Your Name

Mrs. Theo

3/19/21

Notes

6.5

Expanding and Condensing Logarithms

Properties of Logarithms

Product Rule: $\log_m(xy) = \log_m x + \log_m y$

Quotient Rule: $\log_m(x/y) = \log_m x - \log_m y$

Helpful things to remember:

if logs are multiplied you cannot add them inside

Apply property 1 first, then 2 and 3

~~$\log_m x \cdot \log_m y = \log_m(x+y)$~~

If multiple things are divided inside, those things are all subtracted when expanded.

expanding logarithms EXAMPLES

Product Rule: $\log_m xy = \log_m x + \log_m y$
Quotient Rule: $\log_m(x/y) = \log_m x - \log_m y$

log (x/y)

$\log(x) - \log(y)$

$\text{Log}_3(wzy)$

$\log_3 w + \log_3 z + \log_3 y$

$\text{Log}_3 x^4 y$

$\log_3(x^4) + \log_3(y)$

$4 \cdot \log_3(x) + \log_3(y)$

$\text{Log}_7((w^3y)/x)$

$\log_7\left(\frac{w^3y}{x}\right)$

$3\log_7 w + \log_7 y - \log_7 x$

$\text{Log}((4^5x)/(y^3a))$

$\log 4^5 + \log x - (\log y^3 + \log a)$

$5\log 4 + \log x - 3\log y - \log a$

$3.010 + \log x - 3\log y - \log a$

$\text{Log}_2((4^5/8^3)^7)$

$\log_2\left(\frac{4^5}{8^3}\right)^7$

$7\left[\log_2(4^5) - \log_2(8^3)\right]$

$\log_2\left(\frac{4^{35}}{8^{21}}\right)$

condensing logarithms EXAMPLES

$\frac{4\log_6(x)}{2}$

$\frac{4}{2}\log_6 x$

$2\log_6 x$

$\log_6 x^2$

$\frac{4\log_6(x)}{\log_6 2} \rightarrow \frac{\log(2)}{\log(6)}$

$\frac{4\log_6 x}{0.387}$

$\left(\frac{4}{0.387}\right)\log_6 x$

$\log_6 x^{10.336}$

$\text{Log}_3(6) + \log_3 7 + \log_3 y$

$3\text{Log}_7 w + \log_7 y - \log_7 x$

$\log_7 w^3 + \log_7 y - \log_7 x$

$\log_7\left(\frac{w^3 \cdot y}{x}\right)$

$5\text{Log}4 + \log x - 3\log y - \log a$

$\log 4^5 + \log x - \log y^3 - \log a$

$\log\left(\frac{4^5 \cdot x}{y^3 \cdot a}\right)$

$\frac{4^5 \cdot x}{y^3/a} = \frac{\frac{4^5 \cdot x}{y^3}}{\frac{a}{1}}$

$\frac{4^5 \cdot x}{y^3} \cdot \frac{1}{a}$

Oooh weee!

Base b $\log_b b = 1$

$7 + 3\log_4 x - 2\log_4 y$

$7 + \log_5 x - 4\log_5 x$

$7 + \log_5 x - \log_5 x^4$

multiply by 1! $7 \cdot \log_5 5 + \log_5 x - \log_5 x^4$

$\log_{\text{common base}}(\text{common base})$ then bring up exponent

$\log_5 5^7 + \log_5 x - \log_5 x^4$

$\log_5 \left(\frac{5^7 \cdot x}{x^4}\right)$

$\log_5 \left(\frac{5^7}{x^3}\right)$

$7 \cdot \log_4 4 + \log_4 x^3 - \log_4 y^2$

$\log_4 \left(\frac{4^7 \cdot x^3}{y^2}\right)$

$\log_5 \sqrt{125x^2y^8}$

$\log_5 \left(125x^2y^8\right)^{\frac{1}{2}}$

$\frac{1}{2}\left[\log_5 125 + 2\log_5 x + 8\log_5 y\right]$

$\frac{1}{2}\left[3 + 2\log_5 x + 8\log_5 y\right]$

$\log_5 3 = 0.683$ $\log_5 6 = 1.113$

$\log_5 2$

$\log_5 \left(\frac{6}{3}\right)$

$\log_5 6 - \log_5 3$

$1.113 - 0.683$

$\log_5 9$

$\log_5 3^2$

$2 \log_5 3$

$2 \cdot 0.683$

Kuta Software - Infinite Algebra 2 Name_____

Properties of Logarithms Date_____ Period____

Expand each logarithm.

