Your Name

Mrs. Theo

4/22/2021

Notes

Lesson 6.1

Growth and Decay

Objective: To be able to use the exponential growth and compound interest equations to solve problems.

Life Lesson/Math Skills: We are building on our understanding of exponents and the patterns they create. Scientists and paleontologists use exponential functions to calculate growth and decay of bacteria, elements, chemicals, and to see how old fossils are. Investment bankers and money market advisors also use these equations. We need to understand exponential functions before we can understand logarithmic functions or expressions.

Exponential Growth/ Decay Equation

Function described by:

$A = P(1 + r)^t$

A = Accumalated amount (ending amount)

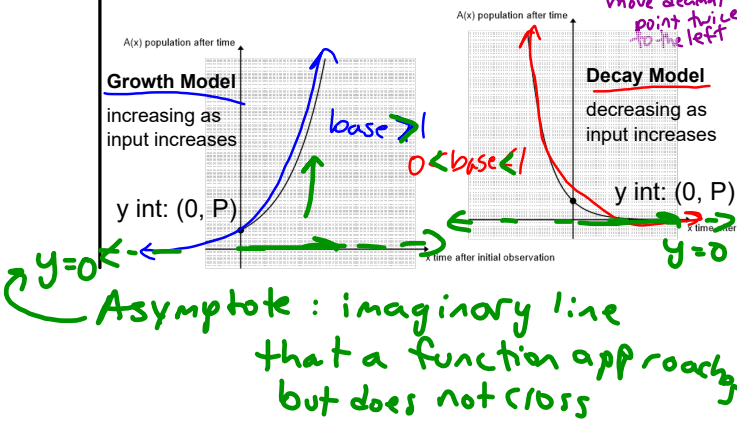
P: Principal amount (starting amount)

r: rate of change as a decimal (positive if growth, negative if decay)

80% → .8

t: time

% → decimal move decimal point twice left



Asymptote: imaginary line that a function approaches but does not cross

In 2005 the town of Flat Creek had a population of about 280,000 and a growth rate of 0.85% per year.

a. Write an equation to represent the population of Flat Creek since 2005

Step 1: A= ? P= 280,000 r= 0.0085 t= ?

Population growth positive r .85%

Step 2: Plug these in $A = P(1+r)^t$

$A = 280,000(1+0.0085)^t$

$A = 280,000(1.0085)^t$

more than 1 → growth

b. According to the equation what will the population of Flat Creek be in 2015?

A = ending population

t =

2015 − 2005

t = 10

$A = 280,000(1.0085)^{10}$

$A = 304,731.2947$ People

round down

$A = 304,731$ population

In 2005 the town of Flat Creek had a population of about 280,000 and a growth rate of 0.85% per year.

rate is +0.0085

a. Write an equation to represent the population of Flat Creek since 2005

Initial Population 0.85%

Step 1: A= ? P=280,000 r=+0.0085 t= ?

Step 2: Plug these in $A = P(1+r)^t$

$A = 280,000(1+0.0085)^t$

$A = 280,000(1.0085)^t$

bigger than 1 growth

b. According to the equation what will the population of Flat Creek be in 2015?

A = ?

$t = 2015 - 2005 = 10$

$t = 10$

$A = 280,000(1.0085)^{10}$

$A = 304,731.2947$ living creatures round down

$A = 304,731$ people in flat creek

During an economic recession, a charitable organization found that its donation dropped by 1.1% per year. Before the recession, its donations were $390,000.

a. Write an equation to represent the charity's donations since the beginning of the recession.

decreased → decay negative r value

starting Donations 0.011

Step 1: A= ? P=390,000 r=−0.011 t= ?

Step 2: Plug these in $A = P(1+r)^t$

$A = 390,000(1-0.011)^t$

$A = 390,000(0.989)^t$

less than 1 → decay

b. Estimate the amount of the donations 5 years after the start of the recession.

$A = 390,000(0.989)^5$ Money

$A = 369016.74$ round to 2 decimal places

During an economic recession, a charitable organization found that its donation dropped by 1.1% per year. Before the recession, its donations were $390,000.

a. Write an equation to represent the charity's donations since the beginning of the recession.

initial donations decrease → decay −1.1%

Step 1: A= ? P=390,000 r= −0.011 t= ?

