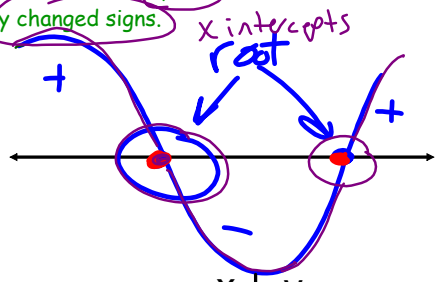
Intermediate Value Theorem Location Principle

If there are positive and negative intervals on the graph, then there is a point where the function P(x) crosses the x axis. Basically, there is a root where y = 0 between when y changed signs.



x	y	
-10	-20	−
-8	-4	−
-6	-1	−
-4	0	
-2	1	+
0	7	+
2	8	+
4	7	+
6	2	+
8	-4	−
10	-13	−

Here is a x intercept/root b/c $x=-4$ b/c $y=0$

Between 6 and 8 there is a root we see change in sign of y

This P(x) has 2 solutions/roots zeros

x	y	
-10	-30	−
-8	-14	−
-6	-1	−
-4	-6	−
-2	-2	−
0	5	+ y intercept
2	0	x intercept $x=2$ $y=0$
4	5	+
6	8	+
8	1	+
10	-3	−

root b/w -2 and 0 • changed sign of y

root b/w 8 and 10 • Changed y signs

This P(x) has 3 roots/zeros/solutions

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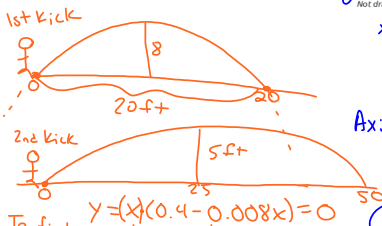
65. MODELING WITH MATHEMATICS A soccer player kicks a ball downfield. The height of the ball increases until it reaches a maximum height of 8 yards, 20 yards away from the player. A second kick is modeled by $y = x(0.4 - 0.008x)$. Which kick travels farther before hitting the ground? Which kick travels higher? *(See Example 5.)*

What information were you given? Draw two pictures. Fill in the information given and indirectly given.



To find x intercepts $y=0$ Set each factor $=0$ and solve

$y = (x)(0.4 - 0.008x) = 0$

$x = 0$ ball at ground at 0 ft away

$0.4 - 0.008x = 0$

$-0.4 \quad -0.4$

$\frac{-0.008x}{-0.008} = \frac{-0.4}{-0.008}$

$x = 50$ ft away

2nd kick traveled Farther

$y = x(0.4 - 0.008x)$

$y = 0.4x - 0.008x^2$

Find Vertex: $x = \frac{-b}{2a} = \frac{-(0.4)}{2(-.008)}$

$x = 25$

or Axis of sym. is halfway between 0 and 50 → 25

$y = 25(0.4 - 0.008 \cdot 25)$

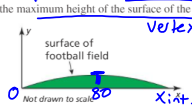
$y = 5$ ft high

1st kick was higher

---

66. MODELING WITH MATHEMATICS Although a football field appears to be flat, some are actually shaped like a parabola so that rain runs off to both sides. The cross section of a field can be modeled by $y = -0.000234x(x - 160)$, where x and y are measured in feet. What is the width of the field? What is the maximum height of the surface of the field?

Vertex



$x = 0 \quad x - 160 = 0$

$+160 \quad +160$

$x = 160$ ft wide

Axis of symmetry $x = 80$

$x = \frac{160 + 0}{2}$

$y = -0.000234(80)(80 - 160)$

$y = 1.4976$ ft max height of surface

Solution/ Root/ Zero

The x intercept, x=____, (_,0) use the factor (x-c) set it =0 and solve for x

Multiplicity

- A solution is repeated
- (x-c) is a factor more than once
- the # of times it is a factor is its multiplicity m

