Your Name

Mrs. Theo

4/26/22

Notes

Complex Zeros

Sum and Difference of Cubes

Day 4

Sum of Cubes

Form: $x^3 + a^3$ (same number)

Binomial of degree 3, addition (2-terms, cube)

Factors into this

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

(same sign; opposite sign; always positive; cubic root of 1st term; cubic root of 2nd term; 1st term squared; product of 1st and 2nd terms; 2nd term squared)

a linear binomial and a quadratic with 2 imaginary solutions which can be found using the quadratic formula

$x^3 + 8 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{-8}$

$x = -2$

and 2 imag.

a) $x^3 + 27$

$\sqrt[3]{27} = 3$

$(x+a)(x^2 - ax + a^2)$

$(x+3)(x^2 - 3x + 3^2)$

$(x+3)(x^2 - 3x + 9)$

b) $x^3 + 64 = 0$ $\sqrt[3]{x^3} = x$ $\sqrt[3]{64} = 4$

$(x+4)(x^2 - 4x + 16) = 0$

$(x+a)(x^2 - ax + a^2)$

$x + 4 = 0$ $x^2 - 4x + 16 = 0$

$x = -4$ $a=1$ $b=-4$ $c=16$

rational solution

must use quadratic formula to solve for imaginary solutions

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$

$x = \frac{4 \pm \sqrt{-48}}{2}$

$x = \frac{4}{2} + \frac{i\sqrt{48}}{2}$ & $x = \frac{4}{2} - \frac{i\sqrt{48}}{2}$

$x = -4, x = 2 + 3.464i, \& x = 2 - 3.464i$

c. $216x^3 + 1 = 0$

Factored

$(6x + 1)(36x^2 - 6x + 1) = 0$

$m + a$ $m^2 - am + a^2$

Side Note: $\sqrt[3]{216x^3}$ and $\sqrt[3]{1}$

$m = 6x$ $a = 1$

Solve for x

separate and set = 0

$6x + 1 = 0$ $36x^2 - 6x + 1 = 0$

$a = 36$ $b = -6$ $c = 1$

$6x = -1$ $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(36)(1)}}{2(36)}$

$x = -\frac{1}{6}$

$x = \frac{6 \pm \sqrt{-108}}{72}$ split into + and −

$x = \frac{6}{72} + \frac{i\sqrt{108}}{72}$ & $x = \frac{6}{72} - \frac{i\sqrt{108}}{72}$

$x = -\frac{1}{6}$, $x = 0.083 + 0.144i$, and $x = 0.083 - 0.144i$

Difference of Cubes

Form: $x^3 - a^3$

Binomial of degree 3, subtraction

Factors into this

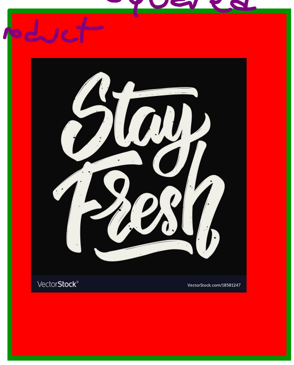
$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

same sign — opposite sign — Always positive

$\sqrt[3]{x^3} = x$ $\sqrt[3]{a^3} = a$

squared — product — squared

a linear binomial and a quadratic with 2 imaginary solutions which can be found using the quadratic formula



$x^3 - 8$

$(x - 2)(x^2 + 2x + 4)$

$\sqrt[3]{x^3} = x$

$\sqrt[3]{8} = 2$

a) $x^3 - 8$

$\sqrt[3]{x^3} = x \quad \sqrt[3]{8} = 2$

$(x-a)(x^2 - ax + a^2)$

$(x-2)(x^2 - 2x + 2^2)$

$(x-2)(x^2 - 2x + 4)$

b) $x^3 - 125 = 0$

$\sqrt[3]{x^3} = x \quad \sqrt[3]{125} = 5 = a$

$(x-a)(x^2 + ax + a^2)$

$(x-5)(x^2 + 5x + 25) = 0$

$x - 5 = 0 \quad x^2 + 5x + 25 = 0$

$x = 5 \quad x = \frac{-5 \pm \sqrt{5^2 - 4(1)(25)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{-75}}{2}$

$x = \frac{-5 \pm 8.660i}{2}$

$x = \frac{-5}{2} \pm \frac{8.660}{2} i$

$x = 5, \quad x = -2.5 + 4.330i, \quad x = -2.5 - 4.330i$

Extension

c) $27x^3 - 8$

$\sqrt[3]{27x^3} = 3x \quad \sqrt[3]{8} = 2$

$(x-a)(x^2 + ax + a^2)$ (S, O, AP)

$(3x-2)((3x)^2 + (2)(3x) + 2^2)$

$(3x-2)(9x^2 + 6x + 4)$

$64x^3 = 125$

$64x^3 - 125 = 0$

Difference of cubes Factored

$(4x - 5)(16x^2 + 20x + 25) = 0$

$m - a \quad m^2 + ma + a^2$ (S, O, AP)