1) $\log (6 \cdot 11)$

$\log 6 + \log 11$

2) $\log (5 \cdot 3)$

$\log 5 + \log 3$

3) $\log \left(\frac{6}{11}\right)^5$

$5\log 6 - 5\log 11$

4) $\log (3 \cdot 2^3)$

$\log 3 + 3\log 2$

5) $\log \frac{2^4}{5}$

$4\log 2 - \log 5$

6) $\log \left(\frac{6}{5}\right)^6$

$6\log 6 - 6\log 5$

7) $\log \frac{x}{y^6}$

$\log x - 6\log y$

8) $\log (a \cdot b)^2$

$2\log a + 2\log b$

9) $\log \frac{u^4}{v}$

$4\log u - \log v$

10) $\log \frac{x}{y^5}$

$\log x - 5\log y$

11) $\log \sqrt[3]{x \cdot y \cdot z}$

$\frac{\log x}{3} + \frac{\log y}{3} + \frac{\log z}{3}$

12) $\log (x \cdot y \cdot z^2)$

$\log x + \log y + 2\log z$

Condense each expression to a single logarithm.

13) $\log 3 - \log 8$

$\log \frac{3}{8}$

14) $\frac{\log 6}{3}$

$\log \sqrt[3]{6}$

15) $4\log 3 - 4\log 8$

$\log \frac{3^4}{8^4} = \log\left(\frac{81}{4096}\right)$

16) $\log 2 + \log 11 + \log 7$

$\log 154$

17) $\log 7 - 2\log 12$

$\log \frac{7}{12^2}$ $\log \frac{7}{144}$

18) $\frac{2\log 7}{3}$

$\log \sqrt[3]{7^2}$ $\frac{2}{3}\log 7$ $\log 7^{2/3}$

19) $6\log_3 u + 6\log_3 v$

$\log_3 (v^6 u^6) = \log_3 (vu)^6$

20) $\ln x - 4\ln y$

$\ln \frac{x}{y^4}$

21) $\log_4 u - 6\log_4 v$

$\log_4 \frac{u}{v^6}$

22) $\log_3 u - 5\log_3 v$

$\log_3 \frac{u}{v^5}$

23) $20\log_6 u + 5\log_6 v$

$\log_6 (v^5 u^{20})$

24) $4\log_3 u - 20\log_3 v$

$\log_3 \frac{u^4}{v^{20}}$

Critical thinking questions:

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

$\log \frac{4x^2}{75y^2}$ $2\log 2x - 2\log y - \log 3 - 2\log 5$

$\log(2x)^2 - \log y^2 - \log 3 - \log 5^2$

$\log\left(\frac{4x^2}{y^2 \cdot 3 \cdot 25}\right)$

26) $\log x \cdot \log 2$

Cannot be simplified

6.5A

Name_____ Date_____ Hour___

Algebra 2 ~ 6.5 A, B, PUZZLE (BIM) ~ 6.5 Properties of Logarithms, p. 327-332

Goal: use properties of logarithm properties to evaluate logarithms, expand and condense logarithms; use change of base formula; apply

In Exercises 1–3, use $\log_8 3 \approx 0.683$ and $\log_8 6 \approx 1.113$ to evaluate the logarithm.

1. $\log_8 2$
2. $\log_8 18$
3. $\log_8 9$

In Exercises 4–6, expand the logarithmic expression.

4. $\log_2 5x$
5. $\log 7x^4$
6. $\log_9 \frac{2x}{y}$

7. Describe and correct the error in expanding the logarithmic expression.

$\log_4 3x = 3\log_4 x$

In Exercises 8–11, condense the logarithmic expression.

8. $\log_7 3 - \log_7 5$
9. $\log 10 - \log 5$
10. $3\ln x + 9\ln y$
11. $\log_2 9 + \frac{1}{2}\log_2 y$

In Exercises 12–14, use the change-of-base formula to evaluate the logarithm.

12. $\log_8 3$
13. $\log_2 11$
14. $\log_6 10$

15. Your friend claims that you can use the change-of-base formula to write the expression $\ln x$ as a common logarithm. Is your friend correct? Explain your reasoning.

16. For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function $L(I) = 10\log \frac{I}{I_0}$, where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). The sound of a coach's whistle is five times greater than the intensity of the referee's whistle. Find the difference in the decibel levels of the sounds made by the coach and the referee.

$L(I) = 10\log \frac{I}{I_0}$

Coach $L(I)$ - Ref $L(I)$

$10\cdot\log \frac{5x}{10^{-12}} - 10\log \frac{x}{10^{-12}}$

$\log\left(\frac{(5x)^{10}}{10^{-120}}\right) - \log\left(\frac{x^{10}}{10^{-120}}\right)$

$\log\left(\frac{5^{10}\cdot x^{10}\cdot 10^{120}}{x^{10}\cdot 10^{120}}\right)$

$\log(5) = 0.699$ decibals greater

6.5B

In Exercises 1–3, use $\log_8 3 \approx 0.683$ and $\log_8 6 \approx 1.113$ to evaluate the logarithm.

