Your Name

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Notes

Lesson 1.4 - Solving Systems of 3 Variable Equations

Applications

Math Skill Objective:

- Write a system of 3 variable equations that represent real life scenarios and be able to solve and interpret your solutions

Life Lesson: Why live life in 2D when you can live it in 3D? Are there other dimensions? Many areas use 3 variable graphs and equations.

Part Six

Who really uses this stuff anyway?

What you need

1. Read the problem

2. Use the question to define your variables specifically (price of or # of or % of)

You need as many equations as you have variables

3. Look for totals and make each equation

   -Object equations are just adding the variables

   - Money equations should have $\boxed{\text{values}}$ multiplied to the variables

4. Look for a described relationship. Tricky

   -Ask yourself: Which is bigger?

   bigger variable should be alone.

   -Write what you think the equation is and then check to make sure it makes sense. Does multiplying that equal the bigger number?

x: # of dolphins
y: # of whales

There are twice as many dolphins as whales

Are there more dolphins or whales?

$d = 2w$ or $\cancel{2d = w}$

$x = 2y$ $2x = y$

Coin Problem

Uncle Scrooge claims he has a bag of 30 coins containing nickels, dimes and quarters. The total value of the coins is $3. There are twice as many nickels as there are dimes. Is Scrooge correct? Explain your reasoning.

x : # of nickels
y : # of dimes
z : # of quarters

Be specific

Totals

Object Equation

$$\boxed{30 = x + y + z}$$

# of coins, # of N, # of D, # of Q

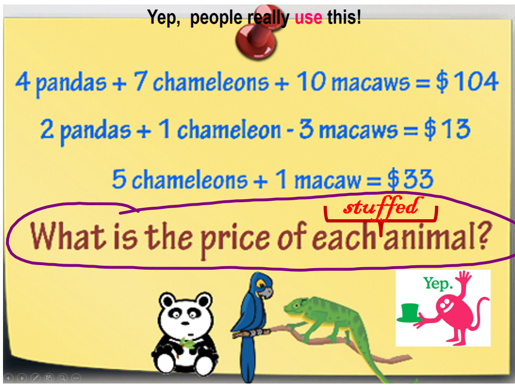
Money Equation

$$\boxed{\$3 = 0.05x + 0.1y + 0.25z}$$

Total Dollars

5¢ is $0.05
10¢ is $0.10
25¢ is $0.25

total objects and cost



Question determines Variable Definition

x : Price of pandas
y : Price of chameleon
z : Price of macaws

be specific

2nd look for Totals

104, 13, 33

become equations

Total spent on P, on Ch., on M

$$4 \cdot x + 7y + 10z = 104$$
$$2x + y - 3z = 13$$
$$5y + z = 33$$

Total spent

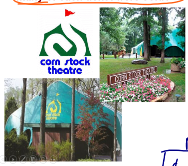
Ticket problem

Cornstock sold a total of 440 tickets for $3940. Each regular ticket cost is $5, each premium ticket is $15 and each elite ticket cost $25. The number of regular tickets was three times the number of premium and elite tickets combined. Write an algebraic model and determine how many of each ticket were sold?

