2y + 15xy - 2xy^2 + 6y^2 \end{aligned}$$

$$5. V = (12 - 2x)(9 - 2x)(x)$$


$$V = (108 - 24x - 18x + 4x^2)x$$

$$V = 108x - 42x^2 + 4x^3$$

$$3. (2\sqrt{x} + 3)(2\sqrt{x} - 3)$$

$$4\sqrt{x^2} - (6\sqrt{x} + 6\sqrt{x}) - 9$$

$$4|x| - 9$$

Multiplying Conjugates

$$(3 + 11i)(3 - 11i)$$

$$9 - 33i + 33i - 121i^2$$

$$9 - 121(-1)$$

$$9 + 121$$

$$130$$

Writing Polynomial Functions

Complex and radical solutions always come in conjugate pairs.

1. Get all solution equations to = 0, check for implied conjugate roots
2. Multiply zero factors together with the given 'a' value, multiply conjugates together first so terms cancel

Quadratic $a = 1, x = 2i$

$x^2 - 2i = 0$ and $x = -2i$

$x - 2i = 0$ and $x + 2i = 0$

① $1(x - 2i)(x + 2i) = 0$

② $1(x^2 + 2ix - 2ix - 4i^2) = 0$

$1(x^2 - 4(-1)) = 0$

$1(x^2 + 4) = 0$

$P(x) = x^2 + 4$

Quadratic $a = -4, x = 3i/2$

$2(x - \frac{3i}{2}) = 0$ and $2(x + \frac{3i}{2}) = 0$

$2x = 3i$ and $2x = -3i$

$-3i - 3i = -6i$ and $+3i + 3i = 6i$

① $2x - 3i = 0$ and $2x + 3i = 0$

② $-4(2x - 3i)(2x + 3i) = 0$

$-4(4x^2 - 6xi - 6xi - 9i^2) = 0$

$-4(4x^2 - 12xi + 9) = 0$

$P(x) = -16x^2 - 36$

Quartic a = -1

$$-1(4x^2 + 5i)(4x^2 - 5i)$$

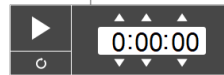
$$-1(16x^4 - 20x^2i + 20x^2i - 25i^2)$$

$$-(16x^4 + 25)$$

$$\boxed{-16x^4 - 25}$$

Complex Number Review Homework Name: _____ Hr: _____

Simplify:			
1. i^{43}	2. i^{98}	3. i^{52}	
4. Simplify: $3i(2i) - 4i$	5. Add and Simplify and Sketch a graph: $(-4 + 2i) + (3i + 6)$		
6. Subtract and Simplify: $(4 + 5i) - (3 - 2i)$	7. Multiply and Simplify: $-3i(4 + 2i)$		
8. Multiply and Simplify: $(2 - 3i)(5 + 4i)$	9. What is the simplified form of $(8 - 3i)^2$?		
10. Multiply $4 - 3i$ by its <u>complex conjugate</u> $(4 - 3i)(4 + 3i)$	11. Multiply $-24 + 13i$ by its complex conjugate $(-24 + 13i)(-24 - 13i)$		
12. Multiply the conjugates together $(3x + 4i)(3x - 4i)$	13. Multiply and Simplify $(x + \sqrt{-4})(x - 7\sqrt{-18})$ $(x + 2i)(x - 7\sqrt{9} \cdot \sqrt{2} \cdot \sqrt{-1})$ $(x + 2i)(x - 21i\sqrt{2})$		



