10.5 - Angles Related to Circles and Arcs

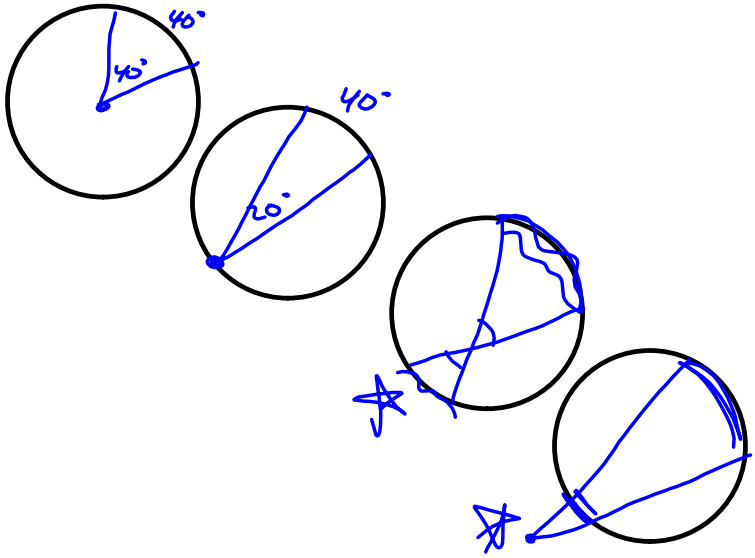
Angles related to circles and arcs have various formulas. In general, they can be classified by the location of the vertex of the angle. The vertex can lie in one of four locations:

1.) Center of the Circle

2.) On the Circle

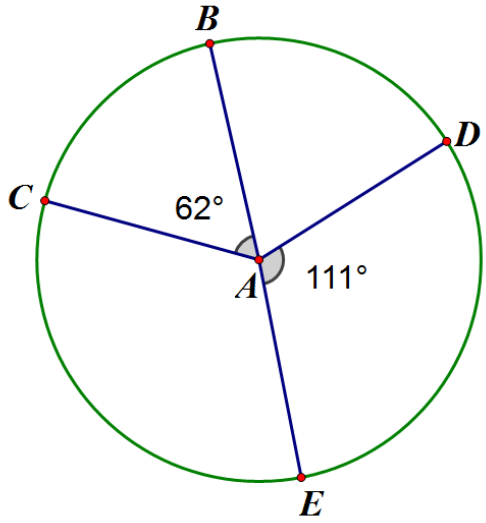
3.) Inside the Circle

4.) Outside the Circle



1.) Center - Central Angles

$\odot A$



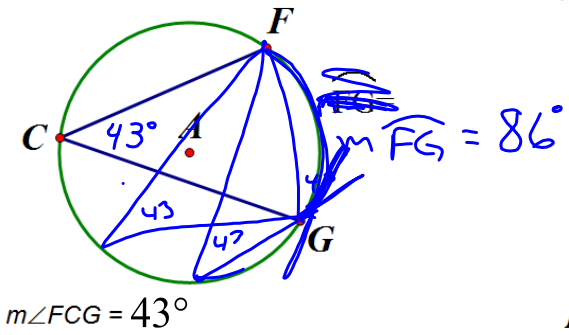
$m\overset{\frown}{BC} = 62°$

$m\overset{\frown}{DE} = 111°$

Center of the Circle — m Central Angle = m Intercepted Arc

2.) On the Circle

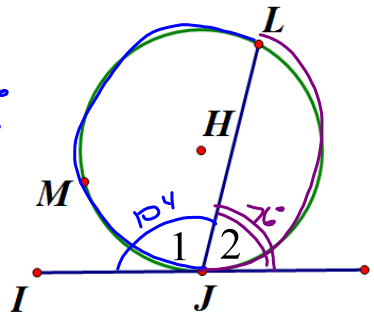
Inscribed Angles



$m\angle FCG = 43°$

Tangent/Chord Angles

$m\overset{\frown}{LMJ} = 208°$ $m\overset{\frown}{LJ} = 152$



$m\angle 1 = 104°$ $m\angle 2 = 76°$

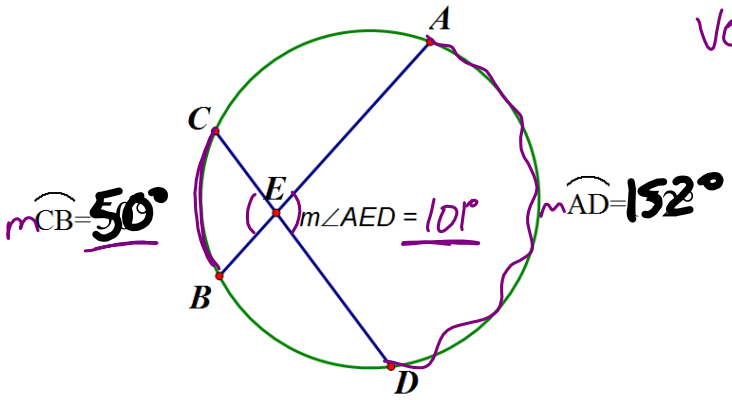
On the Circle

m Angle = $\left(\frac{1}{2}\right)$ m Intercepted Arc

2 · angle = arc

3.) Inside the Circle (Not at Center)

