Your name

Mrs. Theo

11 / 1 / 21

Notes

6.3 Midpoints, Medians, Altitudes

Remember....

Find the length between the following points:
A (3, 4) and B(5, -2)

$a^2 + b^2 = c^2$

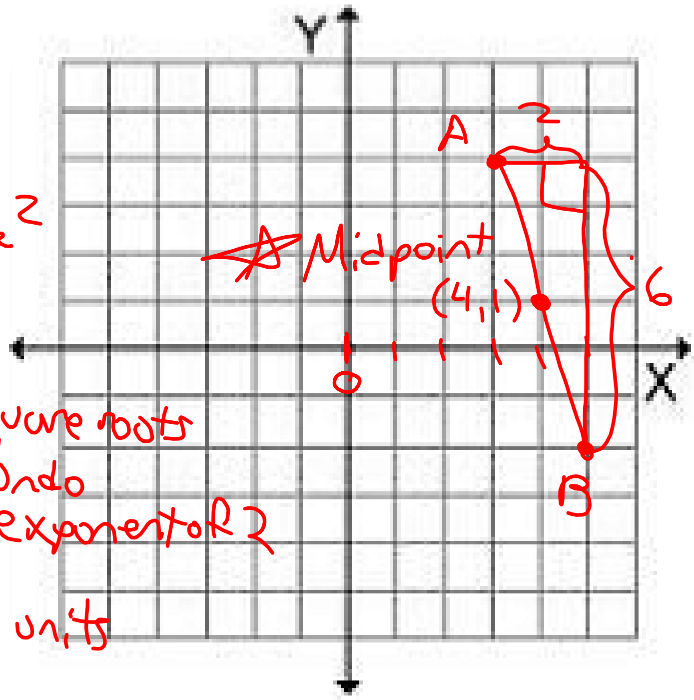
$2^2 + 6^2 = \underline{d}istance^2$

$4 + 36 = d^2$

$40 = d^2$

$\sqrt{40} = \sqrt{d^{\cancel{2}}}$ ☆ square roots undo exponent of 2

$d = 6.325$ units



Midpoint Formula

The midpoint of a segment can be found using the formula:

$$\boxed{M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)}$$

Average of x-values

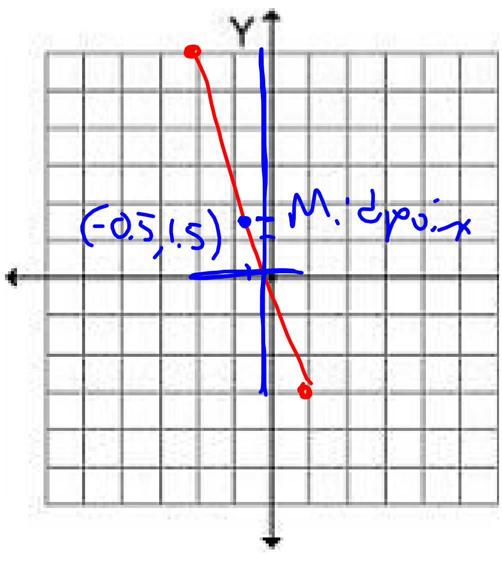
Average of y-values

Example: Find the midpoint of a segment with endpoints at (1,-3) and (-2,6).

x_1 y_1 x_2 y_2

Midpoint: $\left(\frac{1+-2}{2},\frac{-3+6}{2}\right)$

: $(-0.5, 1.5)$



Midpoint

Find the midpoint between the following points:

A (-3, 4) and B (5, 9)

