Your Name

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4/25/22

Notes

Complex Zeros

Difference & Sum of Squares

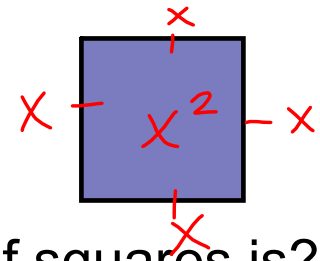
Objective: To be able to find complex roots of a polynomial and write a polynomial given its real and complex roots.

Life Lesson/Math Skill: To be able to solve and find all roots of any polynomial. Sometimes you do things because you can.

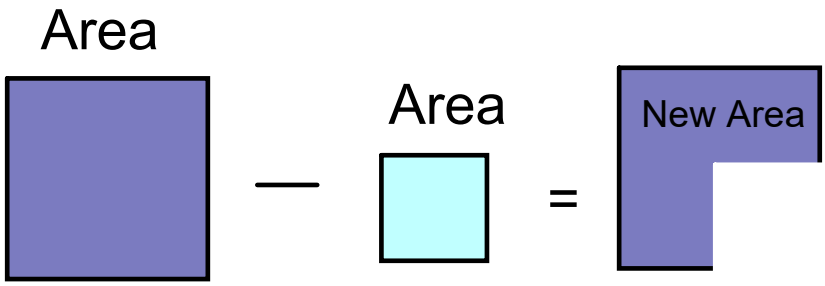
What makes a perfect square?

$A = l \cdot w$

$A = x \cdot x = x^2$



What do you think a difference of squares is?



Difference of Squares

Subtract

$ax^2 - c = (\sqrt{a}x - \sqrt{c})(\sqrt{a}x + \sqrt{c})$

Notice: 2 terms (binomial), and a and c are perfect squares AND c is subtracted

Step 1: Factor out GCF first.

Step 2: Find the square root of the first and last numbers a and c

Step 3. Write the new factors where one is added and the other is subtracted (this would cancel the middle term if you were to check your work and distribute)

ex. $4x^2 - 25$

$(2x+5)(2x-5)$

Check: $4x^2 - 10x + 10x - 25$

ex. $9x^2 - 36$

$(3x+6)(3x-6)$

$9(x^2-4)$

$9(x+2)(x-2)$

ex. $1/64p^2 - 169$

$\frac{1}{64}p^2 - 169$

$(\frac{1}{8}p+13)(\frac{1}{8}p-13)$

*Note: $\sqrt{\frac{1}{64}} = \frac{\sqrt{1}}{\sqrt{64}} = \frac{1}{8}$

ex. $-27p^2 + 48$

you want p^2 to be positive

$-3(9p^2-16)$

$-3(3p+4)(3p-4)$

* factor out a negative *

what! what!

Difference of Squares

$ax^2 - c = (\sqrt{ax} - \sqrt{c})(\sqrt{ax} + \sqrt{c})$

Notice: 2 terms (binomial), and a and c are perfect squares AND c is subtracted

Step 1: Factor out GCF first.

Step 2: Find the square root of the first and last numbers a and c — Terms

Step 3. Write the new factors where one is added and the other is subtracted (this would cancel the middle term if you were to check your work and distribute) — Conjugates!

