Your Name

Mrs. T

1/7/19

Notes

Lesson 6.1

Exponent Rules

Objective: To be able to divide monomials by learning exponent rules.

Virtue/Skill: If we can divide monomials we will find an interesting fact about what negative exponents are. Then we can start having fun simplifying algebraic expressions, multiplying polynomials and factoring.

(Divide)
Quotient of Powers RULE

$$\frac{7^9}{7^3} = \frac{7\cdot7\cdot7\cdot7\cdot7\cdot7\cdot7\cdot7\cdot7}{7\cdot7\cdot7} = 7^{9-3} = 7^6$$

$$\frac{9^9}{9^2} = 9^{9-2} = 9^7$$

$$\frac{12^5}{12^1} = 12^4$$

When dividing powers with the same base, just **subtract the exponents.**

$$\frac{a^{10}}{a^8} = a^{10-8} = a^2$$

$$\frac{x}{x^9} = x^{1-9} = x^{-8}$$

leftovers in denominator

$$\frac{x}{x^9} = \frac{x}{x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x} = \frac{1}{x^8}$$

$$\frac{11^2}{11^5} = \frac{11\cdot11}{11\cdot11\cdot11\cdot11\cdot11} = \frac{1}{11^3} = 11^{-3}$$

$$= 11^{2-5}$$

work to be shown

negatives can happen

Find the quotient, use exponents in your answer.

1. $\frac{7^5}{7^3} = 7^{5-3} = 7^2$

2. $4^9 \div 4 = 4^{9-1} = 4^8$

3. $\frac{6^3}{6^8} = 6^{3-8} = 6^{-5}$

4. $\frac{4a^5b^7}{2a^1b^3} = \frac{4}{2} \cdot a^{5-1} \cdot b^{7-3}$

$= 2a^{-6}b^4$

$= \frac{2b^4}{a^4}$

Can you think of a multiplication or division problem that will have a final answer of $8x^5$

Power of Quotient RULE

$\left(\frac{7^5}{x^3}\right) = \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{x \cdot x \cdot x} \cdot \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{x \cdot x \cdot x} = \frac{7^{10}}{x^6}$

When raising a quotient of monomials to a power/ exponent, multiply the exponent outside with each exponent on the inside on top and bottom.

$\left(\frac{9^9}{9^2}\right)^4 = \frac{9^{9 \cdot 4}}{9^{2 \cdot 4}} = \frac{9^{36}}{9^8} = 9^{36-8} = 9^{28}$

$\frac{3^4}{3^4} = 3^{4-4} = 3^0 = 1$

$\frac{3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = 1$

$(9^{9-2})^4 = (9^7)^4 = 9^{7 \cdot 4} =$

When dividing monomials with a common base, you can simplify after or before distributing the outside exponent

$\left(\frac{a^{10}}{b^8}\right)^4 = \frac{a^{10 \cdot 4}}{b^{8 \cdot 4}} = \frac{a^{40}}{b^{32}}$

$\left(\frac{yx^9}{xz^3}\right)^2 = \left(\frac{yx^8}{z^3}\right)^2 = \frac{(-1)^2y^2x^{16}}{z^6} = \frac{y^2x^{16}}{z^6}$

Find each product or quotient, using exponents.

1. $(ab^5 \div 8a^2b^5)^3 = \left(\frac{ab^5}{8a^2b^5}\right)^3 = \frac{a^{1\cdot3}b^{5\cdot3}}{8^3a^{2\cdot3}b^{5\cdot3}} = \frac{1}{512}a^{3-6}b^{15-15}$

$= \frac{1}{512}a^{-3}b^0 = \boxed{\frac{1}{512a^3}}$

2. $\left(\frac{n^3(p^5)}{m^2}\right)^{11} = \frac{n^{3\cdot11}p^{5\cdot11}}{m^{2\cdot11}} = \boxed{\frac{n^{33}p^{55}}{m^{22}}}$

3. $\left(\frac{-8t^7}{t\cdot t^2}\right)^3 = \frac{(-8)^3t^{7\cdot3}}{t^{1\cdot3}t^{2\cdot3}}$