$$\left(\frac{-3+5}{2}, \frac{4+9}{2}\right)$$

$$\left(\frac{2}{2}, \frac{13}{2}\right)$$

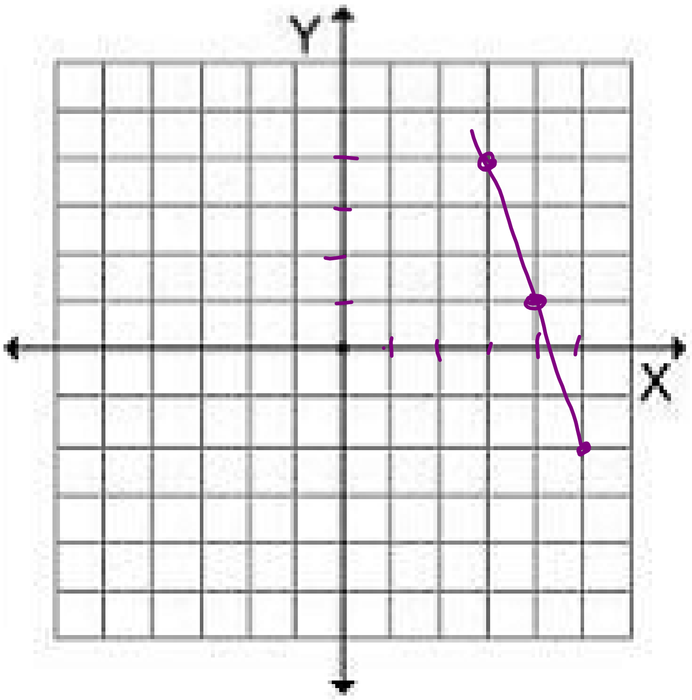
$$(1, 6.5)$$

Find the midpoint between the following points
A (3, 4) and B(5, -2)

$$\left(\frac{3+5}{2}, \frac{4+-2}{2}\right)$$

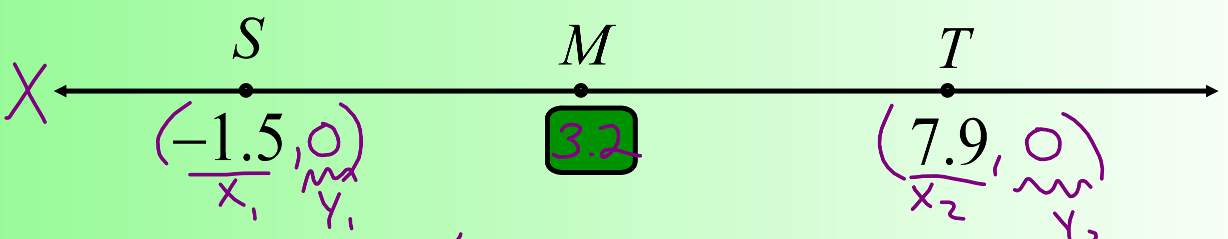
$$\left(\frac{8}{2}, \frac{2}{2}\right)$$

$$(4, 1)$$



Midpoint Formula

2.) Find the coordinate of the midpoint, M, of segment ST.



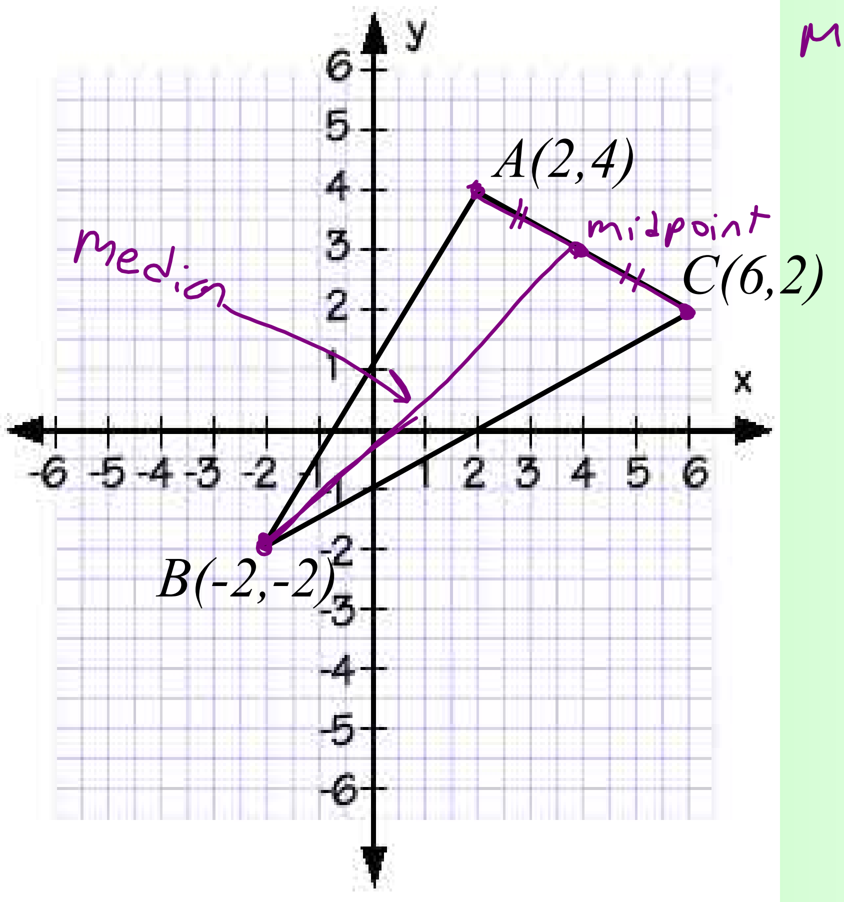
$$\text{Midpoint}: \left(\frac{-1.5+7.9}{2}, \frac{0+0}{2}\right)$$

