Your Name

Mrs. Theo

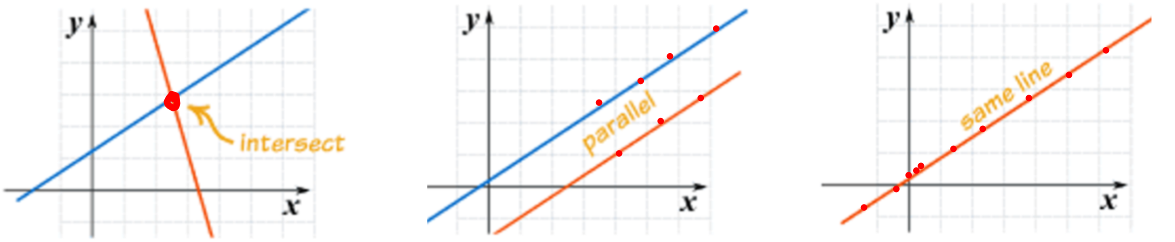
11/18/2020

Notes

Lesson 1.4 - Solving Systems of 3 Variable Equations

No Solution/Infinitely Many Solutions

Systems of 2 Linear Equations



One Solution | No Solution | ∞ Solutions

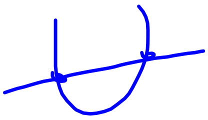
Solve and end up with a solution *(x,y)*.

When solving, both variables are eliminated! Look at the statement that is left when the variables are gone.