$= -512t^{21-3-6}$

$= \boxed{-512t^{12}}$

Zero Power

$\frac{5}{5^1} = 1 = 5^{1-1} = 5^0$ $\quad \frac{a}{a} = a^{1-1} = \boxed{a^0 = 1}$

$\frac{5a^2}{5a^2} = 1$

So... $\frac{x^{10}}{x^{10}} = x^{10-10} = x^0 = 1$

How do you divide fractions? Skip Flip + multiply

$$\frac{\frac{1}{5}}{\frac{5}{1}} = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{5^2} = \frac{1}{25}$$

$$\frac{1}{x^{-2}} = \frac{x^2}{1}$$

$5^3 = 5 \cdot 5 \cdot 5 = 125$

$5^2 = 5 \cdot 5 = 25$

$5^1 = 5$

$5^0 = 1$

$5^{-1} = \frac{1}{5}$

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

+1 exponent

+1 exponent

-1 exponent

-1 exponent

-1 exponent

·5 ·5 ·5 ÷5 ÷5 ÷5 ÷5

Negative Exponents:

Mean the reciprocal or to flip the number and put in the denominator: Aka FRACTIONS

$3^{-7} = \frac{1}{3^7}$

$a^{-8} = \frac{1}{a^8}$

$\frac{1}{b^{-4}} = b^4$

$\frac{a^{-8}}{b^{-4}} = \frac{b^4}{a^8}$ just flip

$\frac{a^{-7}}{b^3} = \frac{1}{a^7 b^3}$

$\frac{x^9}{y^{-3}} = x^9 y^3$

$\frac{a^{-8}}{a^{-4}} = a^{-8--4} = a^{-4} = \frac{1}{a^4}$

$\frac{a^4}{a^8} = a^{4-8}$

Find the quotient, use exponents in your answer.

1. $\left(\frac{7^5}{7^3}\right)^{-1}$ simplify first $= (7^{5-3})^{-1} = (7^2)^{-1} = 7^{2\cdot-1} = 7^{-2}$

2. $\left(4^9 \div 4^{11}\right)^{-2}$ distribute first $= \frac{4^{9\cdot-2}}{4^{11\cdot-2}} = \frac{4^{-18}}{4^{-22}} = 4^{-18--22} = 4^{-16} = \frac{1}{4^{16}}$

3. $\left(\frac{a^4}{a^3}\right)^{-1}$ flip first $= \frac{a^3}{a^4} = a^{3-4} = a^{-1} = \frac{1}{a}$

4. $\left(\frac{9x^3y^5}{5xy^2}\right)^{-2} = \left(\frac{5xy^2}{9x^3y^5}\right)^2$ now positive exponent b/c we flipped

$\left(\frac{5}{9}x^{1-3}y^{2-5}\right)^2$

$\left(\frac{5}{9}x^{-2}y^{-3}\right)^2$

$\frac{5^2}{9^2}x^{-2\cdot2}y^{-3\cdot2}$

$\frac{25}{81}x^{-4}y^{-6}$

$\frac{25}{81x^4y^6}$

Combining like terms

x^2

Words	Formula	Example
Add/subtract coefficients (#s in front) Keep variable the same	$ax + bx$ $(a+b)x$	$3x+2x$ $-2x+4y^3x+6x-7y$ $5x$ $4x-3y^2x$

Product of Powers

add

$(-1ab^5)(8a^0b^5)$

Formula: $x^m \cdot x^n = x^{m+n}$

$-1abbbbb \cdot 8aabbbbb$

$-1 \cdot 8 \cdot a^{1+2} b^{5+5}$

$-8a^3b^{10}$

Power of Product

multiply exponents

$(ab^5)(8a^0b^5)^3$

$(x^m)^n = x^{m \cdot n}$

$abbbbb \cdot 8aabbbbb \cdot 8aabbbbb \cdot 8aabbbbb$

$1 \cdot 8 \cdot 8 \cdot 8 \cdot a^{1+2 \cdot 3} b^{5+5 \cdot 3}$

$512a^7b^{20}$

Quotient of Powers

Subtract exponents

$\frac{3n^8m^2}{21n^2m^3}$

$\frac{x^m}{x^n} = x^{m-n}$

$\frac{3 \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot m \cdot m}{21 \cdot n \cdot n \cdot m \cdot m \cdot m}$