$$\left(\frac{6.4}{2}, \frac{0}{2}\right)$$

$$(3.2, 0)$$

Median is a segment starting at one vertex of a triangle and ending at the midpoint of the opposite side.

Find the and draw of the **median** of triangle ABC that goes to side AC.

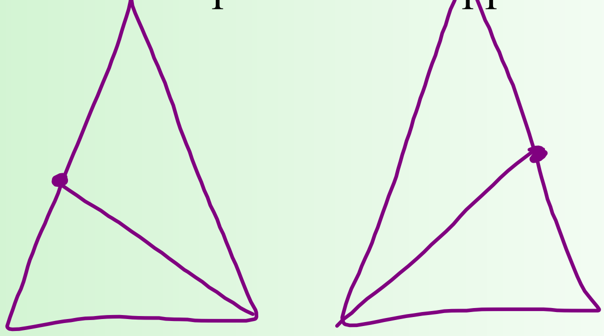


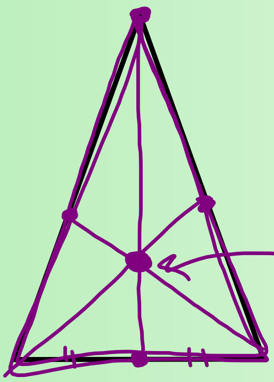
Find Midpoint

Midpoint: $\left(\frac{2+6}{2}, \frac{4+2}{2}\right)$

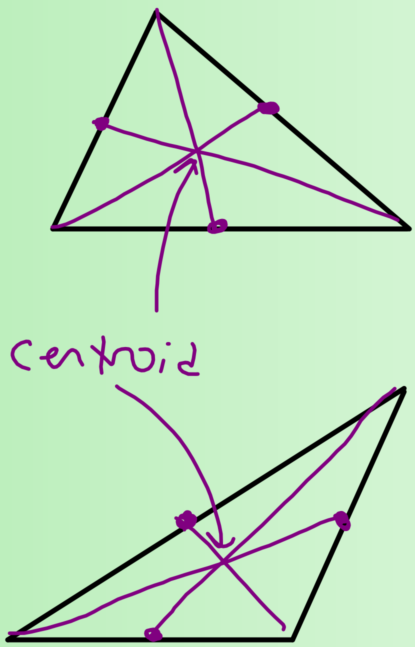
$(4, 3)$

Median = A segment drawn from the vertex of a triangle to the midpoint of the opposite side.





Centroid- meeting point of all medians



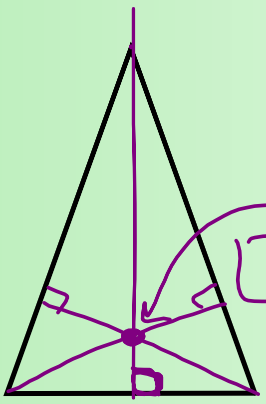
Altitude =

A segment drawn from the vertex of a triangle and is perpendicular to the opposite side (or the opposite side extended).