Complex Number Review Homework Name: _____ Hr: _____

Simply the Radicals and Combine Like terms $3 + \sqrt{-8} - 5 - 7\sqrt{-18}$	Simply the Radicals and Combine Like terms $8 - \sqrt{-75} - 6 + 5\sqrt{-48}$
Solve $5x^2 + 321 = 141$ $\frac{-321 - 321}{5x^2} = \frac{-180}{5}$ $x^2 = -36$ $x = \pm \sqrt{-36}$ $x = 6i \text{ and } x = -6i$	Solve $3x^2 - 105 = -546$
Write a polynomial function given the zeros: $x = -2, x = 3 + \sqrt{10}$ and $a = -4$ $x + 2 = 0$ $x - 3 - \sqrt{10} = 0$ $x - 3 + \sqrt{10} = 0$ $(x + 2)(x - 3 - \sqrt{10})(x - 3 + \sqrt{10}) = 0$ $(x + 2)(x^2 - 6x - 1) = 0$ $(-4x - 8)(x^2 - 6x - 1) = 0$ $-4x^3 + 24x^2 + 4x - 8x^2 + 48x + 8 = 0$ $P(x) = -4x^3 + 16x^2 + 52x + 8$	Write a polynomial function given the zeros: $x = -1, x = 2 + \sqrt{6}$ and $a = -3$ $x + 1 = 0$ $x - 2 - \sqrt{6} = 0$ $x - 2 + \sqrt{6} = 0$ $(x + 1)(x - 2 - \sqrt{6})(x - 2 + \sqrt{6}) = 0$ $(x + 1)(x^2 - 4x - 2) = 0$ $(-3x - 3)(x^2 - 4x - 2) = 0$ $-3x^3 + 12x^2 + 6x - 3x^2 + 12x + 6 = 0$ $P(x) = -3x^3 + 9x^2 + 18x + 6$
Write a polynomial function given the zeros: $x = 1, x = 5i$ and $a = 3$ $x = -5i$ $x - 1 = 0$ $x - 5i = 0$ $x + 5i = 0$ $(x - 1)(x - 5i)(x + 5i) = 0$ $(3x - 3)(x^2 + 25) = 0$	Write a polynomial function given the zeros: $x = 2, x = 4i$ and $a = 5$

Complex Number Review Homework Name: _____ Hr: _____

Simply the Radicals and Combine Like terms $3 + \sqrt{-8} - 5 - 7\sqrt{-18}$	Simply the Radicals and Combine Like terms $8 - \sqrt{-75} - 6 + 5\sqrt{-48}$ $8 - \sqrt{25 \cdot 3} \cdot \sqrt{-1} - 6 + 5\sqrt{16 \cdot 3} \cdot \sqrt{-1}$ $8 - 5i\sqrt{3} - 6 + 5 \cdot 4i\sqrt{3}$ $2 - 5i\sqrt{3} + 20i\sqrt{3} \rightarrow 2 + 15i\sqrt{3}$
Solve $5x^2 + 321 = 141$ $\sqrt{x^2} = \sqrt{-36}$ $x = \pm 6i$	Solve $3x^2 - 105 = -546$
Write a polynomial function given the zeros: $x = -2, x = 3 + \sqrt{10}$ and $a = -4$ $x + 2 = 0$ $x - 3 - \sqrt{10} = 0$ $x - 3 + \sqrt{10} = 0$ $(x + 2)(x - 3 - \sqrt{10})(x - 3 + \sqrt{10}) = 0$ $(-4x - 8)(x^2 - 6x - 1) = 0$ $-4x^3 + 24x^2 + 4x - 8x^2 + 48x + 8 = 0$ $P(x) = -4x^3 + 16x^2 + 52x + 8$	Write a polynomial function given the zeros: $x = -1, x = 2 + \sqrt{6}$ and $a = -3$ $x + 1 = 0$ $x - 2 - \sqrt{6} = 0$ $x - 2 + \sqrt{6} = 0$ $(x + 1)(x - 2 - \sqrt{6})(x - 2 + \sqrt{6}) = 0$ $(-3x - 3)(x^2 - 4x - 2) = 0$ $-3x^3 + 12x^2 + 6x - 3x^2 + 12x + 6 = 0$ $P(x) = -3x^3 + 9x^2 + 18x + 6 = 0$ <i>Only sign in front of radical that changes</i>
Write a polynomial function given the zeros: $x = 1, x = 5i$ and $a = 3$	Write a polynomial function given the zeros: $x = 2, x = 4i$ and $a = 5$

