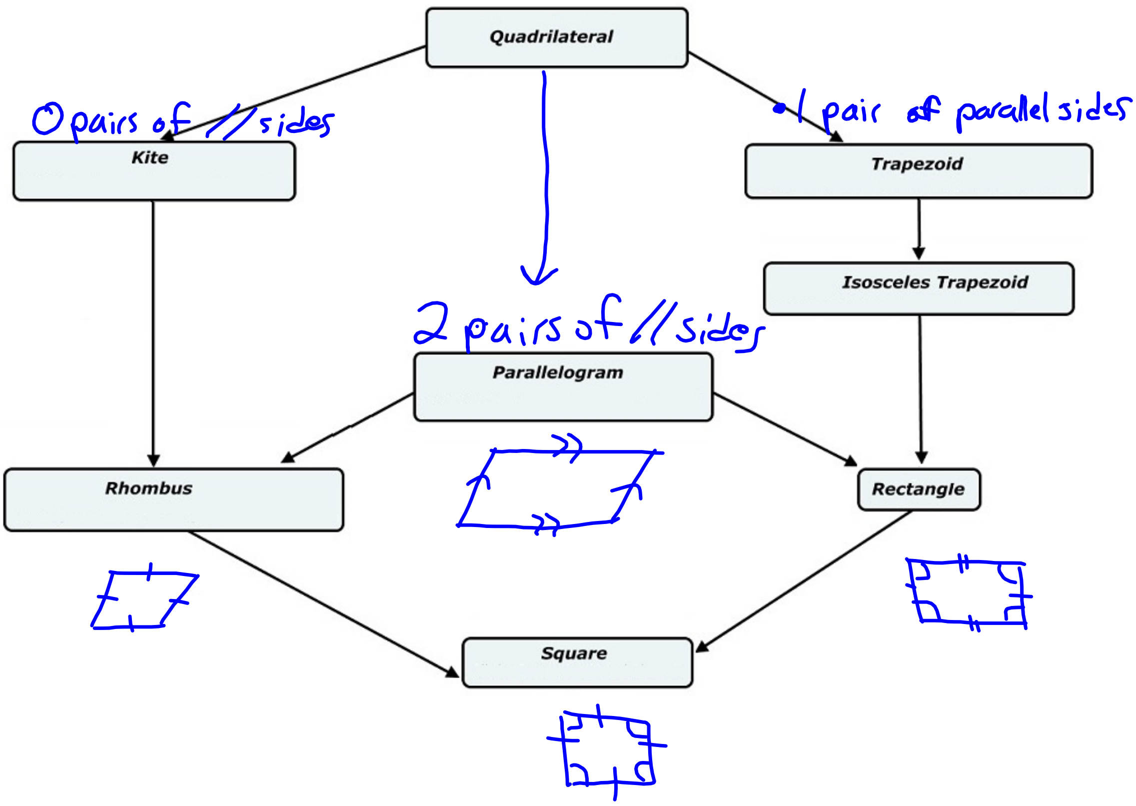
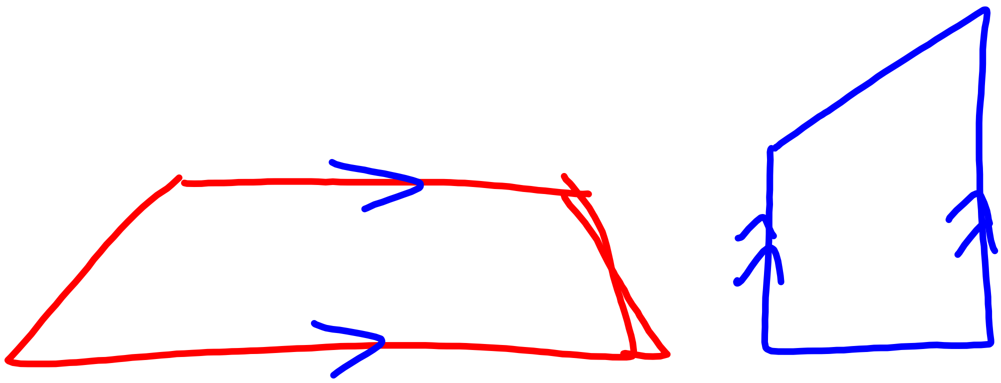
Properties of Quadrilaterals