$\frac{3}{21} n^{8-2} m^{2-3}$

$\frac{1}{7} n^6 m^{-1}$

$\frac{n^6}{7m}$

Power of Quotients

distribute outer exponent

$\left(\frac{n^3(n^5)}{8n^2}\right)^{11}$ $\left(\frac{x^m}{x^n}\right)^y \to x^{m \cdot y}$

$\frac{(-1)^{11} n^{3 \cdot 11} n^{5 \cdot 11}}{8^{11} n^{2 \cdot 11}}$ distribute exponent

$-1^{11} \cdot 8^{-11} n^{33+55-22}$ adding/subtr.

$\frac{-1 n^{66}}{8^{11}}$ rewrite so only positive exponents

Negative Exponents

ex. $\left(\frac{-8t^7r^{-2}}{r^{-3}t^2}\right)^{-3}$

$\frac{(-8)^{-3} t^{-21} r^6}{r^9 t^{-6}}$

$(-8)^{-3} t^{-21-6} r^{6-9}$

$\frac{1}{(-8)^3} \cdot t^{-15} r^{-3}$

$\frac{1}{-512t^{15}r^3}$

ex. $\left(\frac{p}{7^2}\right)^{-1} = \frac{7^2}{p}$

ex. $\frac{p^4m^{-2}}{v^{-3}} = \frac{p^4v^3}{m^2}$

Summary

Objective: To be able to multiply and divide monomials.

Virtue/Skill: If we can divide monomials we will find an interesting fact about what negative exponents are. Then we can start having fun simplifying algebraic expressions.

Assignment: Workbook 7-2

Homework Answers 7-2:

1. 6 2. 9^4 3. x^2 4. $1/s^2$

5. $1/m^2$ 6. 3d 7. $n^4/3$ 8. u^2

9. a^2b^3 10. m^4 11. $(-3w)/u^3$

12. $-4x^2yz^3$ 13. $(16p^{14})/(49s^4)$

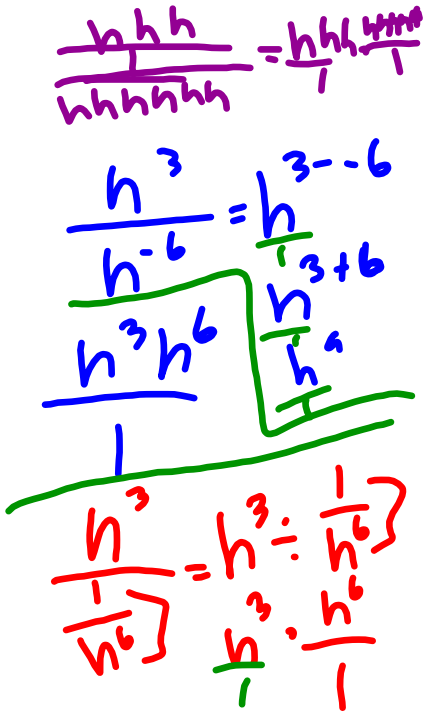
14. 1/256 15. 1/64 16. 9/25

17. 11/9 18. h^9 19. $1/k^2$

20. $m^3/(kl^6)$ 21. $1/f^{11}$ 22. 1

23. (g^4h^2/f^5) 24. $3x^5y^2$ 25. $-3/u^4$

26. $-(8x^5y^2)/z$



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7-1 Study Guide and Intervention (continued)

Multiplying Monomials

Powers of Monomials An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and all integers m and n, $(a^m)^n = a^{mn}$.
Power of a Product	For any number a and all integers m and n, $(ab)^m = a^m b^m$.

Example Simplify $(-2ab^2)^3(a^2)^4$.