Solve for x

$4x - 5 = 0 \quad 16x^2 + 20x + 25 = 0$

$4x = 5 \quad x = \frac{-20 \pm \sqrt{(20)^2 - 4(16)(25)}}{2(16)}$

$x = \frac{5}{4}$

$x = \frac{-20 \pm \sqrt{-1200}}{32}$

$x = \frac{-20}{32} + \frac{i\sqrt{1200}}{32}$ and $x = \frac{-20}{32} - \frac{i\sqrt{1200}}{32}$

$x = \frac{5}{4}, \ x = -0.625 + 1.083i$ and $x = -0.625 - 1.083i$

$0 = 216 + 8y^6$

$8y^6 + 216 = 0$

$\sqrt[3]{8y^6} = 2y^2 \quad \sqrt[3]{216} = 6$

$(2y^2 + 6)(4y^4 - 12y^2 + 36) = 0$

$(\sqrt{2}y + \sqrt{6}i)(\sqrt{2}y - \sqrt{6}i)$

$2y^2 + 6 = 0 \quad 4y^4 - 12y^2 + 36 = 0$

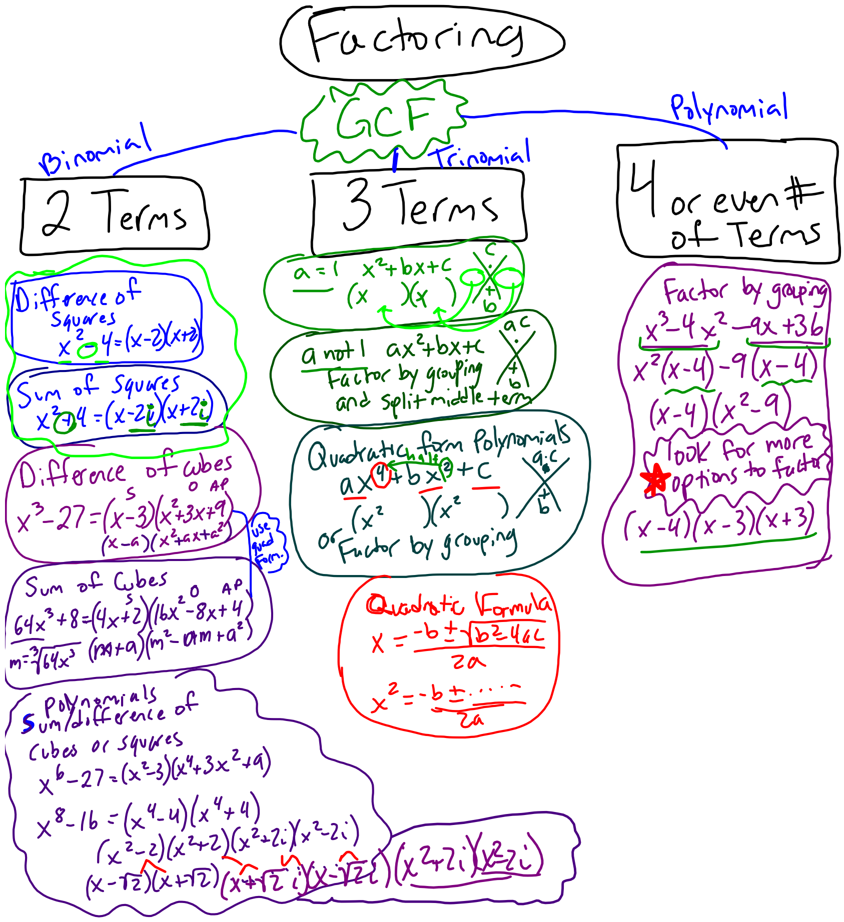
$2y^2 = -6 \quad y^2 = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(36)}}{2(4)}$

$y^2 = -3$

$y = \pm i\sqrt{3} \quad y^2 = \frac{12 + i\sqrt{432}}{8} \quad y^2 = \frac{12 - i\sqrt{432}}{8}$

2 imag 2 imag

4 imaginary solutions



Factor And Solve

$27x^3 - 8$

Factor and Solve :

$64a^3 + 1$

Factor and solve:

$8y^3 - 27$

Factor And Solve

$27x^3 - 8$

x=-1/3 + 0.577i x=-1/3 - 0.577i x=2/3

Factor and Solve :

$64a^3 + 1$

a=-1/4 a= 0.125 + 0.217i a=0.125 - 0.217i

Factor and solve:

8y^3 - 27x°

$(2y - 3)(4y^2 + 6y + 9)$

S O AP

1st term, 2nd Term squared, 1st·2nd, squared

$2y - 3 = 0$ $4y^2 + 6y + 9 = 0$

$y = \frac{3}{2}$

$y = 1.5$

$y = \frac{-6 \pm \sqrt{6^2 - 4(4)(9)}}{2(4)}$

$y = \frac{-6 \pm \sqrt{-108}}{8}$

$y = \frac{-6}{8} \pm \frac{\sqrt{108}}{8} i$

$y = -0.75 \pm \frac{10.39}{8} i$

$y = -0.75 + 1.299i$

$y = -0.75 - 1.299i$

Polynomials - Factoring Sum and Differences of Two Cubes

Draw a line from the cube to its factors. The lines will go through a letter then a number. Write them at the bottom to find what happened when the students went to the Coca-Cola factory.