Step 2: Plug these in $A = P(1+r)^t$

$A = 390{,}000(1+-0.011)^t$

$A = 390{,}000(0.989)^t$

less than 1 so decay

b. Estimate the amount of the donations 5 years after the start of the recession. A=? t=5

$A = 390{,}000(0.989)^5$

$A = \$369{,}016.74$ money round to 100ths

You bought a sculpture in 1985 for $380. It increases in value by 8% each year. What is the value in 1990? 2000?

Is it Decay or Growth? Growth

Equation for Exponential: $A = 380(1.08)^5$

A=? P=$380 r=0.08 t=5

8% 1990 − 1985 = 5

$A = P(1+r)^t$

$A = 380(1+0.08)^5$

$A = 380(1.08)^5$

$A = \$558.34$

A new car in 1990 cost $20,000. The value decreases 16% every year. What is the value in 1996? 1998?

Is it Decay or Growth? Decay

Equation for Exponential: _____

$P=20000 \quad r=.16 \quad t=6, 8$

$\sim 16\%$

$1996 - 1990 = 6 \qquad 1998 - 1990 = 8$

$A=20000(1-.16)^6$

$=20000(.84)^6$

$A=7185.96$

after 6 yrs

$A=\$4957.52$

after 8 yrs

A house originally cost $20,000 in 1950. The value increased 5% a year to 1995. What is the value in 1995?

Is it Decay or Growth? Growth

Equation for Exponential: _____

$A=? \quad P=20,000 \quad r=0.05 \quad t=45$ yrs

$\sim 5\%$

$1995 - 1950 = 45$

$A=P(1+.05)^t$

$A=20000(1.05)^{45}$

$\$179,700.16$

$$A(x) = P(1 + r)^x$$

What if something has a 100% growth, what does that mean?

There are some famous exponential change models in mathematics

- When the rate of change is increasing by **100%, r = 1**
  - we have the **doubling model** : $A(x) = P(1 + 1)^x =$ **$P(2)^x$**
- When the rate of change is increasing by **200%, r = 2**
  - we have the **tripling model** : $A(x) = P(1 + 2)^x =$ **$P(3)^x$**
- When the rate of change is decreasing by **50%, r = - 0.5**
  - we have the **half model** : $A(x) = P(1 - 0.5)^x =$ **$P(0.5)^x$**

In Summary:

When the decimal version of the rate of change is **ADDED**,

we are building an exponential **GROWTH** model.

When the decimal version of the rate of change is **SUBTRACTED**,

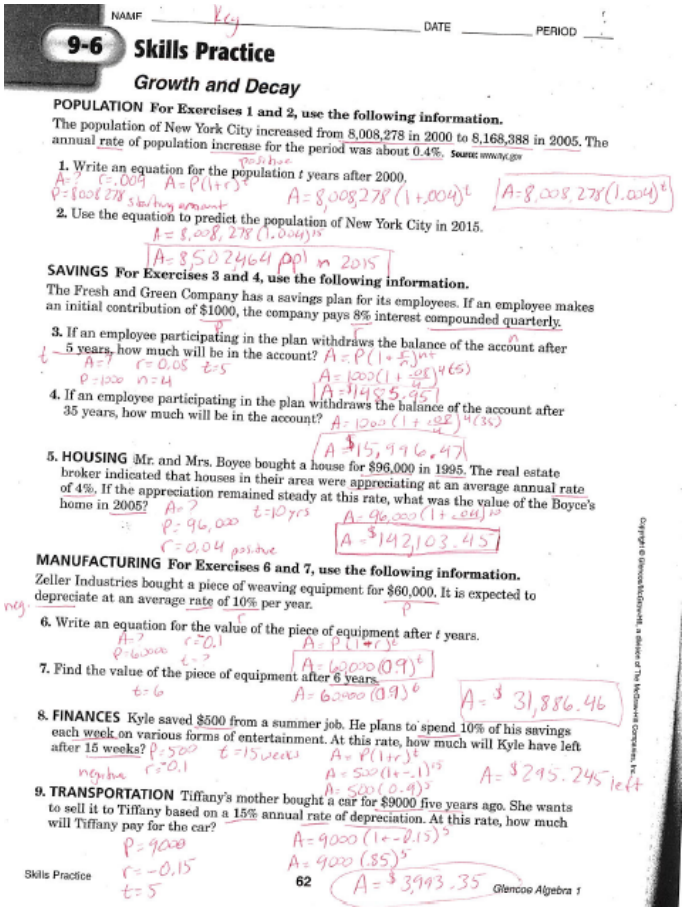
we are building an exponential **DECAY** model.

