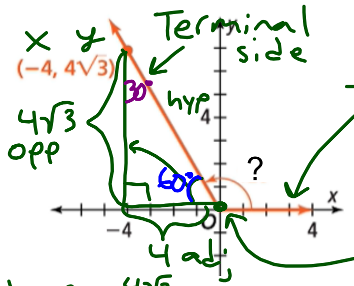
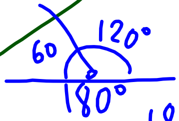
Angles and Radians
Triangle Trigonometry

Mrs. Theo
3 13 23
Notes

Standard Position



Standard Position:
initial side on right x-axis, vertex at (0,0)



$180-60=120°$

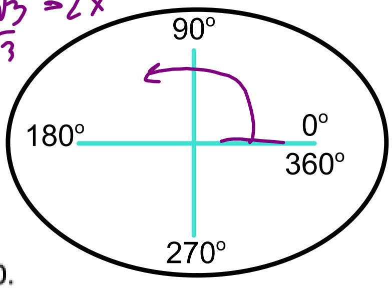
$\tan\theta = \frac{4\sqrt{3}}{4}$

$\theta = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right) = 60$

30-60-90
$x - x\sqrt{3} - 2x$
$4 \quad 4\sqrt{3}$

Types of Angles

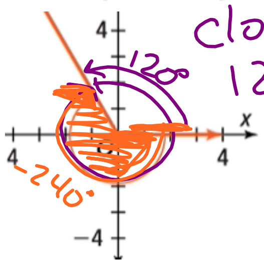
What is the measure of the angle shown?



As a positive angle measure: $0 \le \theta \le 360$.

counterclockwise

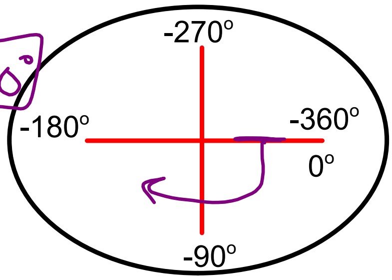
As a negative angle measure: $-360 \le \theta \le 0$.



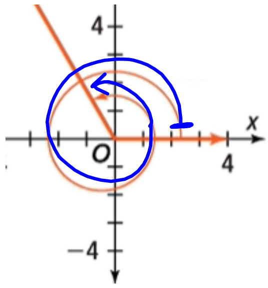
clockwise

$120° - 360° = \boxed{-240°}$

$\theta - 360°$



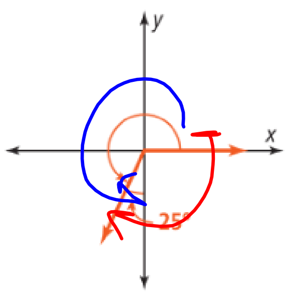
As a positive angle measure greater than 360:
$\theta + 360k$, where k is a natural number representing the number of rotations.



$120° + 360(1) = 480°$
starting 1 rotation

$360 + 120 = 480°$
1 rotation extra

b.



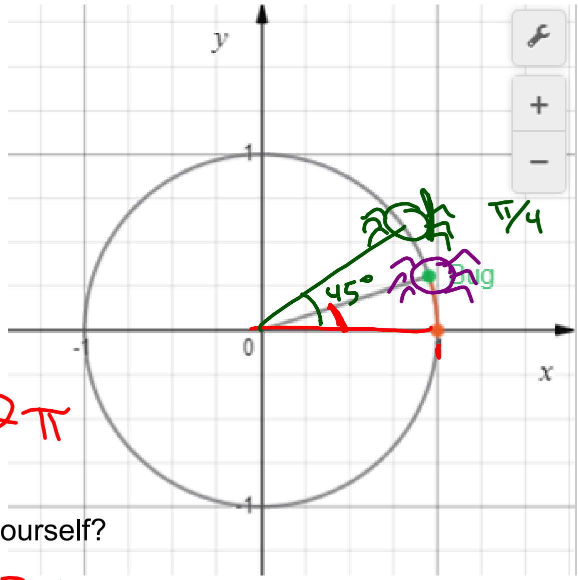
Positive: $180 + (90-25)$
$180 + 65 = 245°$
$-(90+25)+360 = 245°$
$270° - 25° = 245°$

Negative: $-90-25 = -115°$
$-(90+25) = -115°$
$245 - 360 = -115°$

Angle >360°: $245° + 360° = 605°$
$-115° + 360(2) = 605°$

EXPLORE & REASON

Workbooks pg. 173



A bug is placed at the point (1, 0) of the coordinate plane shown. It starts walking counterclockwise along a circle with radius 1.