x : # of regular tickets

y : # of premium tickets

z : # of elite tickets



Total Object Ticket equation

$$\boxed{440 = x + y + z}$$

Total Tickets; # of reg Tickets; # of prem; # of elite

Money equation w/value

$$\boxed{\$3940 = 5x + 15y + 25z}$$

Total $ made; $ from regular Tickets; $ prem ticket; $ from elite

① $x+y+z=440$

② $5x+15y+25z=3940$

③ $x=3y+3z$

Sold more regular tickets, x is alone

$x = 3(y+z)$

Substitution method

$\boxed{x=3y+3z}$ combined so use parentheses

① $(3y+3z)+y+z=440$

ⓐ $(4y+4z=440)(-10)$

② $5(3y+3z)+15y+25z=3940$

$15y+15z+15y+25z=3940$

ⓑ $30y+40z=3940$

ⓐ $-40y-40z=-4400$

ⓑ $+\ 30y+40z=3940$

$\dfrac{-10y}{-10} = \dfrac{-460}{-10}$

$y = \boxed{46 \text{ premium tickets sold}}$

ⓐ $4y+4z=440$

$4(46)+4z=440$

$184+4z=440$

$-184 \qquad -184$

$\dfrac{4z}{4}=\dfrac{256}{4}$

$z = \boxed{64 \text{ elite tickets sold}}$

① $x+y+z=440$

$x+46+64=440$

$x+110=440$

$x = \boxed{330 \text{ regular tickets sold}}$

total objects and cost

Pepe Le Pew has three dates and is ordering three bouquets of flowers from Beck's Florist in Peoria. Three roses, 2 carnations, and 1 tulip cost $14 while 6 roses, 2 carnations and 6 tulips cost $38 and, 1 rose, 12 carnations and 1 tulip cost $18. What is the cost of each individual flower?

Total Equations

x : price of roses

y : price of carnations

z : price of tulips

① $(14 = 3x+2y+z)(-2)$

② $38 = 6x+2y+6z$

③ $(18 = x+12y+z)(-3)$

Step 1

① $14 = 3x+2y+z$

③ $+\ -54 = -3x-36y-3z$

ⓐ $(-40 = -34y-2z)2$

① $-28 = -6x-4y-2z$

② $+\ 38 = 6x+2y+6z$

ⓑ $66 = -2y+4z$

$\dfrac{2y}{2} = \dfrac{4z-66}{2}$

$y = 2z-33$

Step 2

ⓐ $-80 = -68y-4z$

ⓑ $+\ 66 = -2y+4z$

$\dfrac{-14}{-70} = \dfrac{-70y}{-70}$

$y = 0.2$

carnations are 20¢ or $0.20 each

ⓑ $66 = -2y+4z$

$66 = -2(0.2)+4z$

$66 = -0.4+4z$

$\dfrac{66.4}{4} = \dfrac{4z}{4}$

$z = 16.6$

tulips cost $16.60 each

Step 3

$18 = x+12y+z$

$18 = x+12(0.2)+(16.6)$

$18 = x+2.4+16.6$

$18 = x+19$

$-19 \qquad -19$

$-1 = x$

total object cost

17. **MODELING WITH MATHEMATICS** Three orders are placed at a pizza shop. Two small pizzas, a liter of soda, and a salad cost \$14; one small pizza, a liter of soda, and three salads cost \$15; and three small pizzas, a liter of soda, and two salads cost \$22. How much does each item cost?

x : price of small pizzas
y : price of a liter of soda
z : price of a salad

Total Equation

① $14 = 2 \cdot x + 1y + 1z$ (-1)
② $15 = x + y + 3z$
③ $22 = 3x + y + 2z$

Step 1 cancel y, create new system

① $-14 = -2x - y - z$
② $15 = x + y + 3z$
ⓐ $1 = -x + 2z$

① $-14 = -2x - y - z$
③ $22 = 3x + y + 2z$
ⓑ $8 = x + z$

Step 2 cancel a second variable, and solve new system + get 2 answers
choose elimination/substitution

ⓐ $1 = -x + 2z$
ⓑ $8 = x + z$
$\frac{9}{3} = \frac{3z}{3}$
$z = 3$
a salad costs \$3

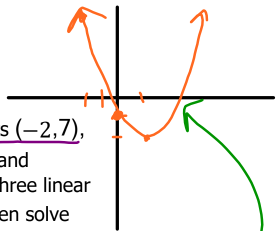
ⓑ $8 = x + z$
$8 = x + (3)$
$5 = x$
Small pizzas cost \$5

Step 3 plug in to original equation to find last variable

② $15 = x + y + 3z$
$15 = 5 + y + 3(3)$
$15 = 5 + y + 9$
$15 = y + 14$
$1 = y$ Sodas cost \$1

Writing functions to model problems

1. A parabola passes through three points $(-2,7)$, $(0,-1)$ and $(1,-2)$. Use these points and $y = ax^2+bx+c$ to construct a system of three linear equations in terms of a, b, and c and then solve the system.