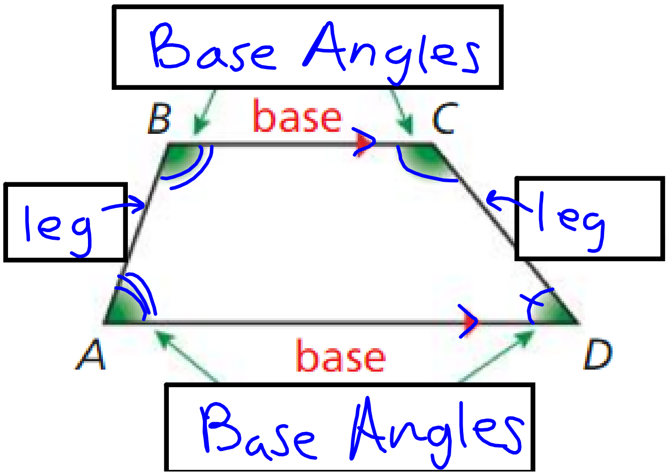


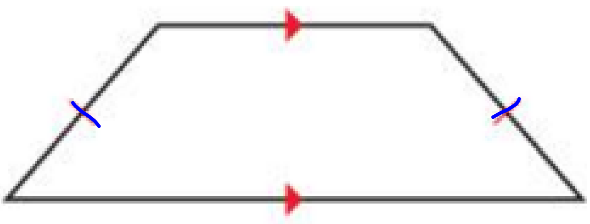


7.5 - More Special Quadrilaterals

Trapezoid = Quadrilateral with exactly **one pair of opposite sides parallel**.

- The parallel sides are called the Bases
- The other sides (non-parallel) are called the Legs
- If the legs are congruent, then the trapezoid is an Isosceles Trapezoid



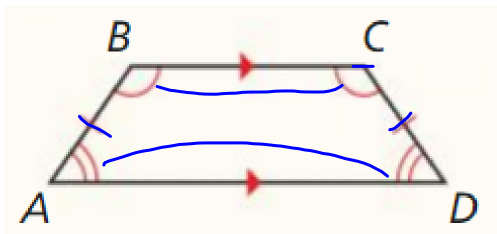


Isosceles trapezoid

Isosceles Trapezoid

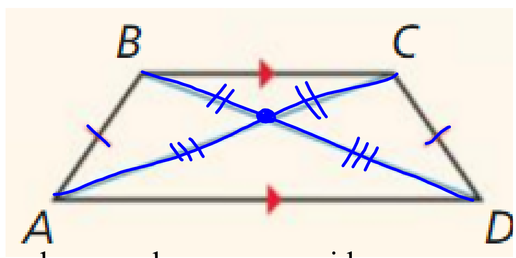
1. Each pair of base angles is congruent.

$\angle B \cong \angle C$	$\angle A \cong \angle D$



2. Diagonals are congruent.

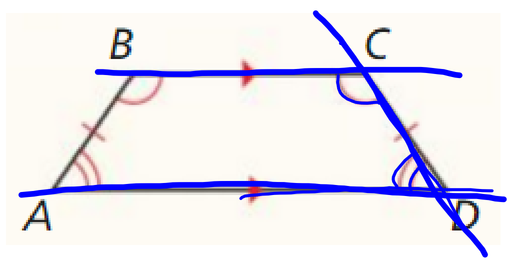
$\overline{BD} \cong \overline{AC}$



3. Leg Angles are supplementary (opposing base angles are same side interior angles to the parallel bases, so they add to 180)

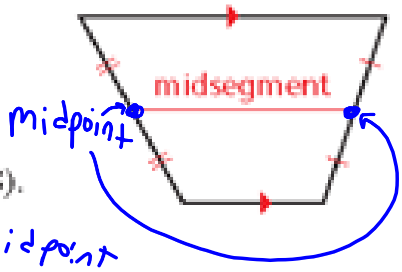
$\angle B + \angle A = 180$

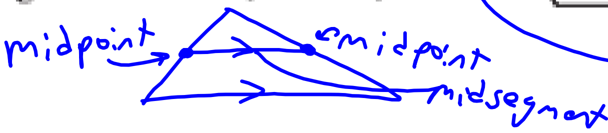
$\angle C + \angle D = 180$



Using the Trapezoid Midsegment Theorem

Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs. The theorem below is similar to the Triangle Midsegment Theorem (Thm. 6.8).