False = No Solution	True = Infinite Solutions
0 = 5 or 1 = 10	*0 = 0 or 5 = 5*

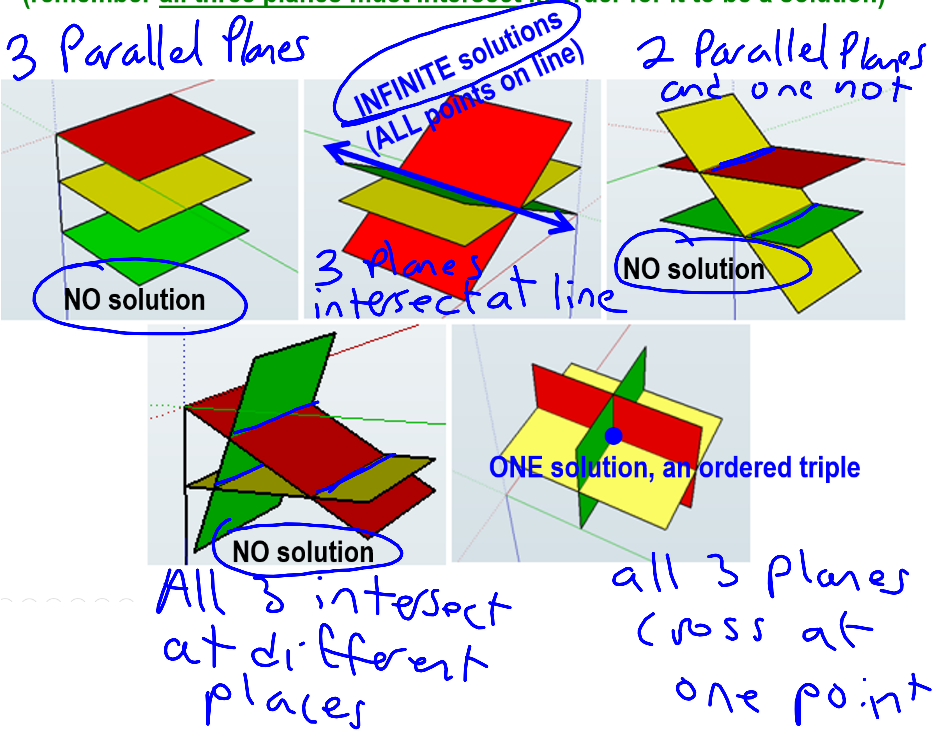
3 Variable System

Types of Solutions

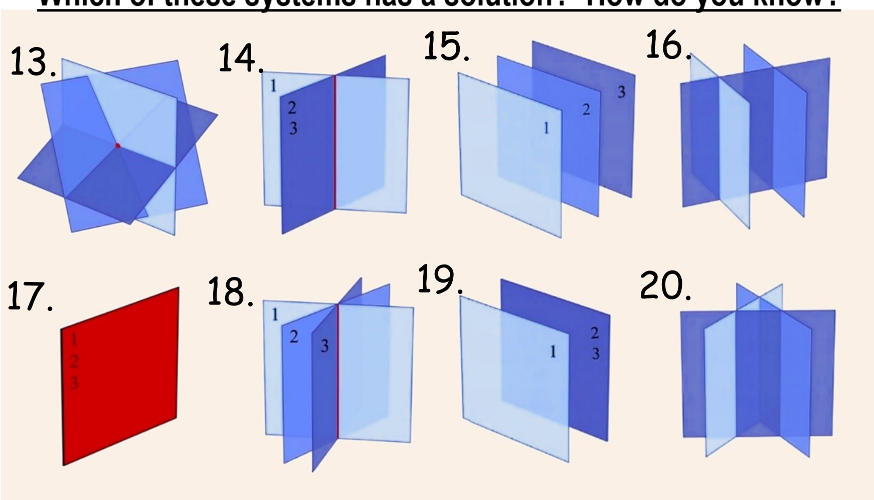


Ways Three Planes Can Intersect & Solution Implications

(remember all three planes must intersect in order for it to be a solution)



Which of these systems has a solution? How do you know?

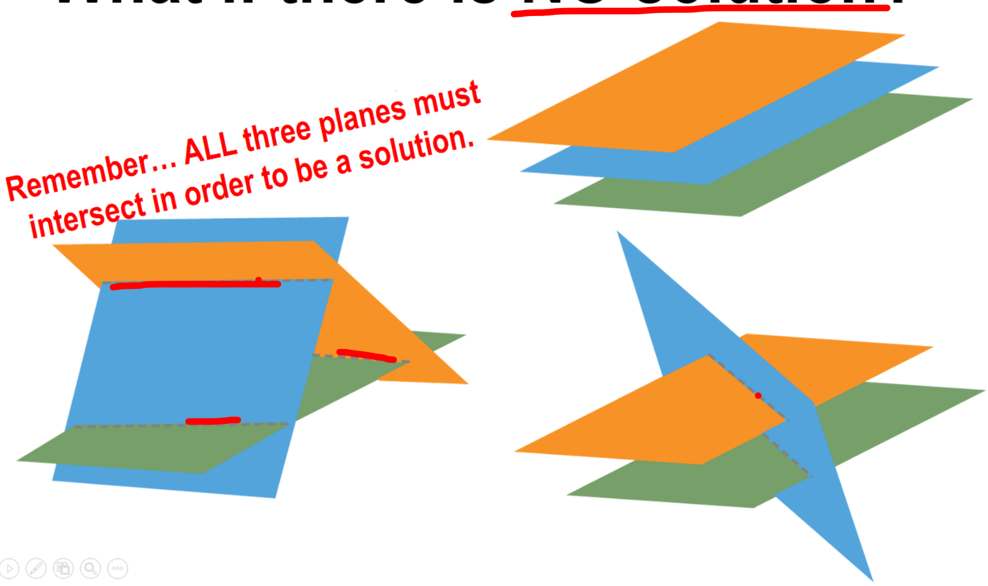


One Solution

Infinite Solutions

No Solutions

What if there is NO solution?



Systems with No Solution

If you eliminate a variable and all three variables cancel and you get a false statement

Three Variable Equations with NO solutions...

Two is company
Three is a crowd
Pick a variable &
KICK IT OUT!

Parallel

$$\begin{cases} x+y+z=2 & \text{Equation 1} \\ 3x+3y+3z=14 & \text{Equation 2} \\ x-2y+z=4 & \text{Equation 3} \end{cases}$$

$\cdot -3$ multiply by -3

$$\begin{cases} \cancel{-3x}-\cancel{3y}-\cancel{3z}=-6 \\ + \; \cancel{3x}+\cancel{3y}+\cancel{3z}=14 \end{cases}$$

0=8

Since this is a ***false equation***, you can conclude the original system of equations has **no solution**.

And... you're done!
There isn't a solution to find.

What is the graphical situation here?

