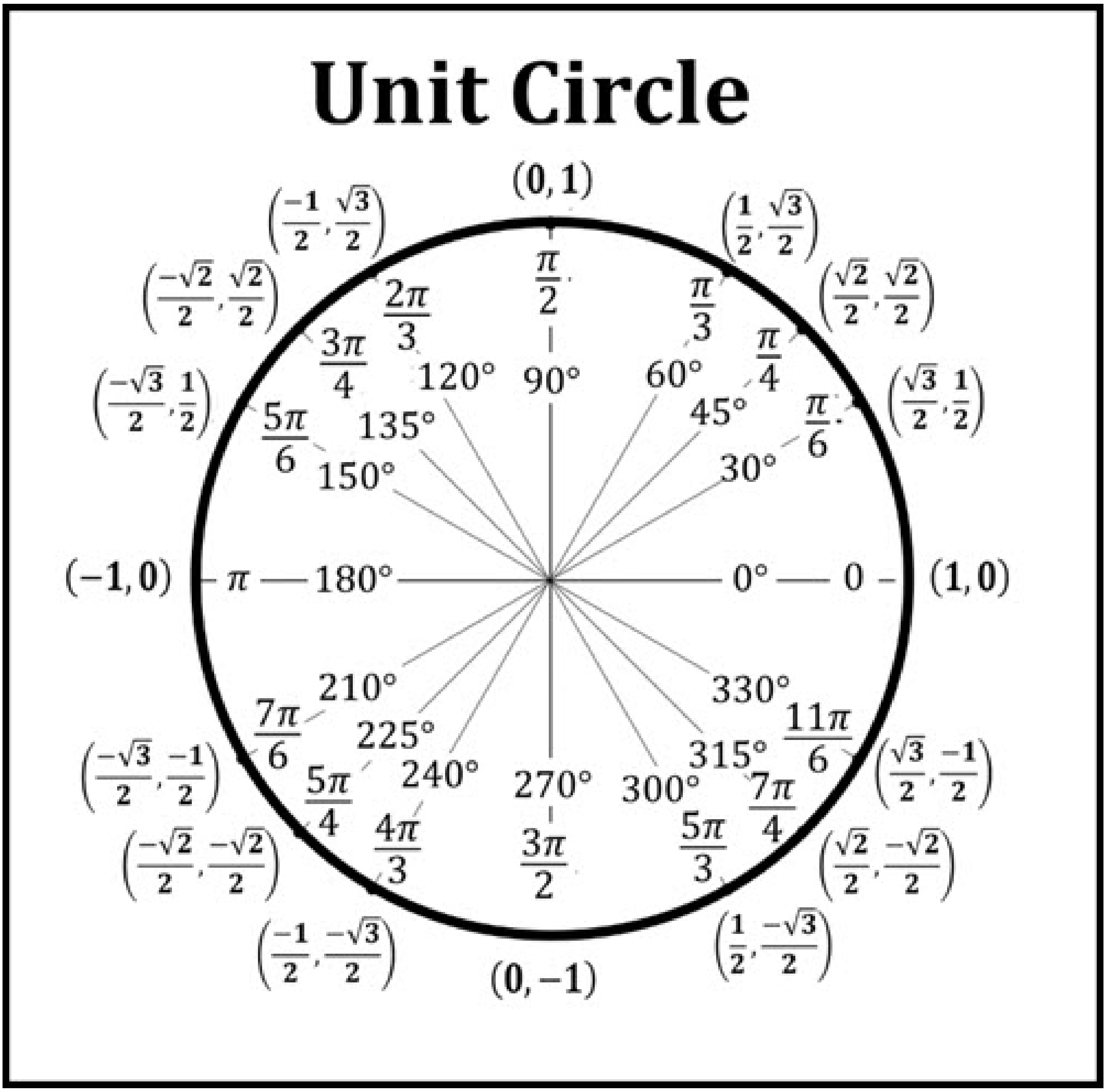
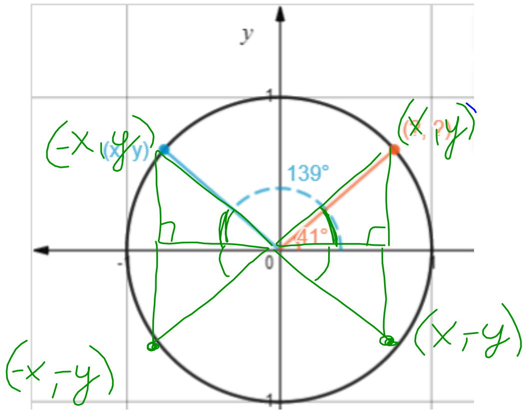