$8x^3-125y^3$	(3x-4y)	$(4x^2+6xy+9y^2)$
$27x^3-125y^3$	(2x-5y)	$(9x^2+12xy+16y^2)$
$27x^3-64y^3$	(3x-5y)	$(9x^2-12xy+16y^2)$
$27x^3-8y^3$	(2x-3y)	$(4x^2+10xy+25y^2)$
$8x^3-27y^3$	(3x+4y)	$(9x^2+15xy+25y^2)$
$27x^3+64y^3$	(3x-2y)	$(16x^2-20xy+25y^2)$
$8x^3+27y^3$	(2x+3y)	$(9x^2+6xy+4y^2)$
$64x^3+125y^3$	(4x+5y)	$(4x^2-10xy+25y^2)$
$27x^3+125y^3$	(2x+5y)	$(4x^2-6xy+9y^2)$
$8x^3+125y^3$	(4x+3y)	$(16x^2-12xy+9y^2)$
$64x^3+27y^3$	(3x+5y)	$(9x^2-15xy+25y^2)$
$64x^3-125y^3$	(4x-5y)	$(9x^2-6xy+4y^2)$
$27x^3+8y^3$	(3x+2y)	$(16x^2+20xy+25y^2)$

T	H	E	R	E	W	A	S	A	P	O	P	Q	U	I	Z
9	6	1	10	1	8	3	12	3	7	5	7	11	2	13	4

There was a pop quiz.

1 $8x^3 - 125$ $(2x-5)(4x^2+10x+25)$

2 a^3b^6+64 $(ab^2+4)(a^2b^4-4ab^2+16)$

3 $27y^9+1$ $(3y^3+1)(9y^6-3y^3+1)$

4 $8a^3-b^3$ $(2a-b)(4a^2+2ab+b^2)$

5 x^6-8 $(x^2-2)(x^4+2x^2+4)$

6 b^9+x^6 $(b^3+x^2)(b^6-b^3x^2+x^4)$

7 $64a^3-1$ $(4a-1)(16a^2+4a+1)$

8 $8a^3b^3-x^6$ $(2ab-x^2)(4a^2b^2+2abx^2+x^4)$

9 x^3+125 $(x+5)(x^2-5x+25)$

10 $27a^{12}+1$ $(3a^4+1)(9a^8-3a^4+1)$

11 $x^{15}-1$ $(x^5-1)(x^{10}+x^5+1)$

12 $8b^3+x^6$ $(2b+x^2)(4b^2-2bx^2+x^4)$

13 y^6b^3+1 $(y^2b+1)(y^4b^2-y^2b+1)$

14 $64a^9+125$ $(4a^3+5)(16a^6-20a^3+25)$

15 b^3+x^3 $(b+x)(b^2-bx+x^2)$

16 $8y^6-27$ $(2y^2-3)(4y^4+6y^2+9)$

17 a^6b^6-8 $(a^2b^2-2)(a^4b^4+2a^2b^2+4)$

Write the GridWord here: SHELL

$2ab-x^2$	b^2-bx+x^2	$2x-5$	$a+2$	$a^4+a^2b^2-2$	b^2-x^2	y^2-1	$5x$	$x^{10}+x^5+1$	b^2+x^2	$2a$	$a^4b^2+4ab^2+16$	$x-y$	$4y^4+6y^2+9$	$ab+4$
$3a^4+1$	$64a^2+4a-1$	$b-x$	$5x^2$	$4x^2+10x+25$	$2a+b$	$27a^6-1$	a	$4a^2+2ab+b^2$	$3x^2-6$	3	a^2b^2+8	b^2+2	$9y^6-3y^3+1$	$x+1$
x^2-2	$x^2-5x+25$	y^2b+1	$bx+1$	$9a^8-3a^4+1$	$4a+1$	$4a^2-a-1$	2	$16a^6-20a^3+25$	$4a^3+5$	b^4	$a^2b^4-4ab^2+16$	$4y^2+3$	$b^6-b^3x^3+x^4$	$4a-1$
$2b-x^2$	b^6-bx+2	$x+5$	$yb+1$	a^2b^2-2	$2a-b$	$2y^2-3$	bx	$4a^2b^2+2abx^2+x^4$	b^6+x^4	$8x$	$a^4b^4+2a^2b^2+4$	$b-2x$	$x^2-25x-25$	x^2-3
$b+x$	x^4+2x^2+4	x^5-1	$4x$	$16a^2+4a+1$	x^5+1	$2b+x^2$	y^6	$y^4b^2-y^2b+1$	$3y^3+1$	xy	$4b^2-2bx^2+x^4$	ab^2+4	$x^{10}-x^5+5$	$a-3$