Complex Number Review Homework Name: _____ Hr: _____

<p>Simplify:</p> <p>1. i^{43} $(i^4)^{10} \cdot i^3 = 1 \cdot i^3 = i^3 = -i$ $(i^4)^{10} \cdot i^3 = 1 \cdot i^3 = i^3 = -i$</p>	<p>2. i^{98} $(i^4)^{24} \cdot i^2 = 1 \cdot i^2 = -1$</p>	<p>3. i^{48} $(i^4)^{12} = 1$</p>
<p>4. Simplify: $3(2i) - 4i$ $6i - 4i = 2i$</p>	<p>5. Add and Simplify and Sketch a graph: $(-4 + 2i) + (3i + 6)$ $(-4 + 6) + (2i + 3i) = 2 + 5i$ Real: 2, Imaginary: 5</p>	
<p>6. Subtract and Simplify: $(4 + 5i) - (3 - 2i)$ $(4 - 3) + (5i - (-2i)) = 1 + 7i$</p>	<p>7. Multiply and Simplify: $-3i(4 + 2i)$ $-12i - 6i^2 = -12i - 6(-1) = -12i + 6 = 6 - 12i$</p>	
<p>8. Multiply and Simplify: $(2 - 3i)(5 + 4i)$ $10 + 8i - 15i - 12i^2 = 10 - 7i - 12(-1) = 10 - 7i + 12 = 22 - 7i$</p>	<p>9. What is the simplified form of $(8 - 3i)^2$? $(8 - 3i)(8 - 3i) = 64 - 24i - 24i + 9i^2 = 64 - 48i + 9(-1) = 64 - 48i - 9 = 55 - 48i$</p>	
<p>10. Multiply $4 - 3i$ by its complex conjugate $(4 - 3i)(4 + 3i) = 16 + 12i - 12i - 9i^2 = 16 + 9 = 25$</p>	<p>11. Multiply $-24 + 13i$ by its complex conjugate $(-24 + 13i)(-24 - 13i) = 576 + 312i - 312i - 169i^2 = 576 + 169 = 745$</p>	
<p>12. Multiply the conjugates together $(3x + 4i)(3x - 4i)$ $9x^2 - 12xi + 12xi - 16i^2 = 9x^2 + 16$</p>	<p>13. Multiply and Simplify $(x + \sqrt{-4})(x - 7\sqrt{-18})$ $(x + 2i)(x - 21i\sqrt{2}) = x^2 - 21xi\sqrt{2} + 2xi - 42i^2\sqrt{2} = x^2 - 29.698xi + 2xi + 42\sqrt{2} = x^2 - 27.698xi + 59.397$</p>	

