Study Guide for Unit Exam on Rational Expressions and Functions Key

15. Complete the table for : $f(x) = \frac{10x+90}{25-5x}$ $\frac{10(x+9)}{-5(x-5)}$

Simplest Form	Domain	X intercept(s)	Y intercept	Vertical Asymptotes	Holes	Horizontal Asymptote	Does this have a slant asymptote?
$\frac{10(x+9)}{-5(x-5)}$	$x \in \mathbb{R}$, $x \neq 5$ or $x \in (-\infty, 5) \cup (5, \infty)$	$10(x+9)=0$, $x=-9$, $(-9, 0)$	$\frac{10(0+9)}{-5(0-5)} = \frac{90}{+25}$, $(0, 3.6)$	$x=5$	NA	Deg = Deg, $y = \frac{10}{-5}$, $y=-2$	No

16. Complete the table for : $g(x) = \frac{x+4}{2x^2-2x-40}$ $\frac{x+4}{2(x^2-x-20)} = \frac{(x+4)}{2(x-5)(x+4)}$

Simplest Form	Domain	X intercept(s)	Y intercept	Vertical Asymptotes	Holes	Horizontal Asymptote	Does this have a slant asymptote?
$\frac{1}{2(x-5)}$	$x \in \mathbb{R}$, $x \neq 5$, $x \neq -4$ or $x \in (-\infty, -4) \cup (-4, 5) \cup (5, \infty)$	$1 \neq 0$, NA	$\frac{1}{2(0-5)}$, $(0, \frac{1}{-10})$	$x=5$	$\frac{1}{2(-4-5)}$, $x=-4$, $(-4, \frac{1}{-18})$	Bottom Heavy, $y=0$	NA

17. Complete the table for : $j(x) = \frac{4x^2-1}{2x^2+5x-3}$ $\frac{(2x+1)(2x-1)}{(2x-1)(x+3)}$

Simplest Form	Domain	X intercept(s)	Y intercept	Vertical Asymptotes	Holes	Horizontal Asymptote	Does this have a slant asymptote?
$\frac{2x+1}{x+3}$	$x \in \mathbb{R}$, $x \neq -3$, $x \neq \frac{1}{2}$	$2x+1=0$, $(-\frac{1}{2}, 0)$	$\frac{2(0)+1}{0+3}$, $(0, \frac{1}{3})$	$x=-3$	$\frac{2(\frac{1}{2})+1}{\frac{1}{2}+3}$, $x=\frac{1}{2}$, $(\frac{1}{2}, \frac{4}{7})$, $\frac{2}{\frac{7}{2}} \to 2 \cdot \frac{2}{7} = \frac{4}{7}$	Deg = Deg, $y = \frac{4}{2}$, $y=2$	No

18. Complete the table for : $m(x) = \frac{2x^3-14x^2}{x^2-4x-21}$ $\frac{2x^2(x-7)}{(x-7)(x+3)}$

Simplest Form	Domain	X intercept(s)	Y intercept	Vertical Asymptotes	Holes	Horizontal Asymptote	Does this have a slant asymptote?
$\frac{2x^2}{x+3}$	$x \in \mathbb{R}$, $x \neq -3$, $x \neq 7$ or $x \in (-\infty, -3) \cup (-3, 7) \cup (7, \infty)$	$2x^2=0$, $(0, 0)$	$(0, 0)$	$x=-3$	$\frac{2(7)^2}{7+3}$, $x=7$, $(7, \frac{98}{10})$, $(7, 9.8)$	No	Yes, $y=2x-6$

$$x+3 \overline{)\,2x^2+0x+0}$$
$$-(2x^2+6x)$$
$$-6x+0$$
$$-(-6x-18)$$
$$18$$

$$x^2-4x-21 \overline{)\,2x^3-14x^2+0x+0}$$
$$-(2x^3-8x^2-42x)$$
$$-6x^2+42x+0$$
$$-(-6x^2+20x+126)$$
$$22x-126$$

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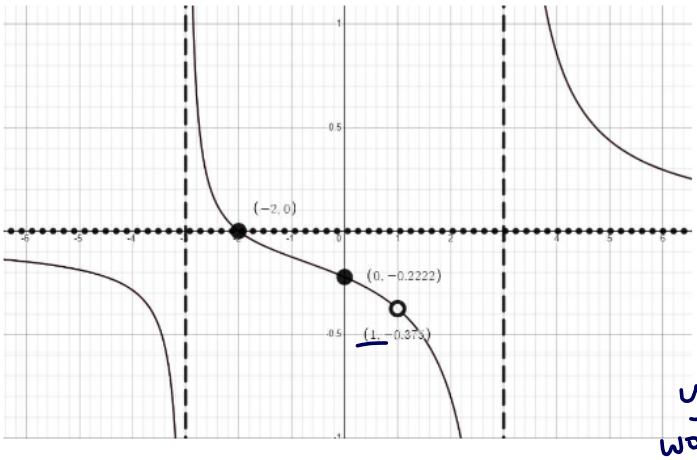
19. Write the rational function that is related to the given graph below.