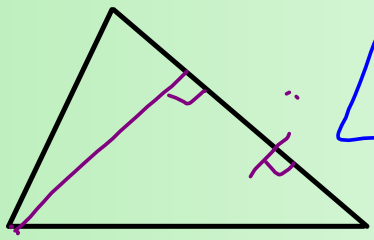
* needed for area

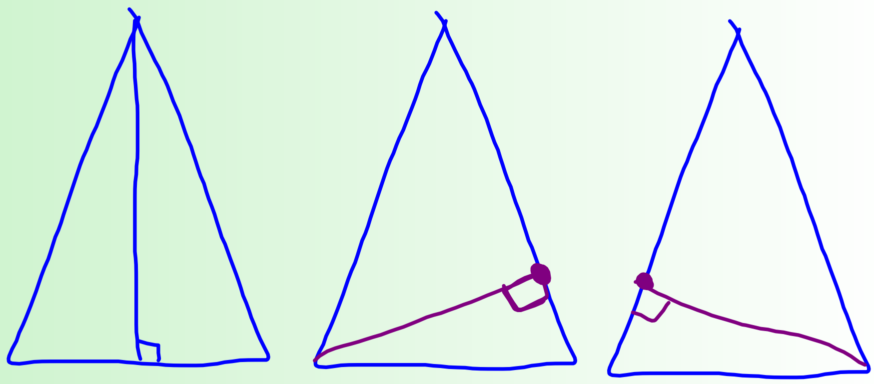
90° angle

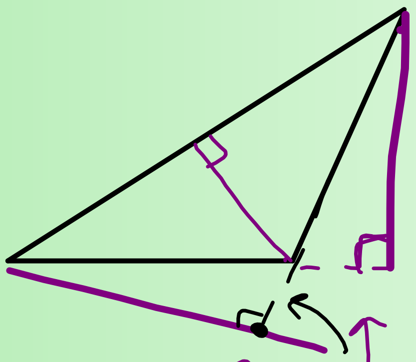
box in corner



Orthocenter- the meeting point of all altitudes







extended base to meet altitude

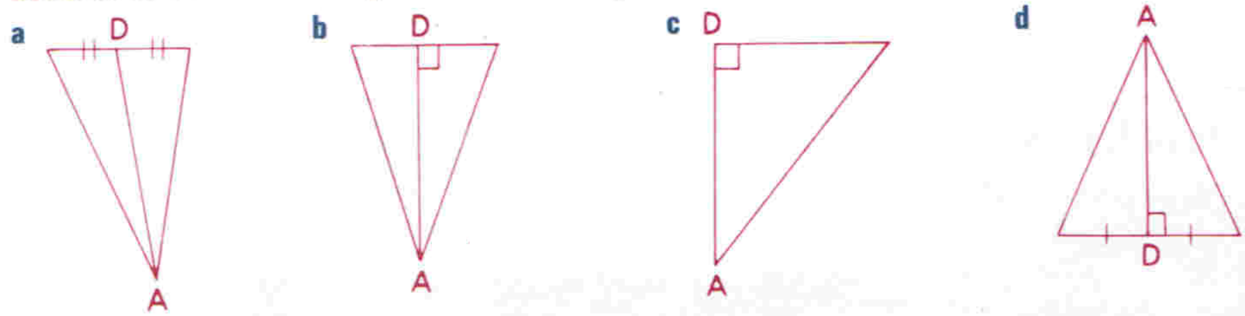
on obtuse triangles altude is outside

Order for using **altitudes** and **medians** in proofs

Altitudes $\leftrightarrow$ Perpendicular $\leftrightarrow$ Right Angles

Medians $\leftrightarrow$ Midpoint $\leftrightarrow$ $\cong$ Segments

1 For the following figures, identify $\overline{AD}$ as a median, an altitude, neither, or both according to what can be proved.

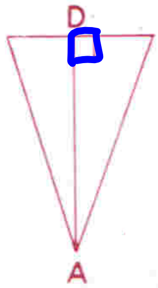


1 For the following figures, identify $\overline{AD}$ as a median, an altitude, neither, or both according to what can be proved.