Complex Number Review Homework Name: _____ Hr: _____

<p>Simply the Radicals and Combine Like terms $3 + \sqrt{-8} - 5 - 7\sqrt{-18}$ $3 + 2i\sqrt{2} - 5 - 21i\sqrt{2}$ $3 - 5 + 2i\sqrt{2} - 21i\sqrt{2}$ $-2 - 19i\sqrt{2}$</p>	<p>Simply the Radicals and Combine Like terms $8 - \sqrt{-75} - 6 + 5\sqrt{-48}$ $8 - 5i\sqrt{3} - 6 + 20i\sqrt{3}$ $2 + 15i\sqrt{3}$</p>
<p>Solve $5x^2 + 321 = 141$ $-321 - 321$ $5x^2 = -180$ $x^2 = -36$ $x = \pm\sqrt{-36}$ $x = 6i$ and $x = -6i$</p>	<p>Solve $3x^2 - 105 = -546$ $3x^2 = -441$ $x^2 = -147$ $x = \pm\sqrt{-147}$ $x = \pm\sqrt{49 \cdot -3} = \pm 7i\sqrt{3}$ $x = 7i\sqrt{3}$ and $x = -7i\sqrt{3}$</p>
<p>Write a polynomial function given the zeros: $x = -2, x = 3 + i\sqrt{10},$ and $a = -4$ $x + 2 = 0, x - 3 - i\sqrt{10} = 0, x - 3 + i\sqrt{10} = 0$ $-4(x + 2)(x - 3 - i\sqrt{10})(x - 3 + i\sqrt{10}) = 0$ $(-4x - 8)(x^2 - 3x + 3i\sqrt{10} - 3x + 9 - 3\sqrt{10}) = 0$ $(-4x - 8)(x^2 - 6x - 1) = 0$ $-4x^3 + 24x^2 + 4x - 8x^2 + 48x + 8 = 0$ $-4x^3 + 16x^2 + 52x + 8 = 0$</p>	<p>Write a polynomial function given the zeros: $x = -1, x = 2 + i\sqrt{6},$ and $a = -3$ $x + 1 = 0, x - 2 - i\sqrt{6} = 0, x - 2 + i\sqrt{6} = 0$ $-3(x + 1)(x - 2 - i\sqrt{6})(x - 2 + i\sqrt{6}) = 0$ $(-3x - 3)(x^2 - 2x - 2x + 4 - 6) = 0$ $(-3x - 3)(x^2 - 4x - 2) = 0$ $-3x^3 + 12x^2 + 6x - 3x^2 + 12x + 6 = 0$ $-3x^3 + 9x^2 + 18x + 6 = 0$</p>
<p>Write a polynomial function given the zeros: $x = 1, x = 5i,$ and $a = 3$ $x - 1 = 0, x - 5i = 0, x + 5i = 0$ $3(x - 1)(x - 5i)(x + 5i) = 0$ $(3x - 3)(x^2 + 5ix - 5ix - 25i^2) = 0$ $(3x - 3)(x^2 + 25) = 0$ $3x^3 + 75x - 3x^2 - 75 = 0$ $3x^3 - 3x^2 + 75x - 75 = 0$</p>	<p>Write a polynomial function given the zeros: $x = 2, x = 4i,$ and $a = 5$ $x = -4i$ $5(x - 2)(x - 4i)(x + 4i) = 0$ $(5x - 10)(x^2 + 16) = 0$ $5x^3 + 80x - 10x^2 - 160 = 0$ $5x^3 - 10x^2 + 80x - 160 = 0$</p>

