Your Name

Mrs. T

4 26 2021

Notes

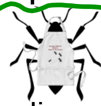
Lesson 6.1

Compound Interest

Compound Interest

If the situation says: "compounded"....

$$A = P(1+\frac{r}{n})^{nt}$$



apron ant

A = Accumalated amount (ending amount)

P: Principal amount (starting amount)

r: rate of interest as a decimal (positive if growth, negative if decay)

n: number of times a year it is compounded

t: time

Compounded Meanings

the percent is taken from the newest amount and added to make a new total

compounded quarterly: 4 times a year $n=4$

compounded monthly: 12 $n=12$

compounded weekly: 52 $n=52$

compounded annually: $n=1$

compounded semiannually: $n=0.5$

compounded biannually: $n=2$

Determine the amount of an investment of $100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years.

Step 1: A= ? P= 100,000 r= .052 n= 4 t= 12

Step 2: Plug these in $A = P(1+\frac{r}{n})^{nt}$

$A = 100{,}000\left(1 + \frac{0.052}{4}\right)^{4 \cdot t}$

$A = 10{,}000\left(1 + \frac{0.052}{4}\right)^{4 \cdot 12}$

$A = \$185{,}888.87$

When Jing May was born her grandparents invested $1000 in a fixed rate savings account at a rate of 7% compounded annually. The money will go to Jing when she Turns 18 to help with her college expenses. What amount of money will Jing receive from the investment?

$A = P(1+r)^t$ or $A = P(1+\frac{r}{n})^{nt}$?

A = ? P = 1000 r = 7% → 0.07

n = 1 t = 18

Determine the amount of an investment of $2500 if it is invested at an interest rate of 5.25% compounded monthly for 4 years?

$A = P(1+r)^t$ or $A = P(1+\frac{r}{n})^{nt}$?

$n = 12$

You have $1000.00 that you want to invest. You find a bank that offers a 5% interest rate and compounds quarterly for savings accounts.

P = 1000 r = .05 n = 4

Equation: $A = 1000(1+\frac{0.05}{4})^{4t}$

What is the balance in the account....

A year? 2 years? 10 years?

$1000(1.0125)^{4(1)}$

$A = \$1,050.94$

$1000(1.0125)^{4(2)}$

$A = \$2,101.89$

$10,509.45

Suppose $5,000 is invested in a savings account at an 6% interest rate. How much is in the account after 10 years if the money is:

a. compounded annually? $n=1$ $A=5000\left(1+\frac{.06}{1}\right)^{1\cdot 10}$ $A=8,954$

b. compounded quarterly? $n=4$ $A=5000\left(1+\frac{0.06}{4}\right)^{4\cdot 10}$ $A=9,070.09$

c. compounded monthly? $n=12$ $A=5000\left(1+\frac{0.06}{12}\right)^{12\cdot 10}$ $A=9,096.98$

d. compounded continuously?

6.2 Natural Exponential Functions And Calculating Interest

So what is *e*???

- The history of mathematics is marked by discovery of special numbers such as zero, imaginary numbers, and π.
- Like π and *i*, the number *e* is denoted by a letter.
- The number is called the natural base *e*, or Euler's number, after its discoverer, Leonhard Euler (1707-1783).

https://www.desmos.com/calculator/qe6f7o8pzf

Investigating the Natural Base *e*

Use a calculator to complete the table:

n	10	100	1000	10000	100000
$(1+\frac{1}{n})^n$	2.5937	2.7048	2.7169	2.7181	2.71826

Do the values appear to be approaching a number?

We discovered as n gets larger and larger, the expression $(1+ 1/n)^n$ gets closer and closer to 2.71828…, which is the **VALUE OF e!!!**

In Calculus terms...

Formula for Continuous Exponential Growth and Decay

2nd Ln

$$\boxed{A = Pe^{rt}}$$

A = Total Amount

P = Initial Amount

r = rate (written as a decimal)

t = time (in years)

\+ growth

\- decay

How much should be invested ($P=$) at an interest rate of 4.5% for 15 years to obtain the accumulated amount of $1,000,000 if the principal was compounded continuously.

$P = ?$ $4.5\% \to r = 0.045$ $t = 15$ years

$A = 1{,}000{,}000$

$$A = Pe^{rt}$$

$$\frac{1{,}000{,}000}{e^{(0.045 \cdot 15)}} = \frac{Pe^{0.045 \cdot 15}}{e^{(0.045 \cdot 15)}}$$

$$P = \$509{,}156.42$$

Ex: The number of flies after t hours is $Q(t) = 20e^{0.03t}$

$A = Pe^{rt}$

a.) Initial number of flies?

$P = 20$ flies

$r = 0.03$ rate 3%

b.) Population of flies after 72 hours? $t = 72$

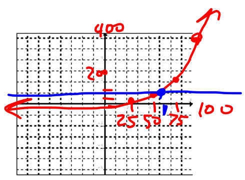
$Q(t) = 20e^{(.03(72))}$

$= 173.42$

$= 173$ flies

c.) Sketch the graph of Q(t).

t	1	2	5	10	100
$Q(t)$	20	21	23	26	401



d.) How long does it take for the number of flies to be more than 100?

$$\frac{100}{20} = \frac{\cancel{20}e^{.03t}}{\cancel{20}}$$

$$5 = e^{.03t}$$

$$\ln(5) = \cancel{\ln}(e^{.03t})$$

$$\frac{\ln(5)}{0.03} = \frac{.03t}{0.03}$$

$$53.65 = t$$

at least 53.65 hours to reach at least 100 flies