a

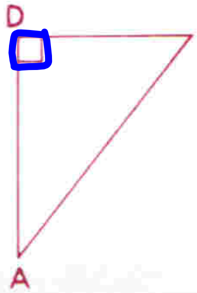
Median

b



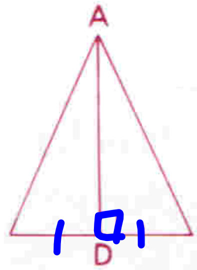
Altitude

c



Altitude

d



Median & Altitude

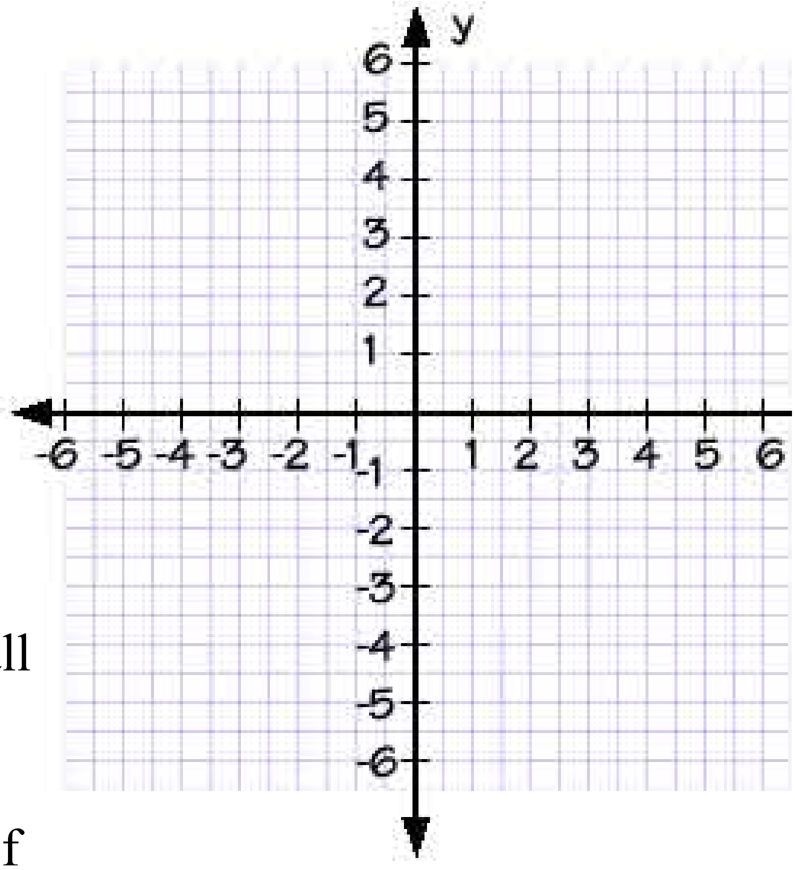
Medians

a.) Plot the following points on a coordinate grid.

$A(-6, 4)$

$B(-2, -6)$

$C(6, -2)$



b.) Find the lengths of all 3 sides of triangle ABC.

c.) Find the midpoints of all three sides

d.) Draw the Medians of all three sides and label the Centroid

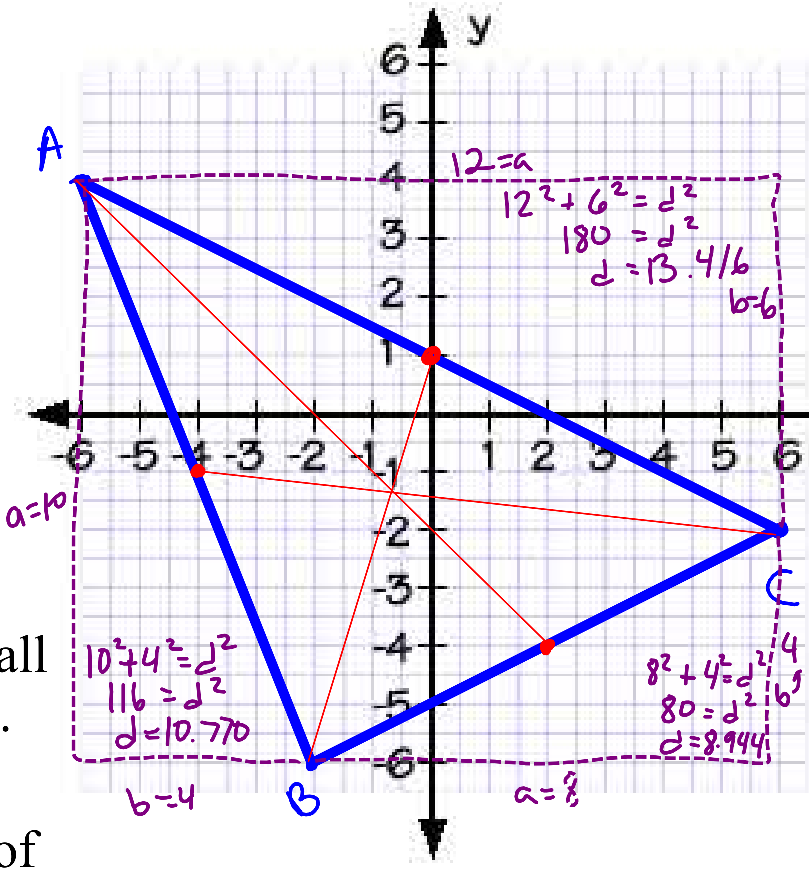
Medians

a.) Plot the following points on a coordinate grid.