1. $\log_8 81$
2. $\log_8 \frac{1}{6}$
3. $\log_8 \frac{1}{2}$

In Exercises 4–6, expand the logarithmic expression.

4. $\log_3 12x^7$
5. $\log_6 \frac{5x^2}{y^3}$
6. $\log_8 6\sqrt{xy}$

7. Describe and correct the error in expanding the logarithmic expression.

$\ln \sqrt[3]{xy} = \frac{1}{3}\ln x + \ln y$

In Exercises 8–11, condense the logarithmic expression.

8. $5\log_9 x - \log_9 4$
9. $\log_8 5 + \frac{1}{4}\log_8 x$
10. $2\ln 4 + 5\ln x + 3\ln y$
11. $\log_6 9 + 2\log_6 \frac{1}{3} - 3\log_6 x$

In Exercises 12–14, use the change-of-base formula to evaluate the logarithm.

12. $\log_8 15$
13. $\log_3 30$
14. $\log_4 \frac{8}{17}$

15. Your friend claims you can use the change-of-base formula to write the expression $\frac{\ln y}{\ln 3}$ as a logarithm with base 3. Is your friend correct? Explain your reasoning.

16. For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function $L(I) = 10\log \frac{I}{I_0}$, where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). The bass guitar player in a band turns up the volume of the speaker so that the intensity of the sound triples. By how many decibels does the loudness increase?

Algebra 2 ~ 6.5 A, B, PUZZLE (BIM) ~ 6.5 Properties of Logarithms, p. 327-332

What Type Of Lizard Loves To Tell Jokes?

Write the letter of each answer in the box containing the exercise number.

Match the expression with the logarithm that has the same value.

1. $\log_2 6 + \log_2 8$
2. $\log_3 10 - \log_3 5$
3. $4 \log_{1/2} 2$
4. $\log_2 2 - \log_2 3$
5. $\log_3 4 + \log_3 2$
6. $6 \log 8$

Condense the logarithmic expression.

7. $4 \log 2 - \log 5$
8. $\log 6 + \log 2 - \log 7$
9. $\frac{1}{2} \log 4 + \log 3$
10. $3 \log 2 - 2 \log 2$
11. $1 - \log 4$
12. $\ln 3 + 4 \ln x - \ln y$

Answers

- **I.** $\log_{1/2} 16$
- **A.** $\log_2 48$
- **M.** $\log \frac{16}{5}$
- **Y.** $\log(8^6)$
- **E.** $\log \frac{5}{2}$
- **S.** $\log_3 2$
- **N.** $\log 6$
- **L.** $\log_2 \frac{2}{3}$
- **L.** $\log_3 8$
- **A.** $\log \frac{12}{7}$
- **D.** $\log 2$
- **R.** $\ln \frac{3x^4}{y}$

1		2	3	4	5	6	7	8	9	10	11	12

6.5 Practice A

1. 0.43 2. 1.796 3. 1.366
4. $\log_2 5 + \log_2 x$ 5. $\log 7 + 4 \log x$
6. $\log_6 2 + \log_6 x - \log_6 y$
7. confuses logarithm rules for products and powers; The log of a product is the sum of the logs, not another product.
$\log_4 3x = \log_4 3 + \log_4 x$
8. $\log_7 \frac{3}{5}$ 9. $\log 2$ 10. $\ln(x^3 y^9)$
11. $\log_2(9y^{1/2})$ 12. 0.683 13. 3.459
14. 1.285 15. yes; $\ln x = \frac{\log x}{\log e}$
16. about 7 decibel levels

6.5 Practice B

1. 2.732 2. −1.113 3. −0.43
4. $\log_3 12 + 7 \log_3 x$
5. $\log_6 5 + 2 \log_6 x - 3 \log_6 y$
6. $\log_8 6 + \frac{1}{2}\log_8 x + \frac{1}{2}\log_8 y$
7. does not distribute the $\frac{1}{3}$ power to y as well as x
$\ln \sqrt[3]{xy} = \frac{1}{3}\ln x + \frac{1}{3}\ln y$
8. $\log_9\left(\frac{x^5}{4}\right)$ 9. $\log_8(5\sqrt[4]{x})$
10. $\ln(16x^5 y^3)$ 11. $\log_6\left(\frac{1}{x^3}\right)$
12. 1.302 13. 3.096 14. −0.544
15. yes; $\frac{\ln y}{\ln 3} = \log_3 y$
16. about 4.77 decibel levels

6.5 Puzzle Time

A SILLYMANDER