HA: $y=0$ Bottom Heavy

Va= $x=3$ $x=-3$

$x-3$ $x+3$

hole $x=1 \rightarrow x-1$

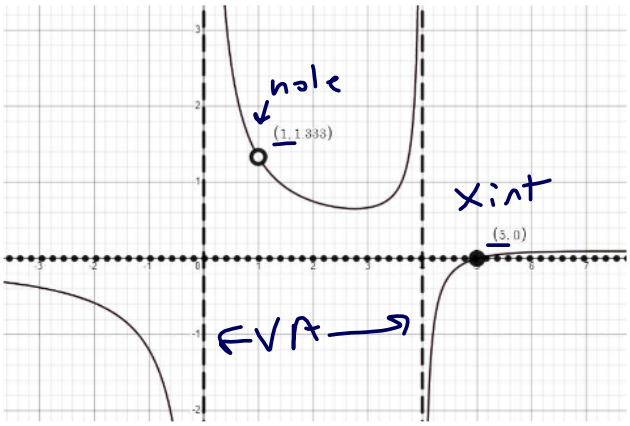
x int $x=-2 \rightarrow x+2$

y int:

$$\frac{(x+2)(x-1)}{(x+3)(x-3)(x-1)}$$

y int works $\frac{(0+2)(0-1)}{(0+3)(0-3)(0-1)} = \frac{-2}{9}$ ✓

20. Write the rational function that is related to the given graph below.



HA $y=0$ Bottom Heavy

1) Find VA

$x=0$ $x=4$

Factors x and $x-4$ in denominator

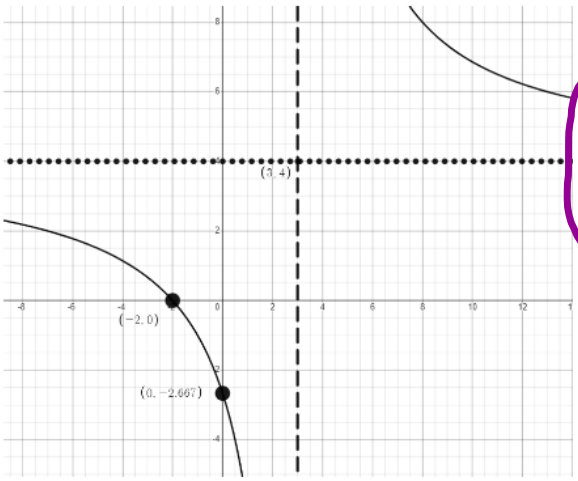
2) Find hole $x=1 \rightarrow x-1$ in top + bottom

3) Find x int $x=5 \rightarrow x-5$ on top

4) Check y int multiply by what you need

$$\frac{(x-5)(x-1)}{x(x-4)(x-1)}$$

21. Write the rational function that is related to the given graph below.



HA: $y=4$ Deg = Deg

VA: $x=3 \rightarrow (x-3)$

Holes None

x int: $x=-2 \rightarrow x+2$

y int: -2.667 check $\frac{4(0+2)}{(0-3)} = \frac{8}{-3}$ ✓

$$\frac{4(x+2)}{(x-3)} = \frac{4x+8}{x-3}$$

Study Guide for Unit Exam on Rational Expressions and Functions Key

Simplify each of the expressions COMPLETELY SHOW ALL OF YOUR WORK IN A CLEAR FASHION