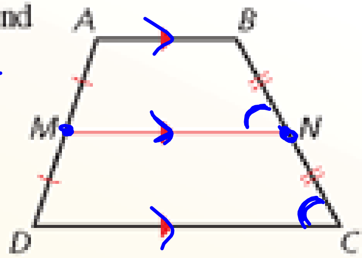
Theorem

Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $\boxed{MN = \frac{1}{2}(AB + CD)}$.

Proof Ex. 49, p. 406



Example #1 Identifying a Trapezoid in the Coordinate Plane

Show that $ORST$ is a trapezoid. Then decide whether it is isosceles.

slope $\frac{\text{rise}}{\text{run}}$

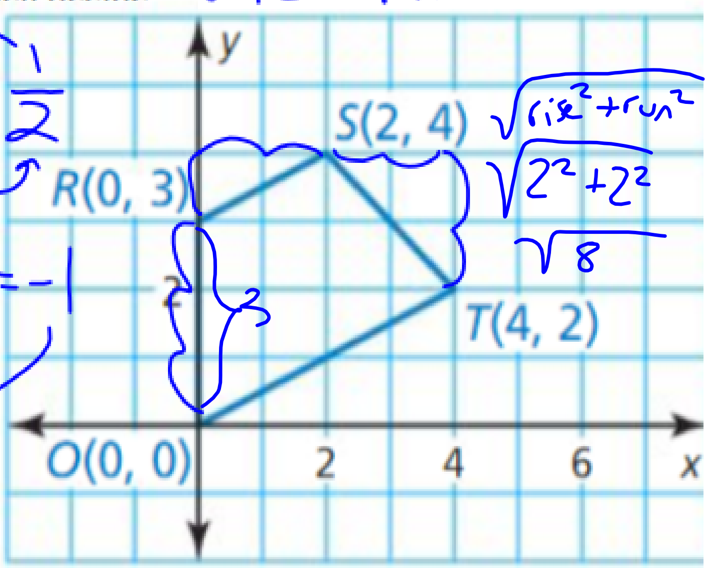
$m_{RS} = \frac{1}{2}$ same $m_{OT} = \frac{2}{4} = \frac{1}{2}$ $\frac{-2}{-4}$

$m_{OR} = \frac{3}{0}$ =undefined $m_{TS} = \frac{2}{-2} = -1$ not same

$\sqrt{rise^2 + run^2}$

$\sqrt{2^2 + 2^2}$

$\sqrt{8}$

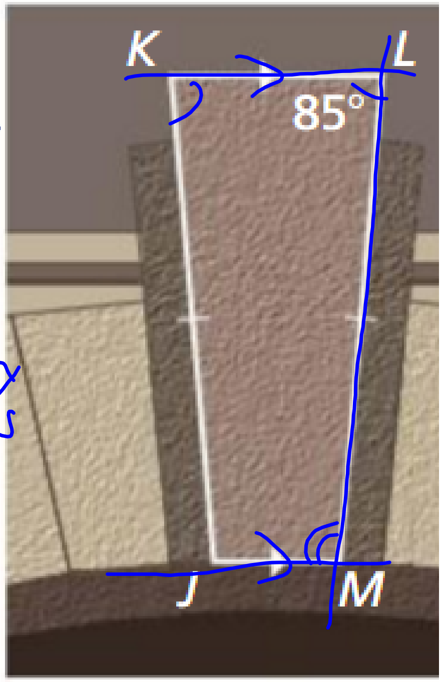


Since only one pair of opposite sides are parallel, this is a trapezoid and not a parallelogram or kite

Not isosceles b/c legs not ≅, $\overline{RO} \ncong \overline{ST}$

Example #2 Using Properties of Isosceles Trapezoids

The stone above the arch in the diagram is an isosceles trapezoid. Find $m\angle K$, $m\angle M$, and $m\angle J$.