Chord/Chord Angles



$m\overset{\frown}{CB} = 50°$ $m\overset{\frown}{AD} = 152°$

Vertical angles are equal

Inside the Circle

add arcs

Chord/Chord Angle = Avg. of Intercepted Arcs

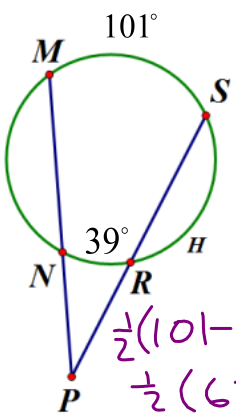
m Angle = $\frac{1}{2}$ (m Arc 1 + m Arc 2)

$m\angle AED = \frac{1}{2}(50 + 152)$

$m\angle AED = \frac{1}{2}(202) = 101°$

4.) Outside the Circle

Secant/Secant

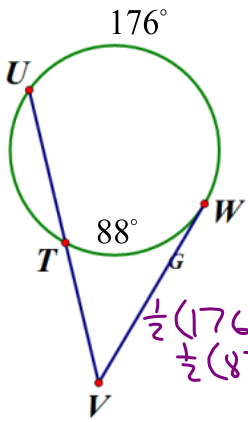


$\frac{1}{2}(101-39)$

$\frac{1}{2}(62)$

$m\angle NPR = 31°$

Secant/Tangent

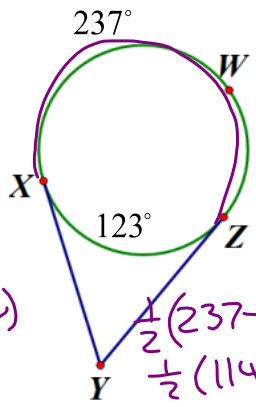


$\frac{1}{2}(176-88)$

$\frac{1}{2}(88)$

$m\angle UVW = 44°$

Tangent/Tangent



$\frac{1}{2}(237-123)$

$\frac{1}{2}(114)$

$m\angle XYZ = 57°$

Outside the Circle

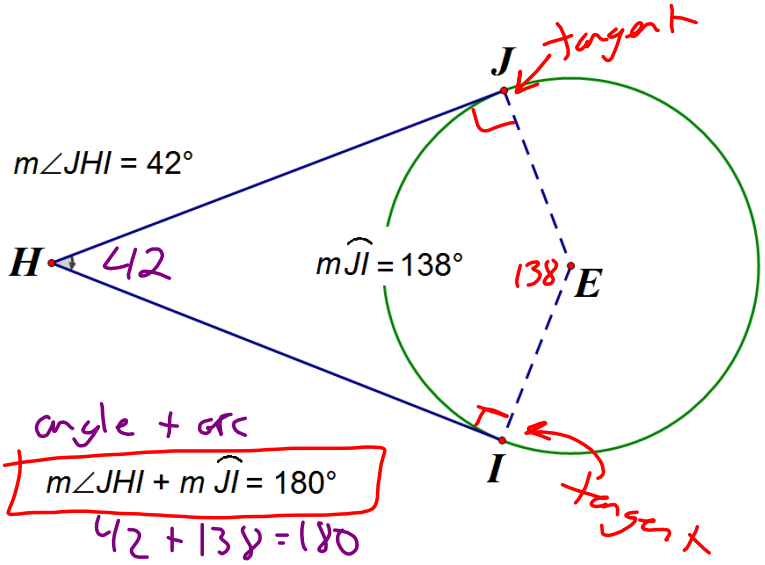
subtract arcs

Secant/Tangent Angle Combinations

Angle = Half the Difference of the Intercepted Arcs

$$m\text{ Angle} = \frac{1}{2}(m\text{ Bigger Arc} - m\text{ Smaller Arc})$$

Theorem: The sum of the measures of a tangent/tangent angle and its minor arc is 180°.