$A(-6,4)$

$B(-2,-6)$

$C(6,-2)$



b.) Find the lengths of all 3 sides of triangle ABC.

c.) Find the midpoints of all three sides

AC Midpt $\left(\frac{-6+6}{2}, \frac{4+-2}{2}\right)$ $(0,1)$

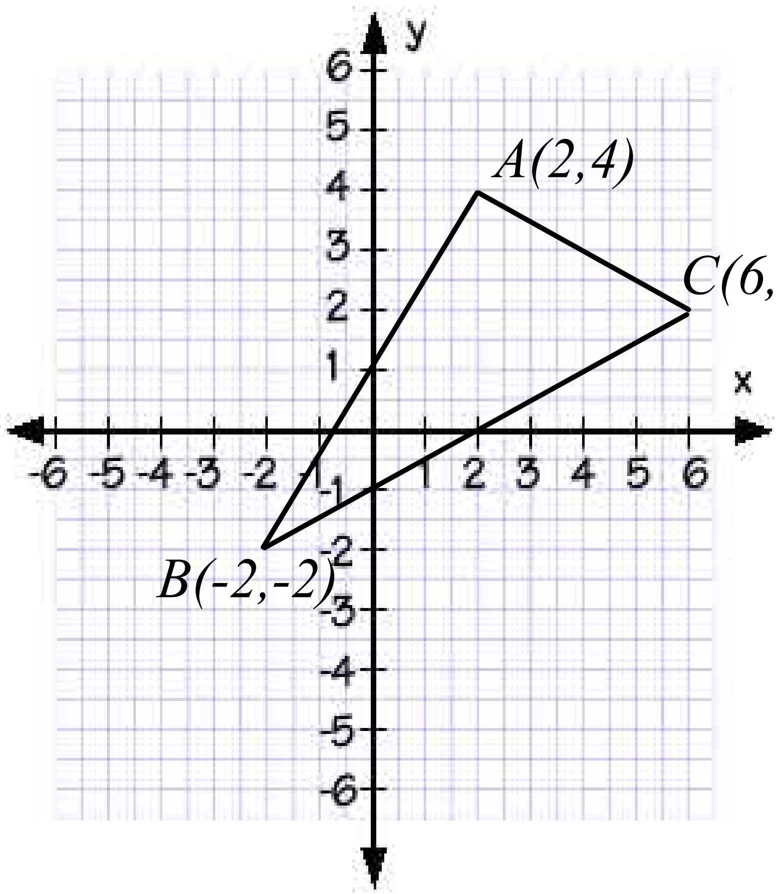
AB Midpt $\left(\frac{-6+-2}{2}, \frac{4+-6}{2}\right)$ $(-4,-1)$

BC Midpt $\left(\frac{-2+6}{2}, \frac{-6+-2}{2}\right)$ $(2,-4)$

d.) Draw the Medians of all three sides and label the Centroid

Altitude | is a segment starting at one vertex of a triangle and ending perpendicular to the opposite side

Find the and draw of the **altitude** of triangle ABC that goes to side AC.