We want to create a quadratic equation $y = \_\, x^2 + \_\, x + \underline{-1}$

Variables are a, b, c

$$y = ax^2 + bx + c$$

Plug in $(-2,7)$: $7 = a(-2)^2 + b(-2) + c$
$\boxed{4a - 2b + c = 7}$

Plug in $(0,-1)$: $-1 = a(0)^2 + b(0) + c$
$\boxed{c = -1}$

Plug in $(1,-2)$: $-2 = a(1)^2 + b(1) + c$
$\boxed{a + b + c = -2}$

Left, Right, and Ambidextrous Handed

The number of left-handed people in the world is one-tenth of the number of right handed people. The percent of right -handed people is nine times the percent of left-handed people and ambidextrous people combined. What percent of people ambidextrous?

x : % of ambidextrous people
y : % of right handed
z : % of lefted handed

$z = \frac{1}{10} y$

$y = 9(z + x)$

100%

Percent Total $x + y + z = 1$

x is a decimal, turn in to percent

$x = 0.05 \rightarrow 5\%$

Percent Mixture Problems

A nurse wishes to prepare a 32-ounce topical antiseptic solution containing 4% hydrogen peroxide. To obtain this mixture, purified water is to be added to the existing 2.5% and 20% hydrogen peroxide products. If only 2 ounces of the 20% hydrogen peroxide solution is available, how much of the 2.5% hydrogen peroxide solution and water is needed?

x: # oz 2.5% solution
y: # oz 20% solution
z: # oz water 0% solution

$x + y + z = 32$

$0.025x + 0.2y + 0z = .04(32)$

$y = 2$

$x + (2) + z = 32$

$x + z = 30$

$\boxed{z = 30 - x}$

$0.025x + 0.2(2) = 1.28 - .04z$

$100(0.025x + 0.04z = 0.88)$

$\boxed{2.5x + 4z = 88}$

$2.5x + 4(30 - x) = 88$

$2.5x + 120 - 4x = 88$

$\frac{-1.5x}{-1.5} = \frac{-32}{-1.5}$

$x = 21.33$ oz 2.5% solution

$x + z = 30$

$21.33 + z = 30$

$z = 8.66$ oz water

Eliot invested his \$6,000 bonus in three accounts earning 4.5%, 3.5%, and 3.6% interest. He invested twice as much in the account earning 4.5% as he did in the other two accounts combined. If the total simple interest for the year was \$234, how much did Joe invest in each account?

Define the Variables and Set up and solve completely:

4. In Meghan's wallet, there are one-dollar, five-dollar, and ten-dollar bills. She has a total of $210. There are 50 bills total where the number of one-dollar bills is one less than twice the number of five-dollar bills. How many of each bill are there?

x : # of $1 bills

y : # of $5 bills

z : # of $10 bills

① $210 = x + 5y + 10z$

② $50 = x + y + z$

③ $x = 2y - 1$

① $210 = 2y - 1 + 5y + 10z$

ⓐ $211 = 7y + 10z$

② $50 = 2y - 1 + y + z$

ⓑ $-10(51 = 3y + z)$

ⓐ $211 = 7y + 10z$

ⓑ $+\ -510 = -30y - 10z$

$-299 = -23y$

$\frac{-299}{-23} = \frac{-23y}{-23}$

$13 = y$

13 $5 bills

ⓑ $51 = 3(13) + z$

$51 = 39 + z$

$12 = z$

12 $10 bills

② $x + y + z = 50$

$x + 13 + 12 = 50$

$x + 25 = 50$

$x =$ 25 $1 bills

8. Ethan sold 82 items at his store for a total of $504. He sold packages of socks for $6, printed t-shirts for $12, and hats for $5. If he sold5times as many hats as he did t-shirts, how many of each item did he sell?