angle + arc

$m\angle JHI + m\overset{\frown}{JI} = 180°$

$42 + 138 = 180$

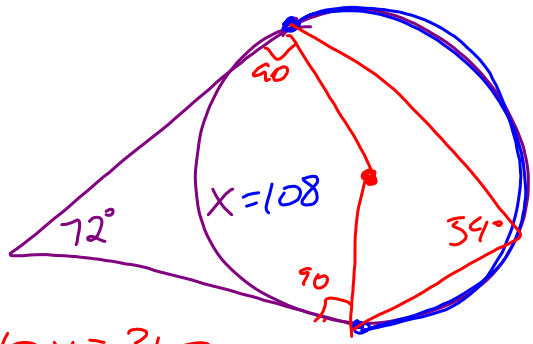
Quadrilateral

Sum = 360

- 90

- 90

180



$90+72+108=360$
$+90$

$72=\frac{1}{2}(360-x-X)$

$72=\frac{1}{2}(360-2x)$

$144=360-2x$

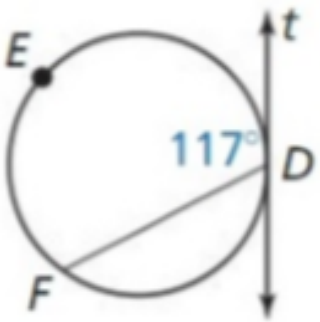
$\frac{-216}{-2}=\frac{-2x}{-2}$

$108=x$

Practice

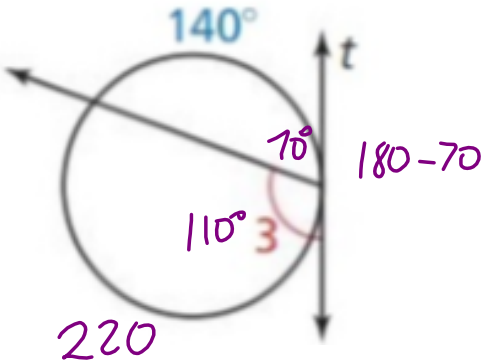
In Exercises 3–6, line *t* is tangent to the circle. Find the indicated measure. *(See Example 1.)*

4. $m\widehat{DEF}$ $234°$



117.2

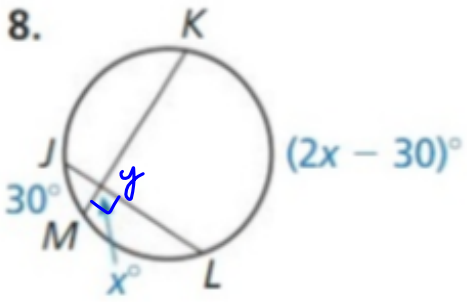
6. $m\angle 3$ $110°$



220
360-140

In Exercises 7–14, find the value of x. *(See Examples 2 an 3.)*

8.



$y = \frac{1}{2}(30 + 2x - 30)$

$y = \frac{1}{2}(2x)$

$y = x$

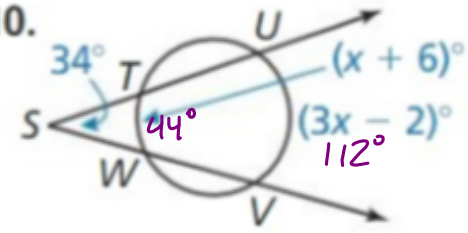
$x + y = 180$

$x + x = 180$

$2x = 180$

$x = 90$

10.



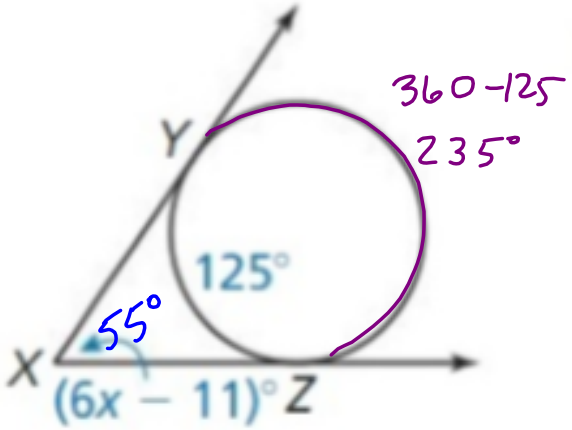
$34 = \frac{1}{2}(3x - 2 - (x + 6))$

$34 = \frac{1}{2}(2x - 8)$

$34 = x - 4$

$38 = x$

12.



$6x - 11 = \frac{1}{2}(235 - 125)$

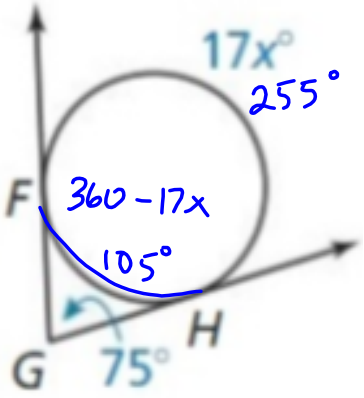
$6x - 11 = \frac{1}{2}(110)$

$6x - 11 = 55$

$6x = 66$

$x = 11$

14.