(18) or 22. $\frac{6}{8x^4y} + \frac{7}{6x^2y^5}$

$\frac{18y^4 + 28x^2}{24x^4y^5}$

$\frac{2(9y^4+14x^2)}{24x^4y^5}$

$\frac{9y^4+14x^2}{12x^4y^5}$

23. $\frac{x+7}{x^2-3x-4} - \frac{x+6}{x^2+2x-24}$

$\frac{x+7}{(x-4)(x+1)} - \frac{x+6}{(x-4)(x+6)}$

$\frac{x+7}{(x-4)(x+1)} - \frac{1}{(x-4)} \cdot \frac{(x+1)}{(x+1)}$

$\frac{x+7-x-1}{(x+4)(x+1)} = \frac{6}{(x+4)(x+1)}$

24. $\frac{5y+3}{y^2-9y} - \frac{6y+2}{9-y}$

$y(y-9) \quad -1(y-9)$

since $\frac{-3}{4} = \frac{3}{-4} = \frac{-3}{4}$

$\frac{5y+3}{y(y-9)} + \frac{(6y+2)y}{(y-9)y}$

$\frac{5y+3+6y^2+2y}{y(y-9)}$

$\frac{6y^2+7y+3}{y(y-9)}$

25. $\frac{1}{x+7} + \frac{6}{5x+5} - \frac{(3x-6)}{x^2+8x+7}$

$5(x+1) \quad (x+7) \quad (x+7)(x+1)$

$\frac{5x+5+6x+42-15x+30}{5(x+1)(x+7)}$

$\frac{-4x+77}{5(x+1)(x+7)}$

Simplify each of the expressions COMPLETELY SHOW ALL OF YOUR WORK IN A CLEAR FASHION

19 or 26. $\left(\frac{(2x^{-5}y)^{-3}}{4x^0y^{-5}}\right)^2$

$\left(\frac{2^{-3} \cdot x^{15} \cdot y^{-3}}{4 \quad y^{-5}}\right)^2$

$\left(\frac{x^{15} \cdot y^5}{2^3 \cdot 4 \cdot y^3}\right)^2$

$\frac{x^{15} \quad y^2}{(8 \cdot 4)^2}$

$\frac{x^{30} \quad y^4}{32^2}$

$\frac{x^{30} \quad y^4}{1024}$

- $x^0 = 1$
- $(\text{product})^{\text{power}}$ - multiply exponents - distribute to all
- Divide variables: Subtract exponents $y^{5-3} = y^2$
- Variables with Negative exponents need to move and sign changes
- Multiply Variables: Add exponents

22 or 27. $\left(\frac{5}{4}xy^8\right)^{-4} \cdot (8x^{-6}y^3)^2$

$\left(\frac{5}{4}\right)^{-4} \cdot x^{-4} \cdot y^{-32} \cdot 8^2 \cdot x^{-12} \cdot y^6$

negative fraction → Flip it

$\left(\frac{4}{5}\right)^4 \frac{8^2 \cdot y^6}{x^4 \cdot y^{32} \cdot x^{12}}$

$\frac{4^4 \cdot 8^2}{5^4} \cdot y^{6-32} \cdot \frac{1}{x^{4+12}}$

$\frac{16384}{625 \quad y^{26} x^{16}}$

$\frac{(2x^{-5}y)^{-6}}{4^2 y^{-10}} \Rightarrow \frac{2^{-6} x^{30} y^{-6}}{4^2 y^{-10}} \rightarrow \frac{x^{30} y^{10}}{2^6 \cdot 4^2 y^6} \rightarrow \frac{x^{30} \quad y^4}{1024}$

Study Guide for Unit Exam on Rational Expressions and Functions Key

Factor 1st!!!!

Simplify each of the expressions COMPLETELY SHOW ALL OF YOUR WORK IN A CLEAR FASHION

28. $\frac{5x+10}{25x^6} \cdot \frac{5x^2+30x-35}{x^2+9x+14}$

$5(x+2)$ $5(x+7)(x-1)$

$\frac{5 \cdot 5 \cdot (x-1)}{25\ x^6}$

$\frac{x-1}{x^6}$

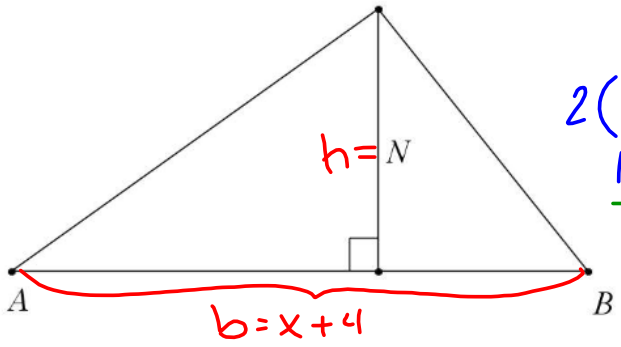
29. $\frac{4x+8}{16x^5} \cdot \frac{x^2-6x+8}{x^2-4} \div \frac{8x^{10}-32x^8}{16x+32}$ Flip

$\frac{4(x+2)}{16x^5} \cdot \frac{(x-4)(x-2)}{(x+2)(x-2)} \cdot \frac{16(x+2)}{8x^8(x^2-4)}$ $(x-2)(x+2)$