Parallel Planes if: ax + by + cz can be the same but with different d

Coinsiding Planes if:ax + by + cz and d can be exactly the same

Ex2: Solve the system.

$$\begin{aligned} A \quad & x+y+z=1 \\ B \quad -2(& 6x+9y-12z=14) \\ C \quad & 12x+18y-24z=-11 \end{aligned}$$

parallel ·2

$$\begin{aligned} (B)\quad & \cancel{-12x} - \cancel{18y} + \cancel{24z} = -28 \\ (C)\quad + & \cancel{12x} + \cancel{18y} - \cancel{24z} = -11 \\ \hline & 0 = -39 \end{aligned}$$

False

No solution

B and C are parallel

Be smart
Check for any parallel planes

Two is company
Three is a crowd
Pick a variable &
KICK IT OUT!

KEEP CALM
THERE MAY BE
NO
SOLUTION

Ex2: Solve the system.

$$\begin{aligned} ① \quad & -6(x+y+z=1) \\ ② \quad & (6x+9y-12z=14)(-2) \\ ③ \quad & 12x+18y-24z=-11 \end{aligned}$$

Parallel eq② ·2

Step 1

$$\begin{aligned} ① \quad & -6x-6y-6z=-6 \\ ② \quad + & 6x+9y-12z=14 \\ \hline (A) \quad & 3y-18z=8 \end{aligned}$$

Step 1

$$\begin{aligned} ② \quad & -12x-18y+24z=28 \\ ③ \quad + & 12x+18y-24z=-11 \\ \hline & 0x+0y+0z=17 \\ & 0=17 \end{aligned}$$

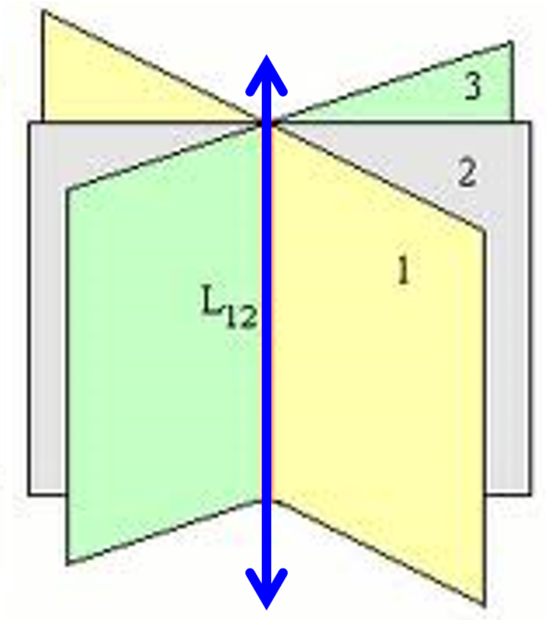
$\boxed{\text{False, then there's No Solution to the system}}$

Be smart
Check for any parallel planes

Two is company
Three is a crowd
Pick a variable &
KICK IT OUT!

KEEP CALM
THERE MAY BE
NO
SOLUTION

What if there are **INFINITE solutions**? How do I find the intersection points if the planes intersect in a **LINE**?

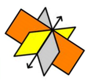


Systems with Infinite Solutions

1. If as you eliminate a variable, and two variables cancel and you are immediately left with one variable = a number

OR 2. you eventually get a true statement like 0 = 0

What happens when three planes intersect in a LINE?



Solve the system.

$2x + y + z = 0$	**Equation 1**
$x - 2y - 2z = 0$	**Equation 2**
$x + y + z = 0$	**Equation 3**

SOLUTION

$x - 2y - 2z = 0$ Equation 2
$\underline{-x - y - z = 0}$ −1 times Equation 3
$-3y - 3z = 0$ New Equation 2

$2x + y + z = 0$ Equation 1
$\underline{-2x - 2y - 2z = 0}$ −2 times Equation 3
$-y - z = 0$ New Equation 1

$$-3y - 3z = 0$$
$$\underline{3y + 3z = 0}$$
$$0 = 0$$

Identity... many solutions

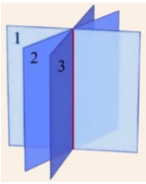
2. Determine what your **infinitely many solutions** will look like: Will they form a **LINE or a PLANE**?