$(-2ab^2)^3(a^2)^4 = (-2ab^2)^3(a^8)$ Power of a Power
$= (-2)^3(a^3)(b^2)^3(a^8)$ Power of a Product
$= (-2)^3(a^3)(a^8)(b^2)^3$ Group the coefficients and the variables
$= (-2)^3(a^{11})(b^2)^3$ Product of Powers
$= -8a^{11}b^6$ Power of a Power

The product is $-8a^{11}b^6$.

Exercises

Simplify.

1. $(y^5)^2$ y^{10}
2. $(n^7)^4$ n^{28}
3. $(x^2)^5(x^3)$ x^{13}
4. $-3(ab^4)^3$ $-3a^3b^{12}$
5. $(-3ab^4)^3$ $-27a^3b^{12}$
6. $(4x^2b)^3$ $64x^6b^3$
7. $(4a^2)^2(b^3)$ $16a^4b^3$
8. $(4x)^2(b^3)$ $16x^2b^3$
9. $(x^2y^4)^5$ $x^{10}y^{20}$
10. $(2a^3b^2)(b^3)^2$ $2a^3b^8$
11. $(-4xy)^3(-2x^2)^3$ $512x^9y^3$
12. $(-3j^2k^3)^2(2j^2k)^3$ $72j^{10}k^9$
13. $(25a^2b)^2\left(\frac{1}{5}abc\right)^2$ $625a^8b^6c^2$
14. $(2xy)^2(-3x^2)(4y^4)$ $-48x^4y^6$
15. $(2x^3y^2z^2)^2(x^2z)^4$ $8x^{17}y^6z^{10}$
16. $(-2n^6y^5)(-6n^2y^2)(ny)^3$ $12n^{12}y^{10}$
17. $(-3a^3n^4)(-3a^2n)^4$ $-243a^{15}n^8$
18. $-3(2x)^4(4x^5y)^2$ $-768x^{14}y^2$

7-1 Practice

Multiplying Monomials

Determine whether each expression is a monomial. Write *yes* or *no*. Explain.

1. $\frac{21a^2}{7b}$ No; this involves the quotient, not the product, of variables.
2. $\frac{b^3c^2}{2}$ Yes; this is the product of a number, $\frac{1}{2}$, and two variables.

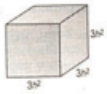
Simplify.

3. $(-5x^2y)(3x^4)$ $-15x^6y$
4. $(2ab^2c^2)(4a^3b^2c^2)$ $8a^4b^4c^4$
5. $(3cd^4)(-2c^2)$ $-6c^3d^4$
6. $(4g^3h)(-2g^5)$ $-8g^8h$
7. $(-15xy^4)\left(-\frac{1}{3}xy^3\right)$ $5x^2y^7$
8. $(-xy^3)(xz)$ $-x^4y^3z$
9. $(-18m^2n)^2\left(-\frac{1}{6}mn^2\right)$ $-54m^5n^4$
10. $(0.2a^2b^3)^2$ $0.04a^4b^6$
11. $\left(\frac{2}{3}p\right)^2$ $\frac{4}{9}p^2$
12. $\left(\frac{1}{4}cd^3\right)^2$ $\frac{1}{16}c^2d^6$
13. $(0.4k^3)^3$ $0.064k^9$
14. $[(4^2)^2]^2$ 4^8 or 65,536

GEOMETRY Express the area of each figure as a monomial.

15.  $18a^3b^6$
16.  $(25x^6)\pi$
17.  $12a^5c^4$

GEOMETRY Express the volume of each solid as a monomial.

18.  $27h^6$
19.  m^4n^5
20.  $(63g^4)\pi$

21. **COUNTING** A panel of four light switches can be set in 2^4 ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set? 2^5 or 32

22. **HOBBIES** Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year? 2^6 or 64

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7-2 Study Guide and Intervention

Dividing Monomials

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b, $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 Simplify $\frac{a^4b^7}{ab^2}$. Assume neither a nor b is equal to zero.

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.
$= (a^{4-1})(b^{7-2})$ Quotient of Powers
$= a^3b^5$ Simplify.

The quotient is a^3b^5.

Example 2 Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that b is not equal to zero.