Unit Circle
(0,1)
(1,0)
(−1,0)
(0,−1)
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(\frac{-1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{-\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{-\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right)$
$\left(\frac{-\sqrt{2}}{2},\frac{-\sqrt{2}}{2}\right)$
$\left(\frac{-1}{2},\frac{-\sqrt{3}}{2}\right)$
$\left(\frac{1}{2},\frac{-\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{-\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)$
$\frac{\pi}{2}$ 90°
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$ 45°
$\frac{\pi}{6}$ 30°
0° 0
$\pi$ 180°
$\frac{2\pi}{3}$ 120°
$\frac{3\pi}{4}$ 135°
$\frac{5\pi}{6}$ 150°
$\frac{7\pi}{6}$ 210°
$\frac{5\pi}{4}$ 225°
$\frac{4\pi}{3}$ 240°
$\frac{3\pi}{2}$ 270°
$\frac{5\pi}{3}$ 300°
$\frac{7\pi}{4}$ 315°
$\frac{11\pi}{6}$ 330°

## EXPLORE & REASON

pg. 177

**A. The graph shows the terminal sides of an angle with measure $\theta$ and its supplement, $180 - \theta$, on the unit circle.**




**A.** How are the coordinates of the intersection of the terminal side of an angle with measure $\theta$ and the unit circle related to the sine and cosine of the angle?

$\cos\theta = \frac{adj}{hyp} = \frac{x}{1} = x$

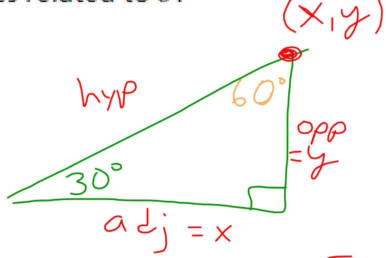
$\sin\theta = \frac{opp}{hyp} = \frac{y}{1} = y$

$\boxed{(\cos\theta, \sin\theta)}$

**B.** What do you notice about $\theta$ and the measure of the acute angle formed by the terminal side of $180 - \theta$ and the $x$-axis? they are equal

**C.** Drag points on the circle to draw terminal sides of angles in Quadrants III and IV that form the same acute angle with the $x$-axis as the angles in Quadrants I and II. How are these angles related to $\theta$?

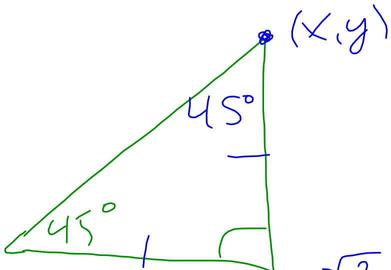
**D.** **Communicate Precisely** How are all four terminal sides related geometrically on the coordinate plane?




$\cos 30° = 0.86 = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

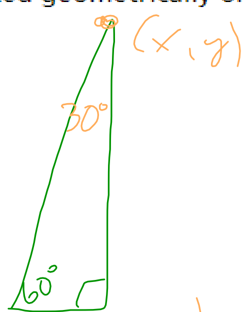
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$




$\cos 45° = 0.70 = \frac{\sqrt{2}}{2}$

$\sin 45 = \frac{\sqrt{2}}{2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$




$\cos(60°) = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

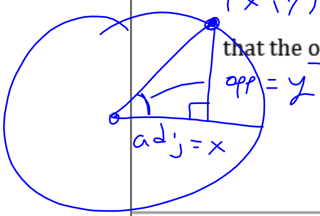
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

# 7-4 Unit Circle Level 1 Name:

Using your Unit Circle find the following values.

| | |
|---|---|
| Cosine is which coordinate? X | Sine is which coordinate? y |
| $cos\ (60°)\ = \frac{1}{2}$ | $sin\ (45°)\ =$ |
| $sin\ (90°) = 1$ | $cos\ \left(\frac{\pi}{6}\right)\ =$ |
| $sin\ \left(\frac{3\pi}{4}\right)\ = \frac{\sqrt{2}}{2}$ | $cos\ (180°)\ =$ |
| $cos\ (315°)\ = -\sqrt{2}/2$ | $sin\ (210°)\ =$ |
| $cos\ (\pi) =$ | $sin\ \left(\frac{4\pi}{3}\right)\ =$ |
| $sin\ \left(\frac{11\pi}{6}\right)\ =$ | $cos\ (\frac{3\pi}{2})\ =$ |

$(\cos\theta, \sin\theta)$
$(x, y)$




$tan(\theta) = \frac{opposite}{adjacent}$ and we have learned that the opposite side is the y coordinate and the adjacent side is the x coordinate.