A. **Model With Mathematics** How can you calculate the distance along the circle the bug has traveled? How can you determine the measure of the central angle?

What is the distance around a circle called?

$C = 2\pi r$ fraction $\cdot 2\pi$

$C = 2\pi$

How many degrees would you spin if you spin around yourself?

$360°$ fraction $\cdot 360°$

B. When the bug has traveled $\frac{1}{8}$ of the way along the circle, how far has it traveled? What central angle does its path travel through?

Distance $\frac{1}{8}\cdot\frac{2\pi}{1}$

central angle $\frac{1}{8}\cdot\frac{360}{1}$

$\div 2\ \frac{2\pi}{8}\ \div 2 = \frac{\cancel{2}\cdot 1\pi}{\cancel{2}\cdot 4}$

$\frac{360}{8}$

$45°$

$\frac{\pi}{4}$ radians

C. What are the distances shown traveled and the central angles when the bug has traveled $\frac{1}{6}$?of the way around the circle and $\frac{4}{5}$ of the way around the circle?

$\frac{4}{5}\cdot\frac{2\pi}{1}$

$\frac{8\pi}{5}$

$\frac{4}{5}\cdot\frac{360}{1}$

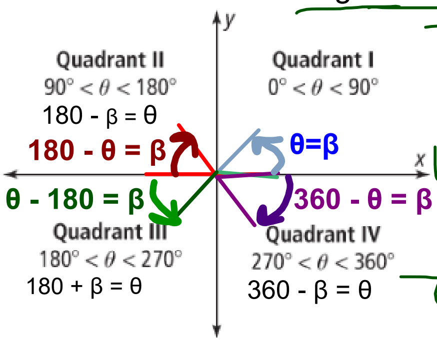
$\frac{4}{\cancel{5}}\cdot\frac{2\cdot\cancel{5}\cdot 36}{1}$

$288°$

Reference Angle

It is the acute angle β formed between the terminal side and the x-axis of an angle θ in standard position

less than 90°



What is the reference angle for a 330°?

$360° - 330° = \boxed{30°}$

Q4

What is the reference angle for a 111°?

$180° - 111° = \boxed{69°}$

Q2

What is the angle if the reference angle 16° is in QIII

$180 + 16° = \boxed{196°}$

16°

Conversion of Angle Measures

One full rotation = 360° = 2π rad

Half a rotation = 180° = π rad

Radian	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Degrees	0	90	180	270	360

Degrees to Radians

$$\boxed{\_\_^{o} \cdot \frac{\pi \text{ rad}}{180^{o}}}$$

a. 112° to radians

$$\cancel{112°} \cdot \frac{\pi \text{ rad}}{\cancel{180°}} =$$

$$\frac{112\pi}{180} = \boxed{\frac{28\pi}{45}} \text{ rad}$$

Radians to Degrees

$$\boxed{\_\_ \cancel{\text{Rad}} \cdot \frac{180^{o}}{\pi \cancel{\text{rad}}}}$$

Note: Units of measure should cancel, dividing diagonally

b. $\frac{\pi}{6}$ radians to degrees

$$\frac{\cancel{\pi}}{6} \cancel{\text{rad}} \cdot \frac{180°}{\cancel{\pi} \cancel{\text{rad}}}$$

$$\frac{180}{6} = \boxed{30°}$$

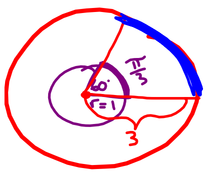
Arc Length

Is the portion of the distance around the circle.

$$S = \theta r$$

Note: given angle must be in Radians or converted to it

ArcLength = Angle in Radians • Radius



similar figures

scale factor=3

distance x 3

π/3 x 3

θr

NASA is tracking a satellite traveling in a circular orbit above Earth. It can only be tracked while it orbits through a $\frac{\pi}{6}$ angle. The radius of Earth is 6,400 km. What is the distance the satellite travels while it is being tracked?

r = 6400 + 320 = 6720

$S = \theta r$

$S = \frac{\pi}{6}(6720)$

S = 3,518.6 km