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient
$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product
$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power
$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{5^5}{5^2}$ 5^3 or 125
2. $\frac{m^6}{m^4}$ m^2
3. $\frac{p^5n^4}{p^2n}$ p^3n^3
4. $\frac{a^2}{a}$ a
5. $\frac{x^5y^3}{x^5y^2}$ y
6. $\frac{-2y^7}{14y^5}$ $-\frac{1}{7}y^2$
7. $\frac{xy^6}{y^4x}$ y^2
8. $\left(\frac{2a^2b}{a}\right)^3$ $8a^3b^3$
9. $\left(\frac{4p^4q^4}{3p^2q^2}\right)^3$ $\frac{64}{27}p^6q^6$
10. $\left(\frac{2v^5w^3}{v^4w^3}\right)^4$ $16v^4$
11. $\frac{16}{81r^4s^8}$
12. $\frac{1}{r^4s^4}$

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7-2 Practice

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{8^5}{8^4}$ 8^4 or 4096
2. $\frac{a^4b^6}{ab^3}$ a^3b^3
3. $\frac{xy^2}{xy}$ y
4. $\frac{m^5np}{m^4p}$ mn
5. $\frac{5c^2d^3}{-4c^2d}$ $-\frac{5d^2}{4}$
6. $\frac{8y^7z^6}{4y^6z^5}$ $2yz$
7. $\left(\frac{4f^3g}{3h^6}\right)^3$ $\frac{64f^9g^3}{27h^{18}}$
8. $\left(\frac{6w^5}{7p^6s^3}\right)^2$ $\frac{36w^{10}}{49p^{12}s^6}$
9. $\frac{-4c^2}{24c^5}$ $-\frac{1}{6c^3}$
10. $x^3(y^{-5})(x^{-8})$ $\frac{1}{x^5y^5}$
11. $p(q^{-2})(r^{-3})$ $\frac{p}{q^2r^3}$
12. 12^{-2} $\frac{1}{144}$
13. $\left(\frac{3}{7}\right)^{-2}$ $\frac{49}{9}$
14. $\left(\frac{4}{3}\right)^{-4}$ $\frac{81}{256}$
15. $\frac{22r^3s^2}{11r^2s^{-3}}$ $2rs^5$
16. $\frac{-15au^0u^{-1}}{5a^3}$ $-\frac{3}{u^4}$
17. $\frac{8c^3d^2f^4}{4c^{-1}d^2f^{-3}}$ $2c^4f^7$
18. $\left(\frac{x^{-3}y^5}{4^{-3}}\right)^0$ 1
19. $\frac{6f^{-2}g^3h^5}{54f^{-2}g^{-5}h^3}$ $\frac{g^8h^2}{9}$
20. $\frac{-12t^{-1}u^5v^{-4}}{2t^{-3}uv^5}$ $-\frac{6t^2u^4}{v^9}$
21. $\frac{r^4}{(3r)^3}$ $\frac{r}{27}$
22. $\frac{m^{-2}n^{-5}}{(m^4n^3)^{-1}}$ $\frac{m^2}{n^2}$
23. $\frac{(j^{-1}k^3)^{-4}}{j^3k^3}$ $\frac{j}{k^{15}}$
24. $\frac{(2a^{-2}b)^{-3}}{5a^2b^4}$ $\frac{a^4}{40b^7}$
25. $\left(\frac{q^{-1}r^3}{qr^{-2}}\right)^{-5}$ $\frac{q^{10}}{r^{25}}$
26. $\left(\frac{7c^{-3}d^3}{e^5de^{-4}}\right)^{-1}$ $\frac{c^8}{7d^2e^8}$
27. $\left(\frac{2x^3y^2z}{3x^4yz^{-2}}\right)^{-2}$ $\frac{9x^2}{4y^2z^6}$

28. **BIOLOGY** A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22^3 white blood cells and 22^5 red blood cells. What is the ratio of white blood cells to red blood cells? $\frac{1}{484}$

29. **COUNTING** The number of three-letter "words" that can be formed with the English alphabet is 26^3. The number of five-letter "words" that can be formed is 26^5. How many times more five-letter "words" can be formed than three-letter "words"? 676