ex. $(x-4)^3$ 4 is a zero of P with multiplicity 3

$(x-4)(x-4)(x-4)=0$

$x-4=0$ $x-4=0$ $x-4=0$

$+4\ +4$ $x=4$ $x=4$

$x=4$

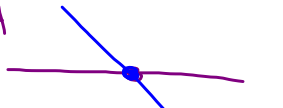
ex. $(x+2)^4$

$x+2=0$

$-2\ -2$

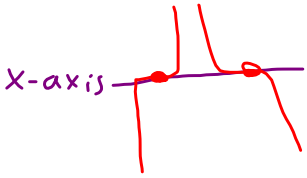
$x=-2$ with $m=4$

Pass • If **m is 1** it goes right through



Bounce • If **m is even** it changes direction (bounces)

Flatten • If **m is odd** it will keep its directions (passes through) but but it will flatten at the zero



Sketching Polynomial Graphs

1. Plot x intercept
2. Plot end behavior
3. Plot y intercept
4. Sketch passes and bounces

$f(x) = x(x-3)(x+3)$

$x=0$ $x-3=0$ $x+3=0$

$+3\ +3$ $-3\ -3$

$x=3$ $x=-3$

Degree:

Zeros and Multiplicities

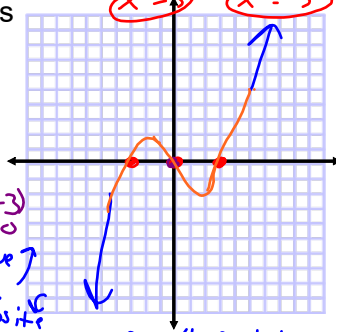
$x=3$ $m=1$ Pass $x=0$ $m=1$ Pass $x=3$ $m=1$ Pass

y intercept: (0, 0) $y=0(0-3)(0+3)$ $0\cdot-3\cdot3=0$

End behavior Lead C: $a=1$ positive

as x-> ∞ y-> ∞ Deg: 3 x^3 odd opposite

as x-> −∞ y-> −∞



Deg: 4 match

$f(x) = 0.4x^2(x-5)(x+2)$

$x^2=0$ $x-5=0$ $x+2=0$

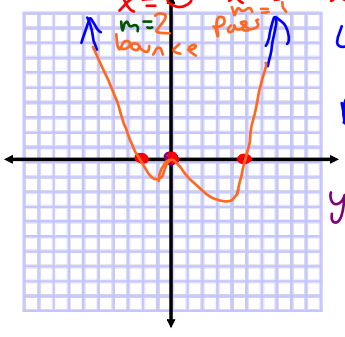
$x=0$ $x=5$ $x=-2$

$m=2$ bounce $m=1$ pass $m=1$ pass

Lead coef: $a=0.4$ positive

Deg: 4 x^4 even same as right

y-int (0,0)