$\frac{x-4}{x^{5} \cdot 2 \cdot x^{8}}$

$\frac{x-4}{2x^{13}}$

Refer to the given figure (NOT DRAWN TO SCALE)



30. Write the equation that you can use to determine the value N

$2(5x^2+14x-24) = \frac{1}{2}(x+4)N \cdot 2$

$\frac{10x^2+28x-48}{x+4} = \frac{(x+4)N}{x+4}$

31. Determine the value of N (be careful)

$2(5x^2+14x-24)$

$\frac{2(5x-6)(x+4)}{x+4} = N$

$2(5x-6) = N$

$10x-12 = N$

Area of Triangle $= \frac{1}{2} b \cdot h$

$A = 5x^2+14x-24 = \frac{1}{2}(x+4)N$

$AB = x+4$

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Solving Rational Equations

32. Where do you find POTENTIAL extraneous solutions? Support with an example

in rational equations, we can't have denominators of 0

33. What is an extraneous solution to a rational equation? Support with an example

it is a answer gotten that doesn't work when you plug it back in.

34. What is a VALID solution to a rational equation?

a solution you get that will satisfy the equation AND not make any denominator 0.

Show ALL of your Work to receive credit, BE SURE to check for extraneous solutions

Rational Equation Problem 35

$$\frac{2\cdot 5}{2\cdot 2x+6}+\frac{x\cdot 2}{x+3\cdot 2}=\frac{4x+10}{4x+12}$$

$2(x+3)$ $\quad 4(x+3)$

$$\frac{10+4x}{4(x+3)}=\frac{4x+10}{4(x+3)}$$

$10+4x=4x+10$

$0=0$

Infinitely many solutions

$x\in\mathbb{R}$ $x\neq -3$

Rational Equation Problem 36

$$\frac{(x-3)}{(x-3)}\frac{2x-5}{x+3}-\frac{5}{3-x}\cdot\frac{(x+3)}{(x+3)}=\frac{x^2+5x+42}{x^2-9}$$

$-1(x-3)$ $\quad (x-3)(x+3)$

$2x^2-11x+15+5x+15=x^2+5x+42$

$x^2-11x-12=0$

$(x-12)(x+1)=0$

$x=12$ $\quad x=-1$

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Show ALL of your Work to receive credit, BE SURE to check for extraneous solutions

Rational Equation Problem 37

$$\frac{x+11}{x^2-5x+4}=\frac{5}{x-4}-\frac{3}{x-1}$$

$(x-4)(x-1)$

$(x-1)$ $(x-4)$

Distribute subtraction

$$\frac{x+11}{(x-4)(x-1)}=\frac{5x-5}{(x-4)(x-1)}-\frac{(3x-12)}{(x-1)(x-4)}$$

$x+11=5x-5-3x+12$

$4=x$ is extraneous

No Solution

Rational Equation Problem 38

$$x\cdot\frac{x}{12}+\frac{x+3}{3x}\cdot\frac{4}{4}=\frac{1}{x}\cdot\frac{12}{12}$$

$$\frac{x^2}{12x}+\frac{4x+12}{12x}=\frac{12}{12x}$$

$x^2+4x+12=12$

$x^2+4x=0$

$x(x+4)=0$

$x=0$ extraneous $\quad$ **$x=-4$**

Rational Equation Problem 39

$$\frac{72}{x^2-36}+\frac{4}{1}\cdot\frac{x^2-36}{x^2-36}=\frac{x}{x+6}\cdot\frac{(x-6)}{(x-6)}$$

$(x-6)(x+6)$

$$\frac{72}{(x-6)(x+6)}+\frac{4x^2-144}{(x+6)(x-6)}=\frac{x^2-6x}{(x+6)(x-6)}$$

$72+4x^2-144=x^2-6x$

$3x^2+6x-72=0$

$3(x^2+2x-24)=0$

$3(x-4)(x+6)=0$

$x=4$ $\quad$ $x=-6$ extraneous

Rational Equation Problem 40

$$\frac{(x-2)}{(x-2)}\cdot\frac{3}{x+8}-\frac{2}{x-2}\cdot\frac{(x+8)}{(x+8)}=1$$

$$\frac{3x-6}{(x-2)(x+8)}-\frac{(2x+16)}{(x-2)(x+8)}=1$$

$$\frac{x-22}{(x-2)(x+8)}=\frac{1}{1}$$ cross products

$x-22=x^2+6x-16$

$0=x^2+5x+6$

$0=(x+3)(x+2)$

$x=-3$ $\quad$ $x=-2$