x : # of sock packages

y : # of t shirts

z : # of hats

① $82 = x + y + z$

② $504 = 6x + 12y + 5z$

③ $z = 5y$

① $82 = x + y + (5y)$

ⓐ $(82 = x + 6y)(-6)$

② $504 = 6x + 12y + 5(5y)$

$504 = 6x + 12y + 25y$

ⓑ $504 = 6x + 37y$

ⓐ $-492 = -6x - 36y$

ⓑ $+\ 504 = 6x + 37y$

$12 = y$

12 printed tshirts

ⓑ $82 = x + 6y$

$82 = x + 6(12)$

$82 = x + 72$

$10 = x$

10 sock packages

① $x + y + z = 82$

$10 + 12 + z = 82$

$22 + z = 82$

$z =$ 60 hats

Name_______________________________________ Additional Practice/ Guided Practice/Project Hour_____

Objective: **Solving Systems of Equations in Three Variables**

Section 1.4 BIM Algebra 2 Text p29-36

https://bit.ly/2NgTPXm

- Set up a system of equations in three variables to model real-life behavior
- Solving system of three variable equations through elimination method

no decimals

1. The sum of three integers is 40. Three times the smaller integer is equal to the sum of the others. Twice the larger is equal to 8 more than the sum of the others. Find the integers.

Study	**Plan**	**Act**
Define Variables:	The 3 Equations are:	Solve
X: first smaller number	A 1) $x + y + \boxed{z} = 40$	Step 1 Sub in z (cancel z)
Y: second middle number	B 2) $3x = y + z \rightarrow \boxed{z} = 3x - y$	A) $x + y + (3x - y) = 40$
Z: third larger number	C 3) $2\boxed{z} = 8 + (x + y)$	$4x = 40$, $\boxed{x} = 10$

C) $2(3x - y) = 8 + x + y$

$6x - 2y = 8 + x + y$

$5x - 3y = 8$

$5(10) - 3y = 8$

$-3y = -42$

$y = 14$

Step 2 solve for x+y

Reflect

Step 3 solve for z

A) $(10) + (14) + z = 40$

$z = 16$

(10, 14, 16)

2. The sum of three integers is 38. Two less than 4 times the smaller integer is equal to the sum of the others. The sum of the smaller and larger integer is equal to 2 more than twice that of the other. Find the integers.

Study	**Plan**	**Act**
Define Variables:	The 3 Equations are:	Solve
X:	1)	
Y:	2)	
Z:	3)	

Reflect

3. The sum of the angles A, B, and C of a triangle is 180°. Angle C is equal to the sum of the other two angles. Five times angle A is equal to the sum of angle C and B. Find the angles.

Study	**Plan**	**Act**
Define Variables:	The 3 Equations are:	Solve
X: measure of Angle A	1) $x + y + z = 180$	
Y: measure of Angle B	2) $z = x + y$	
Z: measure of Angle C	3) $5x = z + y$	

Reflect

Name_______________________________________ Additional Practice/ Guided Practice/Project Hour_____

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- Set up a system of equations in three variables to model real-life behavior
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1. The sum of three integers is 40. Three times the smaller integer is equal to the sum of the others. Twice the larger is equal to 8 more than the sum of the others. Find the integers.

Study	**Plan**	**Act**
Define Variables:	The 3 Equations are:	Solve
X: first smaller number	1) $x + y + z = 40$ (A)	Step 1
Y: second middle number	2) $3x = y + z \rightarrow 3x - y - z = 0$ (B)	A) $x + y + z = 40$ + B) $3x - y - z = 0$ → $4x = 40$, $x = 10$
Z: third larger number	3) $2z = 8 + (x + y) \rightarrow -x - y + 2z = 8$ (C)	A) $x + y + z = 40$ + C) $-x - y + 2z = 8$ → $3z = 48$, $z = 16$

Step 3 $(10) + y + (16) = 40$

$y = 14$

Reflect checked out

(10, 14, 16)

2. The sum of three integers is 38. Two less than 4 times the smaller integer is equal to the sum of the others. The sum of the smaller and larger integer is equal to 2 more than twice that of the other. Find the integers.