So.... $tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

Divide the y coordinate by the x coordinate.

| | |
|---|---|
| $tan\ (30°)\ = \frac{\sin(30)}{\cos(30)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ ) Flip<br>same<br>$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$ | $tan\ \left(\frac{5\pi}{4}\right)\ = 1$ |
| $tan\ (\pi)\ = \frac{0}{-1} = 0$ | $tan\ \left(\frac{\pi}{2}\right)\ = \frac{1}{0} =$ undefined |

# 7-4 Unit Circle Level 2

Name:

Using your Unit Circle find the following values.

| | |
|---|---|
| How do you know if an angle in degrees is more than 1 extra rotation?<br>over 360° → | How do you find the Positive angle version of it? (between 0 and 360)<br>subtract 360° |
| How do you know if an angle in radians is more than 0 extra rotation?<br>over $2\pi$ → | How do you find the Positive angle version of it? (between 0 and $2\pi$)<br>subtract $2\pi$ |
| What do you do if you see an angle in degrees that is negative?<br>add 360° | What do you do if you see an angle in radians that is negative?<br>add $2\pi$ |
| $\cos\left(\frac{9\pi}{4}\right) =$ | $\sin(-45°) =$ |
| $\sin(390°) =$ | $\cos(-135°) = \cos(225°) = \frac{-\sqrt{2}}{2}$<br>$-135° + 360° = 225°$ |
| $\cos\left(\frac{5\pi}{2}\right) =$ | $\tan\left(-\frac{7\pi}{4}\right) =$ |
| $\sin\left(-\frac{3\pi}{2}\right) =$ | $\sin(-225°) =$ |
| $\sin(4\pi) =$ | $\sin\left(\frac{11\pi}{3}\right) =$ |
| $\cos(510°) =$ | $\tan\left(\frac{17\pi}{6}\right) = \frac{\sin\frac{5\pi}{6}}{\cos\frac{5\pi}{6}} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{-\sqrt{3}}$<br>$\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$ |

$\frac{1}{2} \cdot \frac{2}{-\sqrt{3}} = \frac{\cancel{2}}{-\cancel{2}\sqrt{3}} = \frac{1}{-\sqrt{3}}$

# 7-4 Unit Circle Level 2

Name:

If $\cos\theta = \frac{1}{\sec\theta}$ then $\sec\theta = \frac{1}{\cos\theta}$ and....

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}}$$

When you have 1 over a fraction, flip the fraction!

$$\frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

So... $\sec 30^\circ = \frac{2\sqrt{3}}{3}$

In the next block it explains how and why that last change happened.

Now, in math we do not leave roots/radicals in the denominator. We do something called "rationalizing the denominator" which means we make it so there is no root in the denominator, numerator is fine. We do this by multiplying by a special $1 = \frac{\sqrt{a}}{\sqrt{a}}$ It looks like this:

$$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

This is more acceptable than $\frac{2}{\sqrt{3}}$ and the denominator just becomes 3 because

$$\sqrt{3} \cdot \sqrt{3} = \sqrt{9} = 3$$

So this time when doing Tangent, rationalize the denominator.

| | |
|---|---|
| $csc\ (30^\circ) =$ | $sec\ (120^\circ) =$ |
| $sec\left(\frac{\pi}{4}\right) =$ | $csc\left(\frac{3\pi}{4}\right) =$ |
| $csc\left(-\frac{\pi}{2}\right) = csc\left(\frac{3\pi}{2}\right) = \frac{1}{\sin\left(\frac{3\pi}{2}\right)} = \frac{1}{-1} = -1$<br>$-\frac{\pi}{2} + \frac{4\pi}{2} = \frac{3\pi}{2}$ | $tan\ (120^\circ) =$ |
| $sec\left(\frac{4\pi}{3}\right) =$ | $tan\left(-\frac{7\pi}{4}\right) =$ |
| $cot\ (-180) =$ | $tan\left(\frac{17\pi}{6}\right) =$ |
| $csc\ (405^\circ) =$ | $cot\left(-\frac{\pi}{2}\right) =$ |