$m\angle K = 85°$ base angles $\cong$

$m\angle L + m\angle M = 180$ same side int. angles supplementary in // lines

$85 + m\angle M = 180$

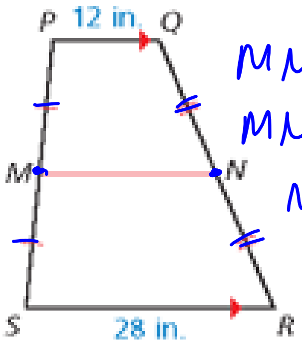
$m\angle M = 95°$

$m\angle J = m\angle M$

$m\angle J = 95°$ base angles $\cong$

Example #3

Find MN Find Midsegment



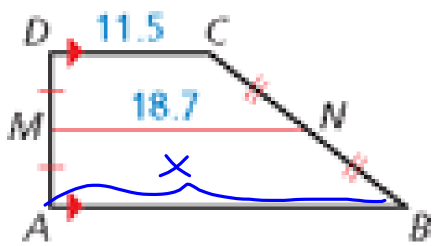
$MN = \frac{1}{2}(PQ + SR)$

$MN = \frac{1}{2}(12 + 28)$

$MN = \frac{1}{2}(40)$

$MN = 20$ in

Find AB Use Midsegment Find Base



$MN = \frac{1}{2}(DC + AB)$

$18.7 = \frac{1}{2}(11.5 + x)$

$18.7 = 5.75 + \frac{1}{2}x$

$-5.75 \quad -5.75$

$2(12.95) = (\frac{1}{2}x)2$

$25.9 = x$

or

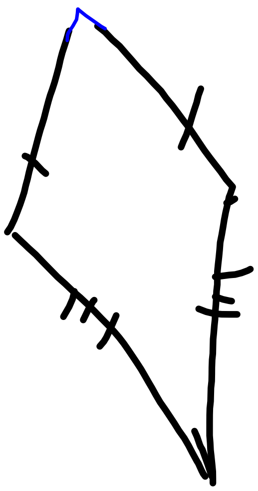
$2(18.7) = (\frac{1}{2}(11.5 + x))2$

$37.4 = 11.5 + x$

$-11.5 \quad -11.5$

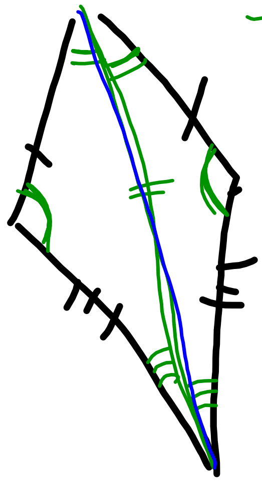
$25.9 = x$

Kites



Kites

SSS



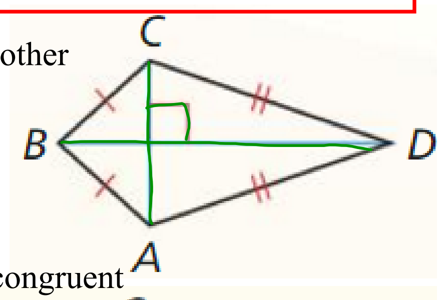
7.5 - More Special Quadrilaterals

Kite = Quadrilateral with **two pairs of consecutive sides congruent**, but opposite sides are not congruent.