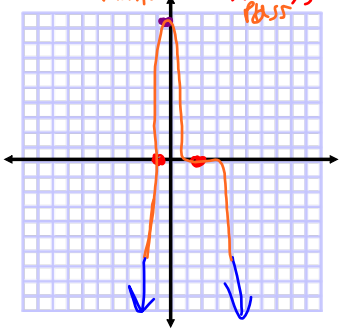
$f(x) = -3(x-2)^3(5x+4)$

y-int: (0, 96) $-3(0-2)^3(5(0)+4)$

$x-2=0$ $5x+4=0$

$x=2$ $5x=-4$

Flat Pass $x=-\frac{4}{5}$ pass



Homework

1. $y = (x-4)^2$

$x-4=0$, $+4$ $+4$, $x=4$ bounce

Lead Coef: 1 right side up

Degree: 2 same end behavior

Zeros and Multiplicities

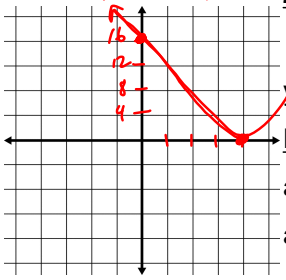
$x=4$ w/a $m=2$

y intercept: $y=(0-4)^2=16$, $(0,16)$

End behavior

as x-> ∞ y-> ∞

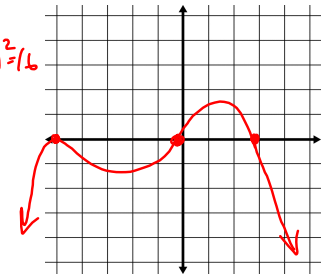
as x-> $-\infty$ y-> ∞



2. $y = -x(x+5)^2(x-3)^1$

$x=0$ pass, $x=-5$ bounce, $x=3$ pass

LC: -1 D: 4



3. $y = -2(x+2)^4(x-3)^3$

$x=-2$ Flat bounce, $x=3$ Flat pass

L.C.: -2 down at right

Degree: 7 opposite left

Zeros and Multiplicities

$x=-2$ $m=4$

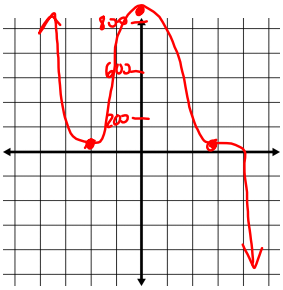
$x=3$ $m=3$

y intercept: $y=-2(0+2)^4(0-3)^3$, $y=864$, $(0,864)$

End behavior

as x-> ∞ y-> $-\infty$

as x-> $-\infty$ y-> ∞



4. $y = -1/2x^2(x-2)^5$

$x=0$ bounce, $x=2$ Flat pass

LC: -½ Deg: 7 opp ↑

ANALYZING RELATIONSHIPS In Exercises 3–6, match the function with its graph.

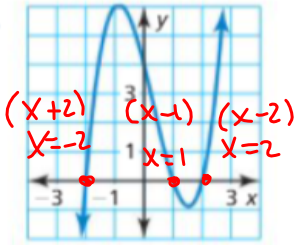
3. $f(x) = (x-1)(x-2)(x+2)$ A

4. $h(x) = (x+2)^2(x+1)$ C

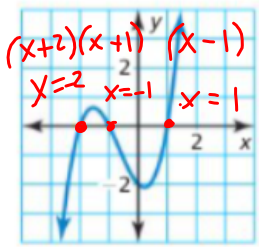
5. $g(x) = (x+1)(x-1)(x+2)$ B

6. $f(x) = (x-1)^2(x+2)$ D

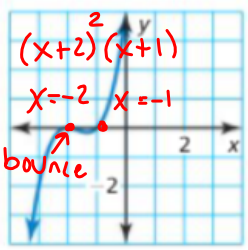
A.



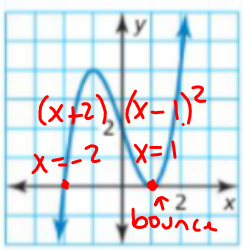
B.



C.

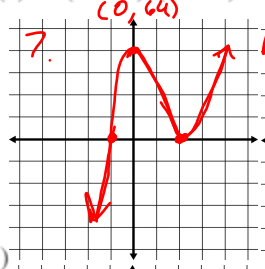


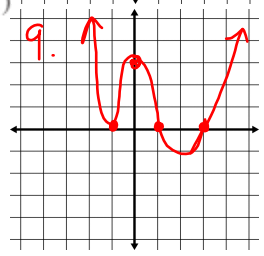
D.

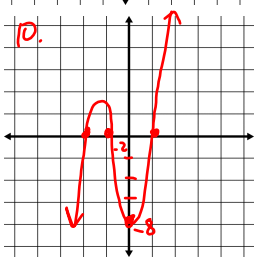


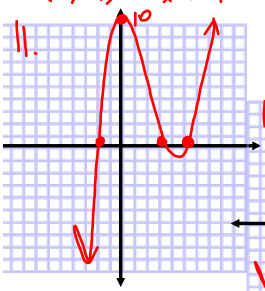
In Exercises 7–14, graph the function. *(See Example 1.)*