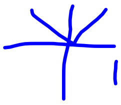
# 7-4 Unit Circle Level 3 Name:

Remember when we used the inverse to find the angle measure in the right triangle? So, if you see the inverse trig function of the coordinate ratio, it is asking you to determine which angles (there will be more than 1) have that ratio as that trig function coordinate. Look at your Unit Circle and your Trig Foldable side flaps to figure out Tangent and Cotangent.

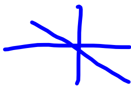
Ex. $\sin^{-1}\left(-\frac{1}{2}\right)$ is asking which angle has its y-coordinate (because sine is y) as negative ½? This will be in quadrant 3 and 4 because the y value is negative. This is $210^0$ and 330° and any other angle with extra rotations. So we write: $\sin^{-1}\left(-\frac{1}{2}\right) = 210^\circ + 360k$ and $330^\circ + 360k$

Evaluate the inverse trig function.

1. $\cos^{-1}\left(\frac{1}{2}\right)$
2. $sin^{-1}(-1)$
3. $cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
4. $\tan^{-1}\left(-\sqrt{3}\right)$
5. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$



$60^\circ + 360K$
$120^\circ + 360K$



$\frac{\cos:\ -\frac{\sqrt{3}}{2}}{\sin:\ \frac{1}{2}}$ $156^\circ + 360K$
$330^\circ + 360K$

Evaluate the expression. Do the inner trig inverse function first to get the angle measure, then do the outer trig function to that angle measure to get the trig ratio.

6. $\sin(\cos^{-1}(0))$
7. $\tan(\sin^{-1}(-1))$
8. $\csc\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$\sin(90^\circ)$ $\sin(270^\circ)$
1 or −1

$\csc(120^\circ)$ $\csc(300^\circ)$
$\frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3} = \frac{1}{-\frac{\sqrt{3}}{2}}$

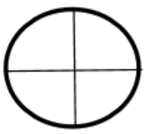
9. What do you get when you type in $\sin^{-1}\left(-\frac{1}{2}\right)$ into a calculator? ______

10. How many angle measures does the calculator give you? ________

This is because the calculator has restricted the domain of the initial function of sine. Instead of being able to get the full unit circle up to 360 and more rotations, it only gives certain outputs.

11. Type in a calculator $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ =________ and what does the calculator say for

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ =________ Which quadrants are these two angles in? _____ and _____



Shade in the Unit Circle given here for the quadrants Sine Inverse is defined in:

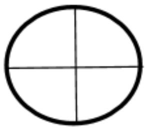
12. From what angle to what angle is Inverse Sine defined in? What angles are not defined for Sine?

# 7-4 Unit Circle Level 3 Name:

13. Type in a calculator $\cos^{-1}\left(\frac{1}{2}\right)$ =________ and what does the calculator say for

$\cos^{-1}\left(-\frac{1}{2}\right)$ =________ Which quadrants are these two angles in? __________

Shade in the Unit Circle given here the quadrants Cosine Inverse is defined in:

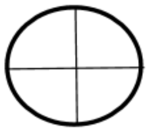


14. From what angle to what angle is inverse cosine defined in? What angles are not defined for Cosine?

15. Type in a calculator $\tan^{-1}(1)$ =________ and what does the calculator say for

$\tan^{-1}(-1)$ =________ Which quadrants are these two angles in? __________

Shade in the Unit Circle given here the quadrants Tangent Inverse is defined in:



16. From what angle to what angle is inverse tangent defined in? What angles are not defined for Tangent?

17. What connections can you make between the quadrants Inverse Sine is defined in and the quadrants Inverse Cosecant is defined in? What angles are not defined for Cosecant?

18. Evaluate the inverse trig function.

Think about it: If sin(30°)= ½ than what is $\csc^{-1}(2)$ ? $= \sin^{-1}\left(\frac{1}{2}\right) = 30°$

$\csc^{-1}(2) = 30°$

19. Evaluate the inverse trig function.

$\sec^{-1}(-2)$ = 120° $\cos^{-1}\left(-\frac{1}{2}\right) = 120°$