- Diagonals are perpendicular to each other

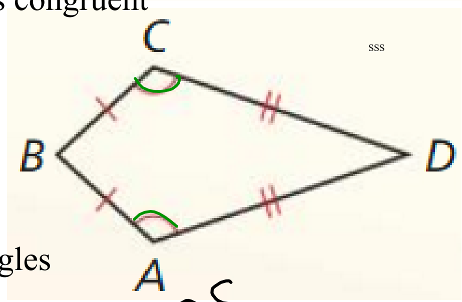
$\boxed{AC \perp BD}$

Perpendicular
90°



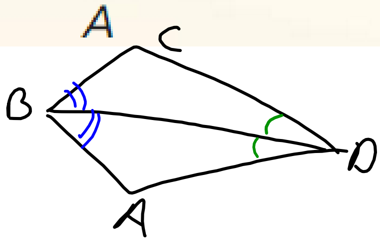
- Exactly one pair of opposite angles congruent

$\boxed{\angle A \cong \angle C}$



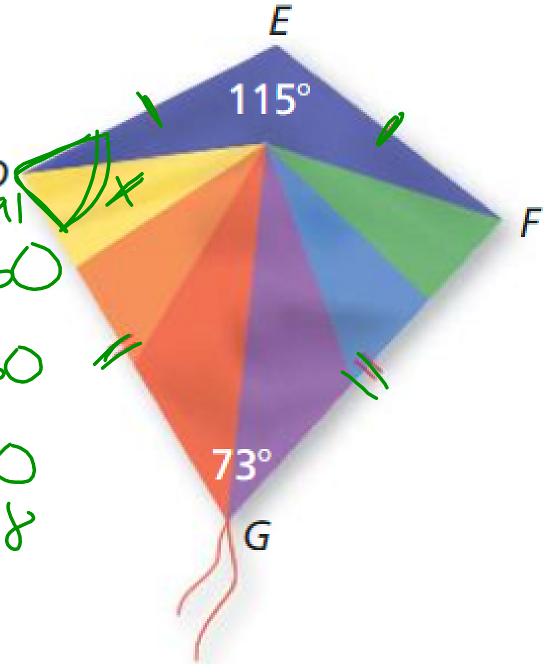
- Diagonals bisect non congruent angles

$\angle ADB \cong \angle CDB$
$\angle CBD \cong \angle ABD$



Example #4 Finding Angle Measures in a kite

Find $m\angle D$ in the kite shown.



$\angle D \cong \angle F = X$ quadrilateral

$m\angle D + m\angle F + m\angle E + m\angle G = 360$

$X + X + 115 + 73 = 360$

$2x + 188 = 360$

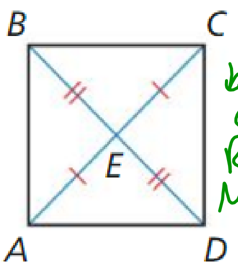
$-188 \quad -188$

$\frac{2x}{2} = \frac{172}{2}$

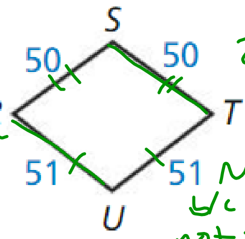
$x = \boxed{86° = m\angle D}$

Example #5 Identifying a Quadrilateral

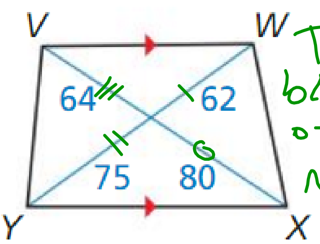
Give the most specific name for the quadrilateral. Explain your reasoning.



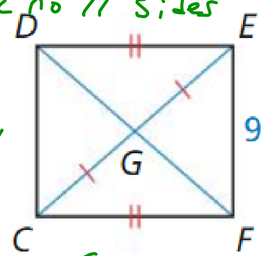
Parallelogram b/c diagonals bisect eachother. Not a Rect b/c diagonals not ≅ Not a rhombus b/c diagonals not ⊥



Kite b/c 2 pairs of adjacent sides ≅ Not parallelogram b/c opposite sides not ≅. Not Trap. b/c no // sides



Trapezoid b/c one pair of opposite sides // not isosceles b/c diagonals not ≅
Not Parallelogram b/c diagonals not bisected
Not kite diagonals not ⊥