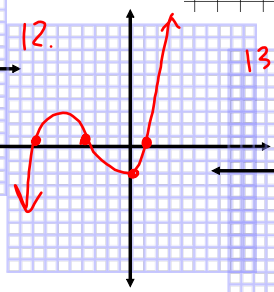
7. $f(x) = (x-2)^2(x+1)$ (0,4) x=2 bounce x=-1 pass
8. $f(x) = (x+2)^2(x+4)^2$ x=-2 B x=-4 B (0,64)
9. $h(x) = (x+1)^2(x-1)(x-3)$ (0,3) x=-1 B x=1 P x=3 P
10. $g(x) = 4(x+1)(x+2)(x-1)$ (0,-8) x=-1 P x=-2 P x=1 P
11. $h(x) = \frac{1}{3}(x-5)(x+2)(x-3)$ (0,10) x=5 P x=-2 P x=3 P
12. $g(x) = \frac{1}{12}(x+4)(x+8)(x-1)$ (0,-2.66) x=-4 P x=-8 P x=1 P
13. $h(x) = (x-3)(x^2+x+1)$ (0,-3) x=3 P Imaginary
14. $f(x) = (x-4)(2x^2-2x+1)$ (0,-4) x=4 Imaginary

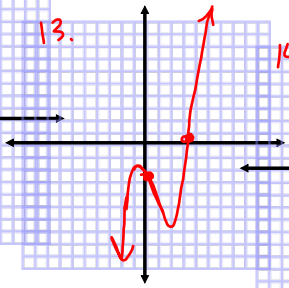
7. 

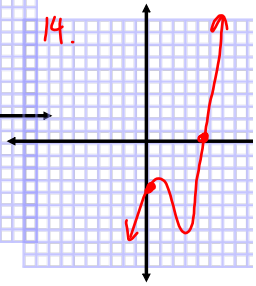

9. 

10. 

11. 

12. 

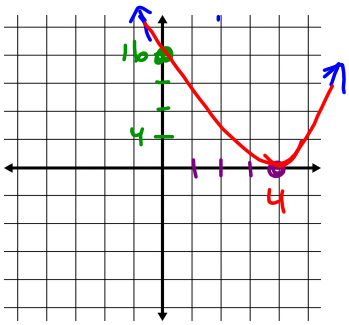
13. 

14. 

Homework Key

$y = (x-4)^2$

$f(0) = (0-4)^2 = 16$



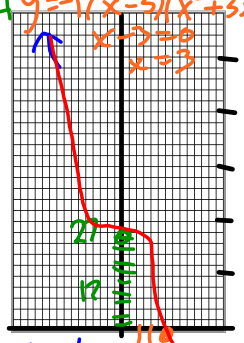
$y = -x^3+27$

$y = -1(x^3-27)$

$y = -1(x-3)(x^2+3x+9)$

$x-3=0$

$x=3$



Lead C: -1 negative

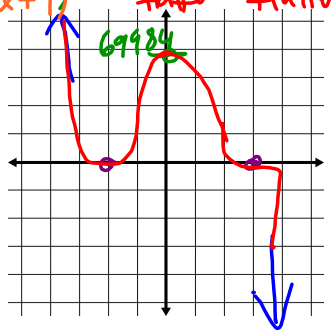
Degree: 3 opposite

$f(0) = -(0)^3+27 = 27$

$y = -2(x+2)^4(x-3)^7$

x^{11}

bounce flatter pass flatter

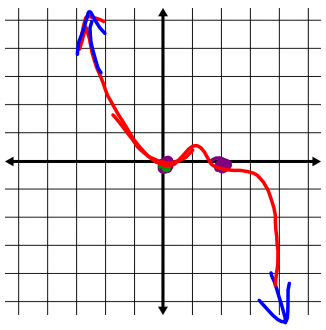


$f(0) = -2(0+2)^4(0-3)^7$

$= -2(2)^4(-3)^7$

$= 69984$

$y = -1/2x^2(x-2)^5$



$f(0) = -\frac{1}{2}(0)^2(0-2)^5$

$= 0$