Homework: Creating Functions given Real and Complex Roots

Write the function of least degree given the solutions

1. $x = \pm 7i$, $x = -3$ and $a = 1$

2. if the function has degree 3
 $a = 1$, $x = 3/2$ and $x = 6 + 2i$

means 3 solutions

solutions

$$2(x - \frac{3}{2})^2 (x - 6 + 2i) (x - 6 - 2i)$$

$$2x^2 - 3x - 6x + 9 - 6x + 12i - 6x + 12i - 36 + 24i - 4$$

$$2x^2 - 3x - 24x + 36 + 24i - 40 = 2x^2 - 27x + 24i - 4$$

Factor

$$(2x - 3)(x - 6 - 2i)(x - 6 + 2i) = 0$$

$$(2x - 3)(x^2 - 6x + 2ix - 6x + 36 - 12i - 2ix + 4(-1) + 12i)$$

$$(2x - 3)(x^2 - 12x + 40) = 0$$

$$2x^3 - 24x^2 + 80x - 3x^2 + 36x - 120 = 0$$

$$P(x) = 2x^3 - 27x^2 + 116x - 120 = 0$$

3. if the function has degree 3 and $x = -4/5$ and $x = -3 \pm i$, $a = -1$

$$S(x) = (-\frac{4}{5})^5 (x + 3 + i) (x + 3 - i) (x + 3 - i)$$

$$5x^2 - 4x - 4x + 12 + 4x + 12i - 4x + 12i - 16 + 24i - 4$$

$$5x^2 + 4x + 12i - 16 + 24i - 4 = 5x^2 + 4x + 24i - 20$$

$$-(5x + 4)(x + 3 - i)(x + 3 + i) = 0$$

$$(-5x - 4)(x^2 + 3ix + 3ix + 9 + i^2) = 0$$

$$(-5x - 4)(x^2 + 6ix + 10) = 0$$

$$-5x^3 - 30x^2 - 50x - 4x^2 - 24ix - 40 = 0$$

$$P(x) = -5x^3 - 34x^2 - 24ix - 40 = 0$$

4. if the function has degree 4
 $a = -1$, $x = 2$, $x = -6$ and $x = -7 - 2i$

5. if the function has degree 5, $a = 1$ and $x = 0$, $x = 1$, $x = -2$ and $x = -3i$

6. if the function has degree 5
 $a = -1$, $x = 2i$, $x = 0$ and $x = 5 - 4i$

$$-1(x)(x - 2i)(x + 2i)(x - 5 + 4i)(x - 5 - 4i)$$

$$-x(x^2 - 4i^2)(x^2 - 5x + 25 - 5x - 16)$$

Homework: Creating Functions given Real and Complex Roots

Write the function of least degree given the solutions

1. $x = \pm 7i$, $x = -3$ and $a = 1$

2. if the function has degree 3
 $a = 1$, $x = 3/2$ and $x = 6 + 2i$

There are 3 solutions

solutions

$$2(x - \frac{3}{2})^2 (x - 6 + 2i) (x - 6 - 2i)$$

$$2x^2 - 3x - 6x + 9 - 6x + 12i - 6x + 12i - 36 + 24i - 4$$

$$2x^2 - 3x - 24x + 36 + 24i - 40 = 2x^2 - 27x + 24i - 4$$

Factor

$$(2x - 3)(x - 6 - 2i)(x - 6 + 2i) = 0$$

$$(2x - 3)(x^2 - 6x + 2ix - 6x + 36 - 12i - 2ix + 4(-1) + 12i)$$

$$(2x - 3)(x^2 - 12x + 40) = 0$$

$$2x^3 - 24x^2 + 80x - 3x^2 + 36x - 120 = 0$$

$$P(x) = 2x^3 - 27x^2 + 116x - 120 = 0$$

3. if the function has degree 3 and $x = -4/5$ and $x = -3 \pm i$, $a = -1$

$$S(x) = (-\frac{4}{5})^5 (x + 3 + i) (x + 3 - i) (x + 3 - i)$$

$$5x^2 - 4x - 4x + 12 + 4x + 12i - 4x + 12i - 16 + 24i - 4$$

$$5x^2 + 4x + 12i - 16 + 24i - 4 = 5x^2 + 4x + 24i - 20$$

$$-(5x + 4)(x + 3 - i)(x + 3 + i) = 0$$

$$(-5x - 4)(x^2 + 3ix + 3ix + 9 + i^2) = 0$$

$$(-5x - 4)(x^2 + 6ix + 10) = 0$$

$$-5x^3 - 30x^2 - 50x - 4x^2 - 24ix - 40 = 0$$

$$P(x) = -5x^3 - 34x^2 - 24ix - 40 = 0$$

4. if the function has degree 4
 $a = -1$, $x = 2$, $x = -6$ and $x = -7 - 2i$

5. if the function has degree 5, $a = 1$ and $x = 0$, $x = 1$, $x = -2$ and $x = -3i$

6. if the function has degree 5
 $a = -1$, $x = 2i$, $x = 0$ and $x = 5 - 4i$

$$-1(x)(x - 2i)(x + 2i)(x - 5 + 4i)(x - 5 - 4i)$$

$$-x(x^2 - 4i^2)(x^2 - 5x + 25 - 5x - 16)$$

$$-x(x^2 + 4)(x^2 - 10x + 9) = 0$$

Name: Key

Homework: Creating Functions given Real and Complex Roots

Write the function of least degree given the solutions

1. $x = \pm 7i, x = -3$ and $a = 1$

$x = 7i \quad x = -7i \quad x = -3$
 $x - 7i = 0 \quad x + 7i = 0 \quad x + 3 = 0$