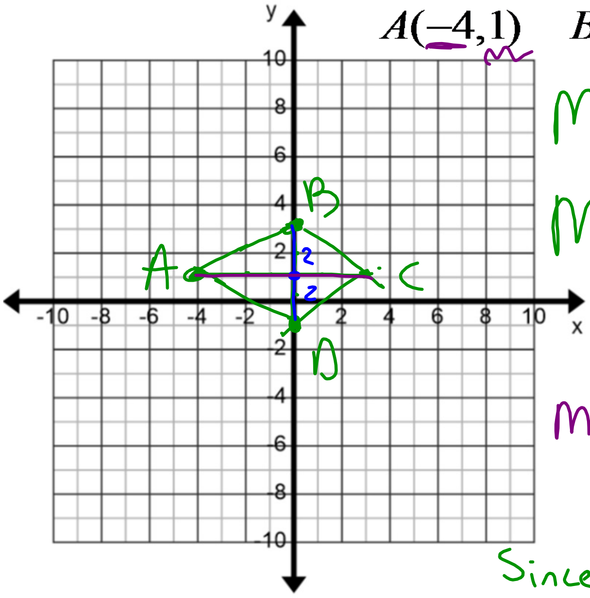


Quadrilateral

Example #6 perpendicular

Give the most descriptive name for quadrilateral ABCD.

$A(-4,1)$ $B(0,3)$ $C(3,1)$ $D(0,-1)$



$m_{AC} = \frac{0}{7} = 0$

$m_{BD} = \frac{-4}{0} = \text{undefined}$ can't ÷ by 0

Midpoint for BD is (0,1)

Midpoint for AC is $\left(\frac{3+-4}{2}, \frac{1+1}{2}\right)$

(-0.5, 1)

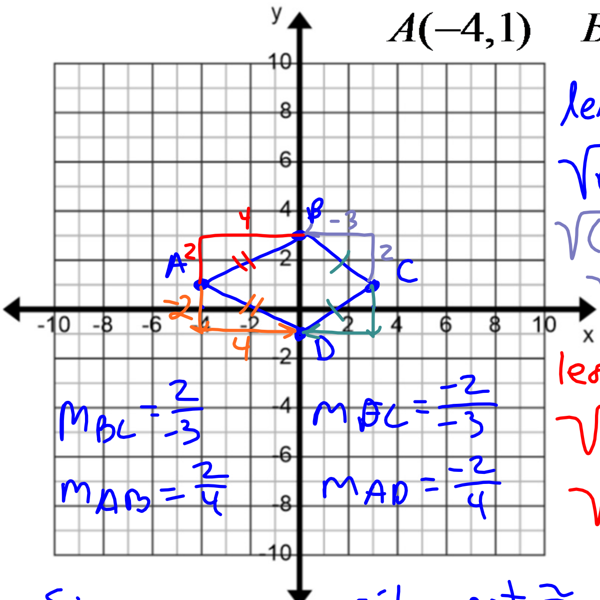
Since diagonals do not bisect each other this is not a Parallelogram, Rhombus, Rect. or Square)

Since diagonals are perpendicular this is a kite and not a trapezoid.

Example #6

Give the most descriptive name for quadrilateral ABCD.

$A(-4,1)$ $B(0,3)$ $C(3,1)$ $D(0,-1)$



length of BC = length of DC

$\sqrt{rise^2 + run^2}$ $\sqrt{(-2)^2+(-3)^2}$

$\sqrt{(2)^2+(-3)^2}$ $\sqrt{4+9}$

$\sqrt{13} = \sqrt{13}$

length of AB length of AD

$\sqrt{(2)^2+(4)^2}$ $\sqrt{(-2)^2+(4)^2}$

$\sqrt{4+16}$ $\sqrt{4+16}$

$\sqrt{20} = \sqrt{20}$

$m_{BC} = \frac{2}{-3}$ $m_{DC} = \frac{-2}{-3}$

$m_{AB} = \frac{2}{4}$ $m_{AD} = \frac{-2}{4}$

Since opposite sides not ≅ (not parallelogram)

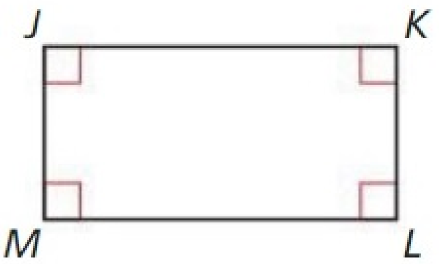
Since no pairs of // sides $m_{BC} \neq m_{AD}$ and $m_{AB} \neq m_{DC}$ (not a trapezoid).