Are any planes coinciding? (multiples overlapping each other)

-All 3 are coinciding: Plane of solutions

-If Not (only 2 are coinciding or none are): Line of solutions

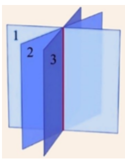
3. Pick one of the variables to write the others in terms of for the coordinate that is actually a line: ex: (x,y,z) =>(x, y= x+2, z= -3x-7)



Solve the system using any algebraic method.

$$x + 2y - z = 4$$
$$3x - y + 4z = -2$$
$$6x + 5y + z = 10$$

Two is company, Three is a crowd
Pick a variable & KICK IT OUT!



Solve the system using any algebraic method.

Two is company, Three is a crowd
Pick a variable & KICK IT OUT!

A $(x + 2y - z = 4)\cdot 4$
B $3x - y + 4z = -2$
C $6x + 5y + z = 10$

Step 1

A $x + 2y - z = 4$
+ C $6x + 5y + z = 10$
(E) $7x + 7y = 14$

A $4x + 8y - 4z = 16$
+ B $3x - y + 4z = -2$
(D) $(7x + 8y = 15)\cdot -1$

$\boxed{(x, -1x+2, -x)}$

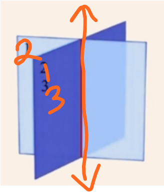
Step 2

(E) $7x + 7y = 14$
+ $-7x - 7y = -14$
$0 = 0$

Infinite solutions

(D) solve for y
$7x + 7y = 14$
$7y = -7x + 14$
$y = -1x + 2$

(A) $(x) + 2(\overset{y=}{-x+2}) - z = 4$
$x + -2x + 4 - z = 4$
$-x + 4 - z = 4$
$-x = z$



2 coinsiding planes
1 intersecting plane
Infinite solutions
Form a Line

① $2x + 2y + 2z = -2$

② $(2x + 3y + 2z = 4)(-1)$

③ $(x + y + z = -1)(-2)$

Multiply equation 3 by 2 and it is exactly the same as eq 1 → coinsiding planes
the same

Step 1

① $2x + 2y + 2z = -2$
\+
② $-2x - 3y - 2z = -4$

$-y = -6$

$y = 6$

① $2x + 2y + 2z = -2$
\+
③ $-2x - 2y - 2z = 2$

$0 = 0$

Infinitely Many Solutions

4. ① $(-3x + 5y + 2z = -19)(-2)$

② $5x - y + 4z = -5$

③ $(4x - 2y + 2z = 2)(-1)$

① $-3x + 5y + 2z = -19$
\+
③ $-4x + 2y - 2z = -2$

$\boxed{-7x + 7y = -21}$

$-7x = -7y - 21$

$x = y + 3$

$\boxed{y = x - 3}$

x is itself

$-3x + 5y + 2z = -19$

$-3x + 5(x-3) + 2z = -19$

$-3x + 5x - 15 + 2z = -19$

$2x - 15 + 2z = -19$

$2z = -2x - 4$

$z = -x - 2$

$\boxed{(x, x-3, -x-2)}$

Same I.M.S.

① $6x - 10y - 4z = 38$
\+
② $5x - y + 4z = -5$

$\boxed{11x - 11y = 33}$

$11x = 11y + 33$

$\boxed{x = y + 3}$

Solve for x or y in terms of y

$-3x + 5y + 2z = -19$

$-3(y+3) + 5y + 2z = -19$

$-3y - 9 + 5y + 2z = -19$

$2y - 9 + 2z = -19$

$2z = -2y - 10$

$z = -y - 5$

$\boxed{(y+3, y, -y-5)}$

?

?

Objective: Vocabulary Associated with Three Variable Systems — Resource Credit: Section 1.4

Using Technology to Model Mathematics & Vocabulary — BIM Algebra 2 Text p29-36

Unfortunately, sometimes the special cases of 3 x 3 systems of equations are the hardest to understand. If a system of equations does not have a single solution, then it either has NO solutions or it has infinitely many solutions. This is hard to determine which scenario is present using determinants or matrix inverse because the determinant of both types of systems is 0.

System 1 — System 2 — System 3 — System 4 — System 5

Use the numbers under each representation of a three variable system to answer the questions below

1. ____________Which of the systems has exactly one solution?
2. ____________Which of the systems has infinitely many solutions?
3. ____________Which of the systems has no solutions?