Study	**Plan**	**Act**
Define Variables:	The 3 Equations are:	Solve
X:	1)	
Y:	2)	
Z:	3)	

Reflect

3. The sum of the angles A, B, and C of a triangle is 180°. Angle C is equal to the sum of the other two angles. Five times angle A is equal to the sum of angle C and B. Find the angles.

Study	**Plan**	**Act**
Define Variables:	The 3 Equations are:	Solve
X: measure of Angle A	A) $x + y + z = 180$	AB AC
Y: measure of Angle B	B) $z = x + y \rightarrow x + y - z = 0$	
Z: measure of Angle C	C) $5x = z + y \rightarrow 5x - y - z = 0$	

Reflect

4. The sum of the angles A, B, and C of a triangle is 180°. The larger angle C is equal to twice the sum of the other two. Four times the smallest angle A is equal to the difference of angle C and B. Find the angles.

Study	**Plan**	**Act**
Define Variables:	The 3 Equations are:	Solve
X:	1)	
Y:	2)	
Z:	3)	

Reflect

5. A parabola passes through three points $(-2,11)$, $(-1,4)$, and $(1,2)$. Use these points and $\boxed{y = ax^2+bx+c}$ to construct a system of three linear equations in terms of a, b, and c and solve it.

x y   x y   x y

Study
Define Variables:
A: the coefficient a for x^2
B: the coefficient b for x
C: the constant c term

Plan
The 3 Equations are:
$11 = a(-2)^2 + b(-2) + c$
Ⓐ $11 = 4a - 2b + c$
$4 = a(-1)^2 + b(-1) + c$
Ⓑ $(4 = a - b + c)\cdot 2$
$2 = a(1)^2 + b(1) + c$
Ⓒ $2 = a + b + c$

Act
Solve

step 1

AB
Ⓐ $11 = 4a - 2b + c$
Ⓑ $-8 = -2a + 2b - 2c$
Ⓓ $(3 = 2a - c)\cdot 2$

BC
Ⓑ $4 = a - b + c$
Ⓒ $2 = a + b + c$
Ⓔ $6 = 2a + 2c$

step 2
Ⓓ $6 = 4a - 2c$
Ⓔ $6 = 2a + 2c$
$12 = 6a$
$a = 2$

$6 = 2(2) + 2c$
$6 = 4 + 2c$
$2 = 2c$
$c = 1$

step 3
Ⓒ $2 = 2 + b + 1 \rightarrow b = -1$

Reflect
The Parabola that goes through these points is:

$\boxed{y = 2x^2 - x + 1}$

6. A parabola passes through three points $(-1,7)$, $(1,-1)$ and $(2,-2)$. Use these points and $y = ax^2+bx+c$ to construct a system of three linear equations in terms of a, b, and c and then solve the system.

Study	**Plan**	**Act**
Define Variables:	The 3 Equations are:	
A:	1)	
B:	2)	
C:	3)	

Answers:

Plot your equations on a 3d graph and the key below to see if your equations get the correct solution.

If they do not fix your equations.
https://www.geogebra.org/3d?lang=en

2 8, 12, 18
4 $A = 20^\circ$, $B = 40^\circ$, and $C = 120^\circ$
~~The amount invested at 2% was $6,000, the amount invested at 4% was $2,000, and the amount invested at 5% was [illegible].~~
~~72 nickels, 23 dimes, and 10 quarters~~
~~10 ounces of the 1.5% hydrogen peroxide solution and 2 ounces of water~~
~~96 adult tickets, 20 child tickets, and 12 senior tickets were sold.~~
6 $a = 1, b = -4$, and $c = 2$