Since it has 2 pairs of adjacent sides ≅ it is a kite.

Example #7

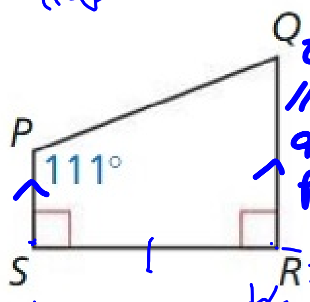
In Exercises 21–24, give the most specific name for the quadrilateral. Explain your reasoning. *(See Example 5.)*

21.



22.

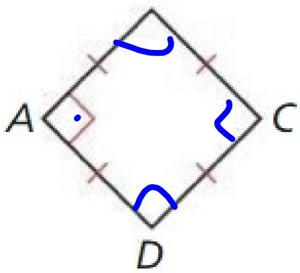
Trapezoid



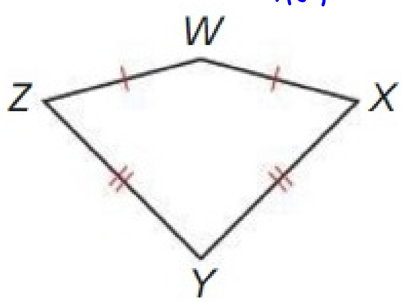
one pair // sides
90+90=180
PS//QR
Not Isosceles b/c base angles no

23.