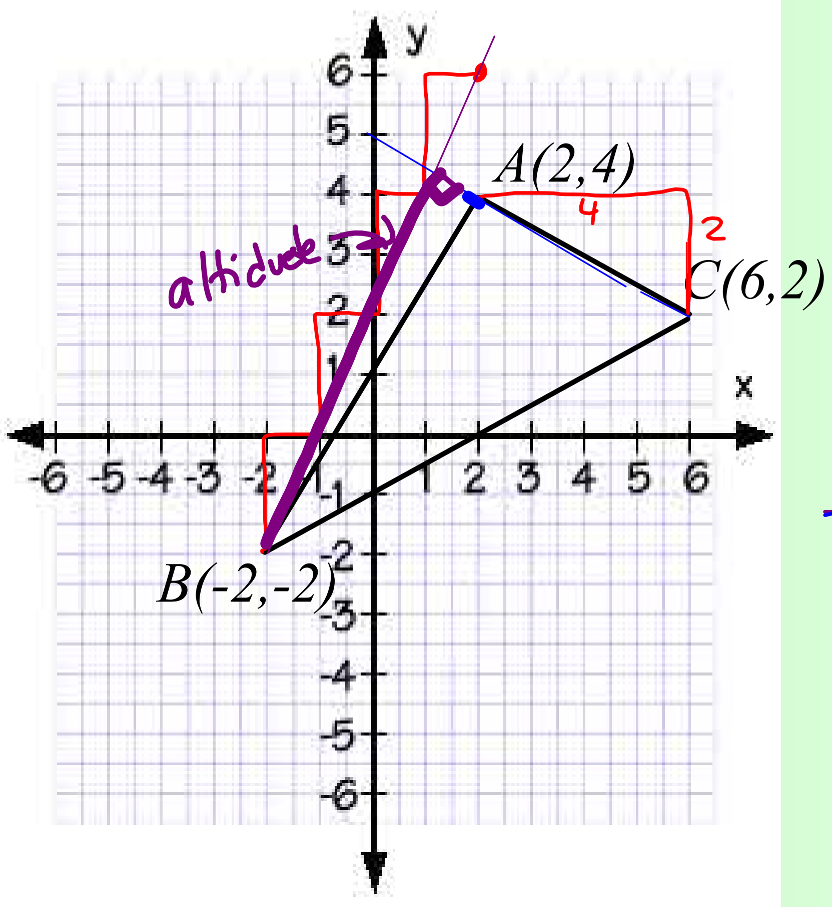
Step1: Find the slope of AC

Step 2: Determine perpendicular slope to AC

Step 3: From point B, move the slope, extend line AC if needed.

Altitude is a segment starting at one vertex of a triangle and ending perpendicular to the opposite side

Find the and draw of the **altitude** of triangle ABC that goes to side AC.



Step1: Find the slope of AC

$m = \frac{2}{4} = \frac{1}{2}$

Step 2: Determine perpendicular slope to AC

opposite reciprocal

$m_{\perp} = -\frac{2}{1} = -2$

Step 3: From point B, move the slope, extend line AC if needed.

M is the midpoint of CD

C= (1,8)

M= (3, 5)

Find the location

of D. $(5, 2)$

$$(3,5) = \left(\frac{1+x}{2}, \frac{8+y}{2}\right)$$

$$3 = \frac{1+x}{2} \qquad 5 = \frac{8+y}{2}$$

$$6 = 1+x \qquad 10 = 8+y$$

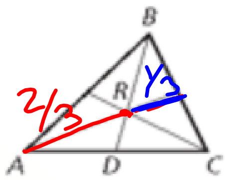
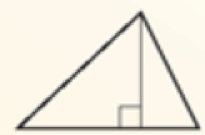
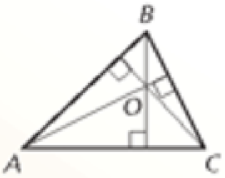
$$\boxed{5 = x} \qquad \boxed{2 = y}$$

Medians and Altitudes Practice

Name:

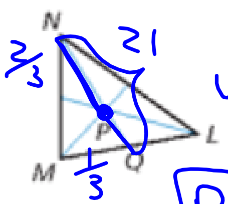
Period: Date:

where points cross

	Example	Point of Concurrency	Property	Example
median		centroid all medians meet	The centroid R of a triangle is two thirds of the distance from each vertex to the midpoint of the opposite side.	
altitude		orthocenter all altitudes meet	The lines containing the altitudes of a triangle are concurrent at the orthocenter O.	

In Exercises 3–6, point P is the centroid of $\triangle LMN$. Find PN and QP. *(See Example 1.)* medians

4. $QN = 21$



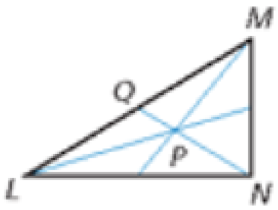
Take whole and cut into 3 parts

what $\frac{21}{3} \rightarrow 7$ is $\frac{1}{3}$ of 21

14 is $\frac{2}{3}$ of 21

$\boxed{PN = 14}$ $\boxed{PQ = 7}$

5. $QN = 30$



$QP = 10$

$PN = 20$

There are three paths through a triangular park. Each path goes from the midpoint of one edge to the opposite corner. The paths meet at point P. medians is centroid

1. Find PS and PC when $SC = 2100$ feet. $\frac{}{3} = 700$ ft
2. Find TC and BC when $BT = 1000$ feet. $TC = 1000$ ft $BC = 2000$ ft
3. Find PA and TA when $PT = 800$ feet. $PA = 1600$ $TA = 2400$ ft