Directions: Match the systems A, B, C and D to the numbered system examples above and give an ALGEBRAIC reason for the selection of system

System A	System B	System C	System D	System E
$x-2y+3z=2$	$x-2y+3z=2$	$-2(-4x+3y+7z=25)$	$5x+5y+5z=-20$	$x+2y-7z=-4$
$2x-4y+6z=48$	$2x-4y+6z=7$	$4x-2y+12z=34$	$4x+3y+3z=-6$	$2x+y+z=13$
$-5x+10y-15z=-50$	$-5x-10y-15z=-50$	$-8x-5y+3z=-5$	$-4x+3y+3z=9$	$3x+9y-36z=-33$

$8x-6y-14z=-50$

$-11y-11z=-55$

$5y+18z=55$

4. System A is an example like System ~~2~~ 3
   Algebraically how do you know this classification?______________________
5. System B is an example like System 5
   Algebraically how do you know this classification?______________________
6. System C is an example like System 4
   Algebraically how do you know this classification?______________________
7. System D is an example like System 1
   Algebraically how do you know this classification?______________________
8. System E is an example like System 3
   Algebraically how do you know this classification?______________________

2

Link to GeoGebra 3D plotting https://www.geogebra.org/3d?lang=en

Solve these systems of equations. Show all work or thinking. If there is no solution say so, if there are infinitely many solutions say so and write as an ordered triple in terms of y.

9. A $(5x+5y+5z=-20)4$
   B $4x+3y+3z=-6$
   C $(-4x+3y+3z=9)5$

B $\cancel{4x}+3y+3z=-6$
C $\cancel{-4x}+3y+3z=9$
D $6y+6z=3$ ÷ 6

A $\cancel{20x}+20y+20z=-80$
C $\cancel{-20x}+15y+15z=45$
E $35y+35z=-35$ ÷ 35

D $\cancel{y}+\cancel{z}=\frac{1}{2}$
E $-1(y+z=-1)$
E $-\cancel{y}-\cancel{z}=+1$ Parallel

$0=1\frac{1}{2}$

False No Solution

10. $6x+10y+4z=-38$
    $-5x+y-4z=5$
    $-4x-2y+2z=2$

11. A $3x+3y+3z=-12$
    B $2x+3y+5z=9$
    C $3(-x-y-z=3)$

C $-\cancel{3x}-\cancel{3y}-\cancel{3z}=9$
A $\cancel{3x}+\cancel{3y}+\cancel{3z}=-12$

$0=-3$

False

No Solution

12. A $3x+3y+3z=-12$
    B $2x+3y+5z=9$
    C ~~3~~ $2(-x-y-z=4)$

C $-\cancel{3x}-\cancel{3y}-\cancel{3z}=12$
A $\cancel{3x}+\cancel{3y}+\cancel{3z}=-12$
$0=0$

IMS.

C $-\cancel{2x}-2y-2z=8$
B $\cancel{2x}+3y+5z=9$
E $y+3z=17$
$-3z\quad-3z$
$\boxed{y}=-3z+17$

$(2z-21,-3z+17,z)$

C $-x-(-3z+17)-z=4$
$-x+3z-17-z=4$
$+17\quad+17$
$-x+2z=21$
$+x\quad+x$
$2z=x+21$
$-21\quad-21$
$2z-21=\boxed{x}$

To Solve Word Problems

What you need

1. Read the problem
2. Use the question to define your variables specifically (price of or # of or % of)

You need as many equations as you have variables

3. Look for totals and make each equation
   - -Object equations are just adding the variables
   - - Money/Value equations should have values multiplied to the variables
4. Look for a described relationship. Tricky
   - -Ask yourself: Which is bigger?
     bigger variable should be alone.
   - -Write what you think the equation is and then check to make sure it makes sense. Does multiplying that equal the bigger number?

$x = 2y$

$2x = y$

$2\cancel{x} - y = 0$

x: # of dolphins

y: # of whales

There are twice as many dolphins as whales

Are there more dolphins or whales?

$d = 2w$ or $\cancel{2d = w}$

$x = 2y$

$2x = y$

twice bigger is smaller? $\cancel{x}$