Summary

Objective: To be able to use the exponential growth and compound interest equations to solve problems.

Virtue/Skills: We are building on our understanding of exponents and the patterns they create. Scientists and paleontologists use exponential functions to calculate growth and decay of bacteria, elements, chemicals, and to see how old fossils are. Investment bankers and money market advisors also use these equations. We need to understand exponential functions before we can understand logarithmic functions or expressions.

Assignment: Workbook 9-6



NAME Key DATE PERIOD

9-6 Skills Practice

Growth and Decay

POPULATION For Exercises 1 and 2, use the following information.

The population of New York City increased from 8,008,278 in 2000 to 8,168,388 in 2005. The annual rate of population increase for the period was about 0.4%. Source: www.nyc.gov

positive

1. Write an equation for the population t years after 2000.

A=? r=.004 $A=P(1+r)^t$ $A=8,008,278(1+.004)^t$ $A=8,008,278(1.004)^t$

P=8008278 starting amount

2. Use the equation to predict the population of New York City in 2015.

$A=8,008,278(1.004)^{15}$

A=8,502,464 ppl in 2015

SAVINGS For Exercises 3 and 4, use the following information.

The Fresh and Green Company has a savings plan for its employees. If an employee makes an initial contribution of $1000, the company pays 8% interest compounded quarterly.

3. If an employee participating in the plan withdraws the balance of the account after 5 years, how much will be in the account? $A=P(1+\frac{r}{n})^{nt}$

A=? r=0.08 t=5

P=1000 n=4

$A=1000(1+\frac{.08}{4})^{4(5)}$

A=1485.95

4. If an employee participating in the plan withdraws the balance of the account after 35 years, how much will be in the account? $A=1000(1+\frac{.08}{4})^{4(35)}$

A=$15,996.47

5. **HOUSING** Mr. and Mrs. Boyce bought a house for $96,000 in 1995. The real estate broker indicated that houses in their area were appreciating at an average annual rate of 4%. If the appreciation remained steady at this rate, what was the value of the Boyce's home in 2005?

A=? t=10 yrs

P=96,000

r=0.04 positive

$A=96,000(1+.04)^{10}$

A=$142,103.45

MANUFACTURING For Exercises 6 and 7, use the following information.

Zeller Industries bought a piece of weaving equipment for $60,000. It is expected to depreciate at an average rate of 10% per year.

neg.

6. Write an equation for the value of the piece of equipment after t years.

A=? r=0.1

P=60000 t=?

$A=P(1+r)^t$

$A=60000(.09)^t$

7. Find the value of the piece of equipment after 6 years.

t=6

$A=60000(.09)^6$

A=$31,886.46

8. **FINANCES** Kyle saved $500 from a summer job. He plans to spend 10% of his savings each week on various forms of entertainment. At this rate, how much will Kyle have left after 15 weeks?

P=500 t=15 weeks $A=P(1+r)^t$

negative r=-0.1

$A=500(1+-.1)^{15}$

$A=500(0.9)^5$

A=$295.245 left

9. **TRANSPORTATION** Tiffany's mother bought a car for $9000 five years ago. She wants to sell it to Tiffany based on a 15% annual rate of depreciation. At this rate, how much will Tiffany pay for the car?

P=9000

r=-0.15

t=5

$A=9000(1+-0.15)^5$

$A=9000(.85)^5$

A=$3993.35