ex. $4x^2 - 25$

$\sqrt{4x^2}$ $\sqrt{25}$

$(2x+5)(2x-5)$

ex. $9x^2 - 36$

$9(x^2-4)$

$9(x+2)(x-2)$

ex. $1/64p^2 - 169$

$\sqrt{\frac{1}{64}} = \frac{\sqrt{1}}{\sqrt{64}} = \frac{1}{8}$ $\sqrt{169}=13$

$(\frac{1}{8}p + 13)(\frac{1}{8}p - 13)$

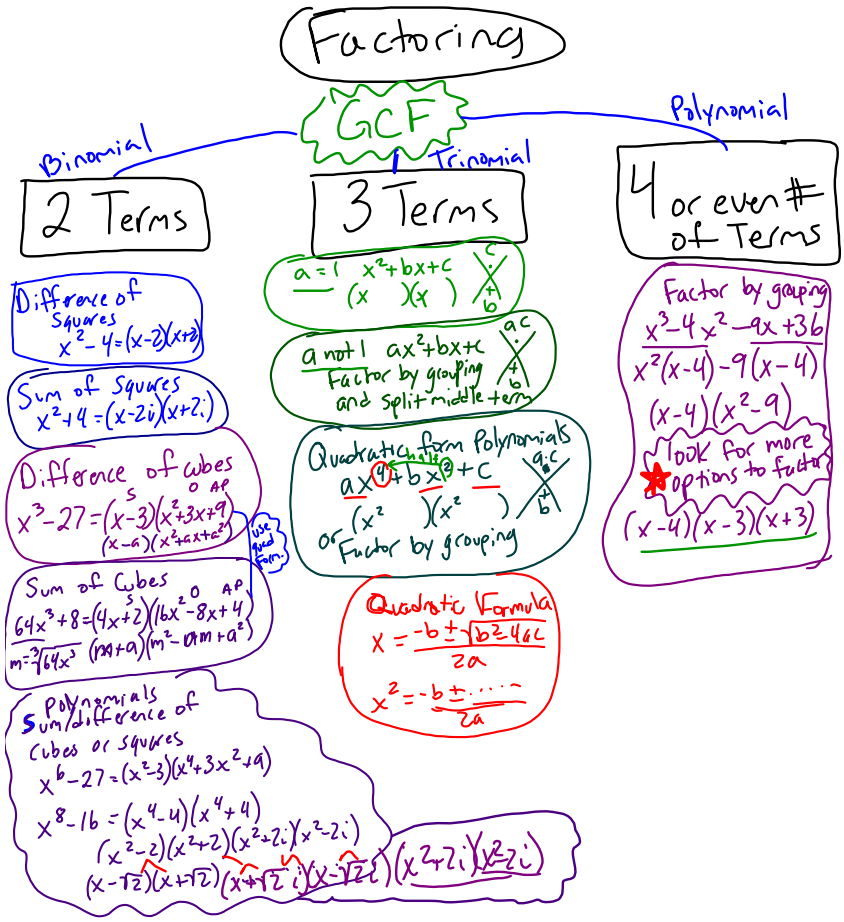
ex. $-18p^2 + 48$

$48 - 18p^2$

$2(24 - 9p^2)$

$2(\sqrt{24}-3p)(\sqrt{24}+3p)$

what! what!



Irreducible Factors

Factors that cannot be factored into linear factors without using imaginary numbers

ex. $x^4 + 2x^2 - 8$

$(x^2-2)(x^2+4)$

Difference of squares

Sum of squares ?

$(x'-\sqrt{2})(x'+\sqrt{2})$

-8, -2, 4, +, 2

Sum of Squares

$ax^2 + c = (\sqrt{a}x - i\sqrt{c})(\sqrt{a}x + i\sqrt{c})$

Notice: 2 terms (binomial), and a and c are perfect squares AND c is added

Step 1: Factor out GCF first.

Step 2: Find the square root of the first and last numbers a and c

Terms

Step 3. Write the new factors as complex conjugates with i (this would cancel the middle term if you were to check your work and distribute)

$16x^2 + 49 = 0$

$x^2 = -\frac{49}{16}$

$x = \pm\sqrt{-\frac{49}{16}}$

$x = \pm\frac{7}{4}i$

ex. $16x^2 + 49$

$\sqrt{16x^2}$ $\sqrt{49}$

$(4x+7i)(4x-7i)$

$16x^2 - 28xi + 28xi - 49i^2$

$+49$

ex. $81x^6 + 121$

$\sqrt[2]{81x^6} = \sqrt[2]{81}\cdot x^{6/2} = 9x^3$

$(9x^3+11i)(9x^3-11i)$

Sum and difference of cubes! More to learn!