$(x+3)(x-7i)(x+7i) = 0$
 $(x+3)(x^2+49) = 0$

$P(x) = x^3 + 3x^2 + 49x + 147$

2. if the function has degree 3
 $a = 3/2, x = 3/2$ and $x = 6 + 2i$

$x = 3/2 \quad x = 6 + 2i \quad x = 6 - 2i$
 $2x - 3 = 0 \quad x - 6 - 2i = 0 \quad x - 6 + 2i = 0$

$(2x-3)(x-6-2i)(x-6+2i) = 0$
 $(2x-3)(x^2-6x-6x+36+4) = 0$
 $(2x-3)(x^2-12x+40) = 0$
 $2x^3 - 24x^2 + 80x - 3x^2 + 36x - 120 = 0$

$P(x) = 2x^3 - 27x^2 + 116x - 120$

3. if the function has degree 3
and $x = -4/5$ and $x = -3 + i, a = -1$

$x = -4/5 \quad x = -3 + i \quad x = -3 - i$
 $5x + 4 = 0 \quad x + 3 - i = 0 \quad x + 3 + i = 0$

$-1(5x+4)(x+3-i)(x+3+i) = 0$
 $(-5x+4)(x^2+3x+3x+9+1) = 0$
 $(-5x+4)(x^2+6x+10) = 0$
 $-5x^3 - 30x^2 - 50x + 4x^2 + 24x + 40 = 0$

$P(x) = -5x^3 - 26x^2 - 46x + 40$

4. if the function has degree 4
 $a = -1, x = 2, x = -6$ and $x = -7 - 2i$

$x = 2 \quad x = -6 \quad x = -7 - 2i \quad x = -7 + 2i$
 $x - 2 = 0 \quad x + 6 = 0 \quad x + 7 + 2i = 0 \quad x + 7 - 2i = 0$

$-1(x-2)(x+6)(x+7+2i)(x+7-2i) = 0$
 $-1(x^2+4x-12)(x^2+14x+49+4) = 0$
 $-1(x^2+4x-12)(x^2+14x+53) = 0$
 $-1(x^4+14x^3+53x^2+4x^3+56x^2-12x^2-168x-636) = 0$

$P(x) = -x^4 - 18x^3 - 97x^2 - 44x + 636$

5. if the function has degree 5, $a = 1$
and $x = 0, x = 1, x = -2$ and $x = -3i$

$x = 0 \quad x - 1 = 0 \quad x + 2 = 0 \quad x + 3i = 0 \quad x - 3i = 0$

$x(x-1)(x+2)(x+3i)(x-3i) = 0$
 $x(x^2+x-2)(x^2+9) = 0$
 $x(x^4+9x^2+x^3+9x-2x^2-18) = 0$
 $x^5 + 9x^4 + 7x^3 + 9x^2 - 18x = 0$

$P(x) = x^5 + 9x^4 + 7x^3 + 9x^2 - 18x$

6. if the function has degree 5
 $a = -1, x = 2i, x = 0$ and $x = 5 - 4i$

$x = 2i \quad x = -2i \quad x = 0 \quad x = 5 - 4i \quad x = 5 + 4i$

$-1(x+2i)(x-2i)(x+5-4i)(x-5+4i) = 0$
 $-x(x^2+4)(x^2-5x-5x+25-16i^2) = 0$
 $-x(x^2+4)(x^2-10x+41) = 0$
 $-x(x^4-10x^3+41x^2-40x+164) = 0$

$P(x) = -x^5 + 10x^4 - 41x^3 + 40x^2 - 164x$