$75 = \frac{1}{2}(17x - (360 - 17x))$

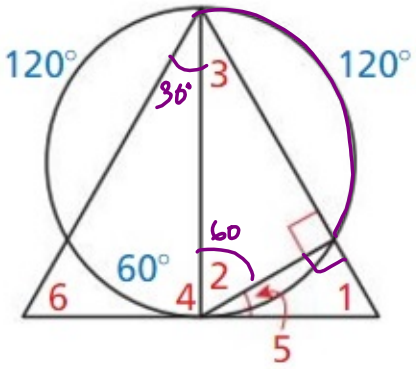
$2(75) = \frac{1}{2}(17x + 17x - 360) \cdot 2$

$150 = 34x - 360$

$\frac{510}{34} = \frac{34x}{34}$

$15 = x$

In Exercises 17–22, find the indicated angle measure. Justify your answer.



17. $m\angle 1 = 60°$ 90-30

18. $m\angle 2 = 60°$

19. $m\angle 3$

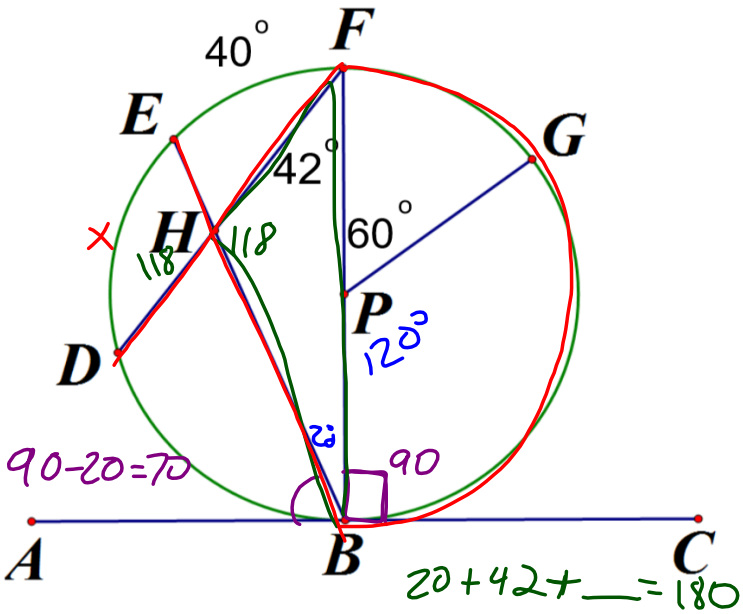
20. $m\angle 4 = 90°$

21. $m\angle 5 = 30°$ 90-60

22. $m\angle 6 = 60°$ $m\angle 6 + 30 = 90$

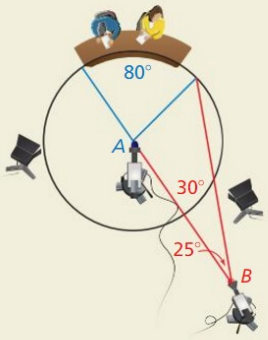
Given: $\odot P$
Diameter $\overline{FB}$
Tangent $\overline{AC}$ at B
Measures as shown

Find: *a.)* $m\angle FBC = 90°$
b.) $m\angle EBF = 20°$
c.) $m\overset{\frown}{GB} = 120°$
d.) $m\overset{\frown}{ED} = 56°$
e.) $m\angle ABE = 70°$
f.) $m\angle EHD = 118°$



$\frac{1}{2}(x+180) = 118$
$x + 180 = 236$
$x = 56$

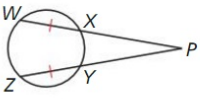
34. HOW DO YOU SEE IT? In the diagram, television cameras are positioned at A and B to record what happens on stage. The stage is an arc of $\odot A$. You would like the camera at B to have a 30° view of the stage. Should you move the camera closer or farther away? Explain your reasoning.

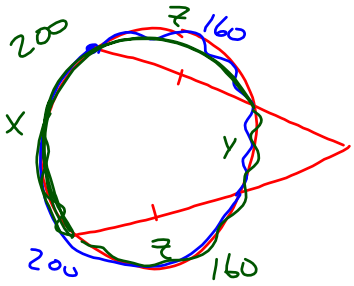


To increase the view from 25° to 30° move camera closer. this will increase both intercepted arcs and so increase the difference will

In Exercises 39 and 40, find the indicated n
Justify your answer

39. Find $m\angle P$ when $m\overset{\frown}{WZY} = 200°$.





$z + x = 200$ $z + y = 160$

$160 - y + x = 200$ $z = 160 - y$

$x - y = 40$

$\frac{1}{2}(x - y) = P$

$x - y = 2P$

$40 = 2P$

$20° = m\angle P$