ex. $-18p^2 - 200$

$-2(9p^2+100)$

$-2(3p+10i)(3p-10i)$

what! what!

ex. $p^2 + 1/144$

$(p+\frac{1}{12}i)(p-\frac{1}{12}i)$

$$3b^3 - 27b = 0$$

$$3b(b^2 - 9) = 0$$

$$3b(b-3)(b+3) = 0$$

$3b = 0$ $\quad b-3=0$ $\quad b+3=0$

$b = 0$ $\quad b = 3$ $\quad b = -3$

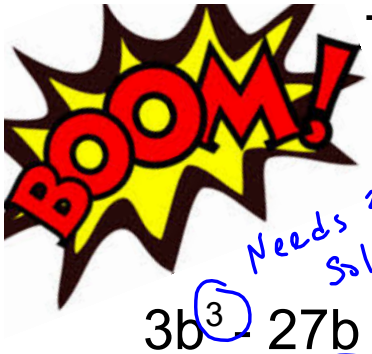
Remember: It is the simple things in life. . . .

The Zero Product Property

Anything times 0 equals 0

$a(0) = 0$ $\quad (0)b = 0$

if ab = 0 then **either b was 0 or a was 0**



There it is! Use your new skills to solve the equation.

Needs 3 solutions

$3b^3 - 27b = 0$

$3b(b^2 - 9) = 0$

$3b(b-3)(b+3) = 0$

$3b = 0 \quad b-3 = 0 \quad b+3 = 0$

$b = 0 \quad b = 3 \quad b = -3$

Needs 4 answers

$d^4 = 16$

$d^4 - 16 = 0$

$(d^2 + 4)(d^2 - 4) = 0$

$(d+2i)(d-2i)(d-2)(d+2) = 0$

$d+2i = 0 \quad d-2i = 0 \quad d-2 = 0 \quad d+2 = 0$

$d = -2i \quad d = 2i \quad d = 2 \quad d = -2$

Difference of Squares

$x^2 - 144$

$(x+12)(x-12)$

-144, 12, -12, +, 0

$25d^2 - 100$

$25(d^2 - 4)$

$25(d-2)(d+2)$

$4a^3 - 64a$

$4a(a^2 - 16)$

$4a(a-4)(a+4)$

$3b^3 - 27b = 0$

$3b(b^2 - 9) = 0$

$3b(b-3)(b+3) = 0$

$3b = 0 \quad b-3 = 0 \quad b+3 = 0$

$b = 0 \quad b = 3 \quad b = -3$

$9x^3 = 25x$

$9x^3 - 25x = 0$

$x(9x^2 - 25) = 0$

$x(3x-5)(3x+5) = 0$

$x = 0 \quad 3x-5 = 0 \quad 3x+5 = 0$

$x = 5/3 \quad x = -5/3$

$7a^3 = 175a$

$7a^3 - 175a = 0$

$7a(a^2 - 25) = 0$

$7a(a-5)(a+5) = 0$

$7a = 0 \quad a-5 = 0 \quad a+5 = 0$

$a = 0 \quad a = 5 \quad a = -5$

Warm Up: **Factor** each by using the Difference or Sum of Squares **then** Solve using 0 product property.