① $\frac{2100}{3} = 700$ is $\frac{1}{3}$

$PS = 700$ ft

$PC = 1400$ ft

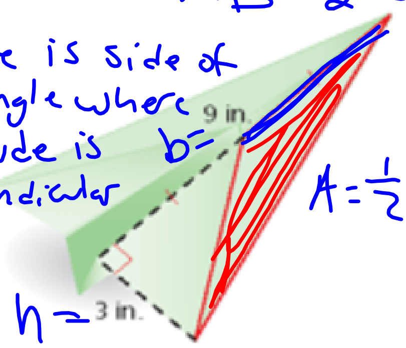
Area space inside

$A_{\square} = b \cdot h$

39. MODELING WITH MATHEMATICS Find the area of the triangular part of the paper airplane wing that is outlined in red. Which special segment of the triangle did you use?

$A_{\triangle} = \frac{1}{2} b \cdot h$

base is side of triangle where altitude is perpendicular

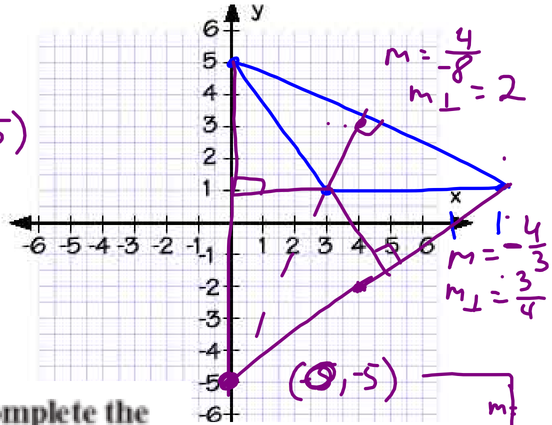


$A = \frac{1}{2}(9\text{ in})(3\text{ in})$

in^2

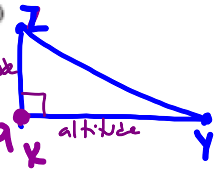
In Exercises 19–22, tell whether the orthocenter is *inside*, *on*, or *outside* the triangle. Then find the coordinates of the orthocenter. (See Example 3.)

19. $L(0, 5), M(3, 1), N(8, 1)$ Outside $(0, -5)$

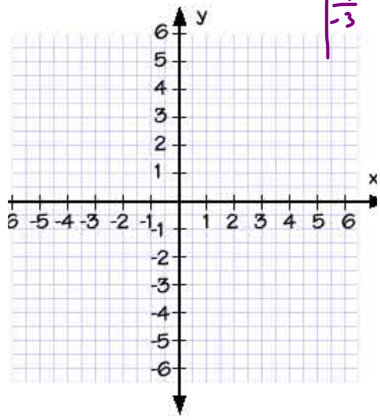


20. $X(-3, 2), Y(5, 2), Z(-3, 6)$

On the triangle $(-3, 2)$



CRITICAL THINKING In Exercises 31–36, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.



31. The centroid is always on the triangle.

32. The orthocenter is sometimes outside the triangle.

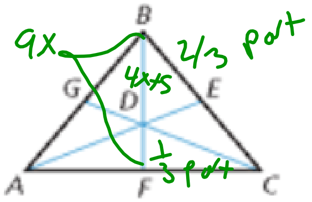
33. A median is sometimes the same line segment as a perpendicular bisector.

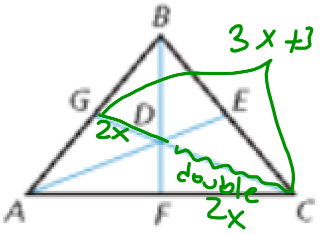
34. An altitude is sometimes the same line segment as an angle bisector.

35. The centroid and orthocenter are sometimes the same point.

36. The centroid is Always formed by the intersection of the three medians.

MATHEMATICAL CONNECTIONS In Exercises 41–44, point D is the centroid of $\triangle ABC$. Use the given information to find the value of x.





41. $BD = 4x + 5$ and $BF = 9x$

Part + Part = whole

$4x+5 + \frac{4x+5}{2} = 9x$

$4x+5+2x+2.5 = 9x$

$6x + 7.5 = 9x$

$\frac{7.5}{3} = \frac{3x}{3}$

$\boxed{x = 2.5}$

42. $GD = 2x - 8$ and $GC = 3x + 3$

Part + Part = whole

$2x + 2(2x) = 3x+3$

$2x + 4x = 3x + 3$

$6x = 3x + 3$

$3x = 3$

$\boxed{x = 1}$