1. $81x^2 - 49 = 0$

$\sqrt{81x^2} = 9x \quad \sqrt{49} = 7$

$(9x + 7)(9x - 7) = 0$

$9x + 7 = 0$ or $9x - 7 = 0$

$9x = -7 \quad 9x = 7$

$x = -\frac{7}{9} \quad x = \frac{7}{9}$

2. $36n^2 = -1$

$i \cdot (-i)$

$-i^2$

$-(-1)$

$+1$

$+1 \quad +1$

$36n^2 + 1 = 0$

$\sqrt{36n^2} = 6n \quad \sqrt{1} = 1$

$(6n + 1i)(6n - 1i) = 0$

$6n + i = 0 \quad 6n - i = 0$

$6n = -i \quad 6n = i$

$n = \frac{-i}{6} \quad n = \frac{i}{6}$

3. $49 - 25x^2 = 0$

4. $\frac{1}{4}x^2 + 104 = 4$

$\sqrt{\frac{1}{4}x^2} = \frac{1}{2}x$

$\sqrt{100} = 10$

$-4 \quad -4$

$\frac{1}{4}x^2 + 100 = 0$

$(\frac{1}{2}x + 10i)(\frac{1}{2}x - 10i) = 0$

$\frac{1}{2}x + 10i = 0 \quad \frac{1}{2}x - 10i = 0$

$-10i \quad -10i$

$\frac{1}{2}x = 10i$

$2(\frac{1}{2}x) = (-10i)2$

$x = 20i$

$x = -20i$

5. $\frac{81}{256} = 16m^4$

6. $48n^7 = 147n$

Homework: Factor and Solve each Difference or Sum of Squares.

1. $81x^2 - 49 = 0$

$(9x - 7)(9x + 7) = 0$

$9x - 7 = 0 \quad 9x + 7 = 0$

$x = \frac{7}{9} \quad x = -\frac{7}{9}$

2. $36n^2 = -1$

$36n^2 + 1 = 0$

$(6n + i)(6n - i) = 0$

$6n + i = 0 \quad 6n - i = 0$

$n = -\frac{i}{6} \quad n = \frac{i}{6}$

3. $49 - 25x^2 = 0$

$(7 - 5x)(7 + 5x) = 0$

$7 - 5x = 0 \quad 7 + 5x = 0$

$x = \frac{7}{5} \quad x = -\frac{7}{5}$

4. $\frac{1}{4}x^2 + 104 = 4$

$-4 \quad -4$

$\frac{1}{4}x^2 + 100 = 0$

$(\frac{1}{2}x + 10i)(\frac{1}{2}x - 10i) = 0$

$\frac{1}{2}x + 10i = 0 \quad \frac{1}{2}x - 10i = 0$

$2(\frac{1}{2}x) = (-10i)2 \quad \frac{1}{2}x = 10i$

$x = -20i \quad x = 20i$

5. $\frac{81}{256} = 16m^4$

$16m^4 - \frac{81}{256} = 0$

$(4m^2 - \frac{9}{16})(4m^2 + \frac{9}{16}) = 0$ sum squares

$(2m - \frac{3}{4})(2m + \frac{3}{4})(2m + \frac{3}{4}i)(2m - \frac{3}{4}i) = 0$

$2m - \frac{3}{4} = 0 \quad 2m + \frac{3}{4} = 0 \quad 2m + \frac{3}{4}i = 0 \quad 2m - \frac{3}{4}i = 0$

$\frac{1}{2}(2m) = (\frac{3}{4})\frac{1}{2}$

$m = \frac{3}{8} \quad m = -\frac{3}{8} \quad m = -\frac{3}{8}i \quad m = \frac{3}{8}i$

$2m + \frac{3}{4} = 0$

$-\frac{3}{4} \quad -\frac{3}{4}$

$\frac{1}{2}(2m) = (-\frac{3}{4})\frac{1}{2}$

$m = -\frac{3}{8}$

$2m - \frac{3}{4}i = 0$

$+\frac{3}{4}i \quad +\frac{3}{4}i$

$\frac{1}{2}(2m) = (\frac{3}{4}i)\frac{1}{2}$

$m = \frac{3}{8}i$

6. $48n^7 = 147n$

$48n^7 - 147n = 0$

$3n(16n^6 - 49) = 0$

$3n(4n^3 + 7)(4n^3 - 7) = 0$

Sum + Difference of cubes!

$3n = 0 \quad 4n^3 + 7 = 0 \quad 4n^3 - 7 = 0$

$n = 0 \quad n = \sqrt[3]{-\frac{7}{4}} \quad n = \sqrt[3]{\frac{7}{4}}$

4 imaginary solutions

There is still more to learn!

$\sqrt{n^6} = n^{6/2}$

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