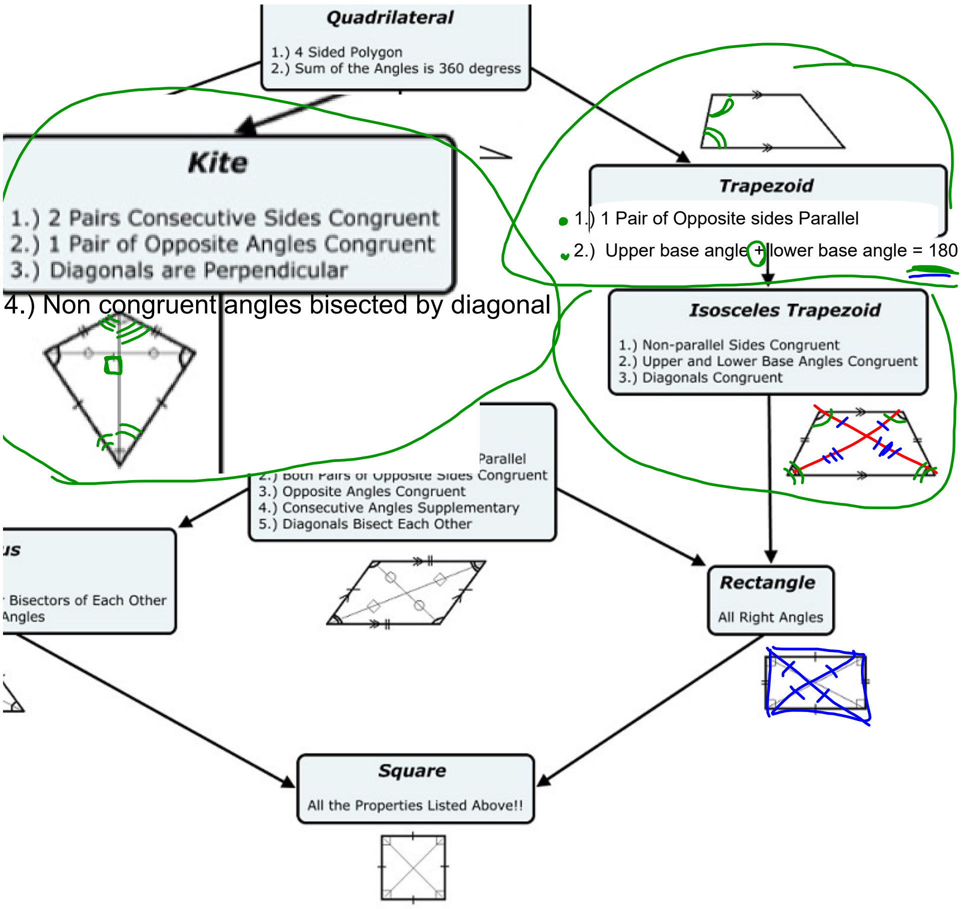
Square

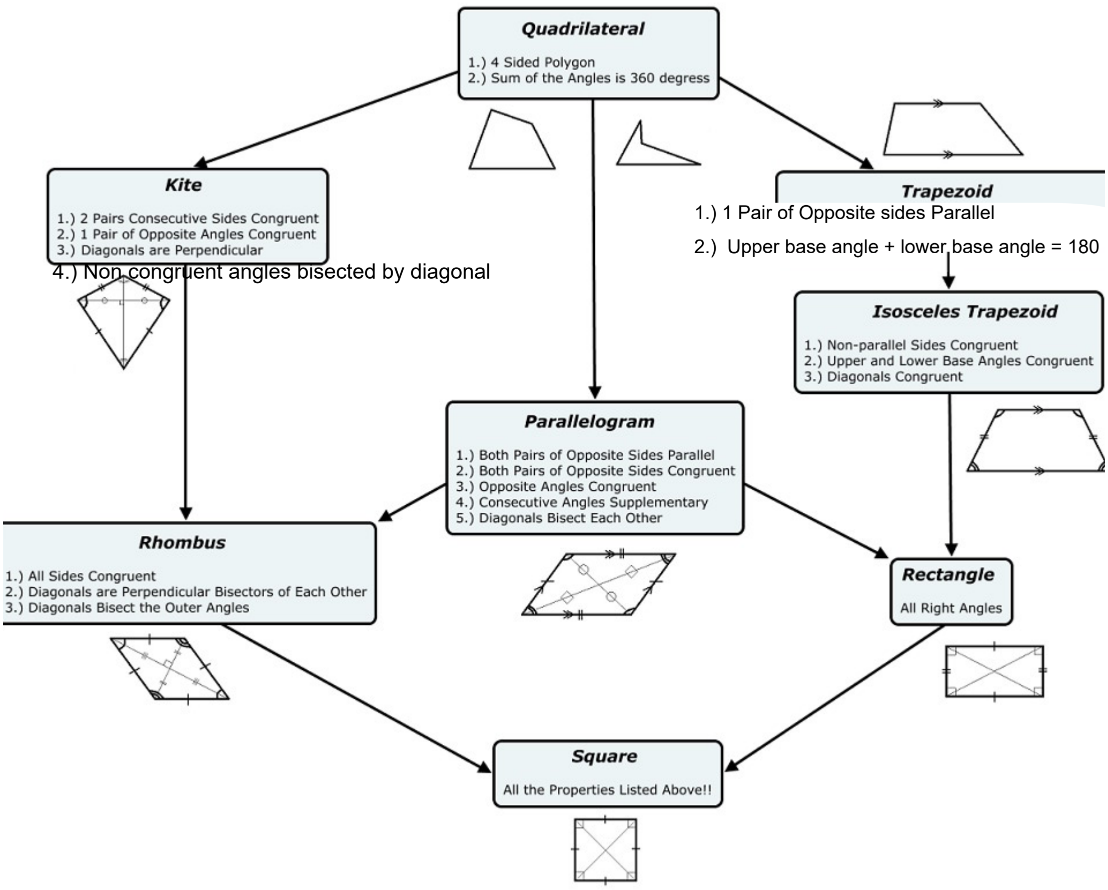


24.



Fill in your chart with the properties and drawings circled on the next page!


Quadrilateral
1.) 4 Sided Polygon
2.) Sum of the Angles is 360 degress
Kite
1.) 2 Pairs Consecutive Sides Congruent
2.) 1 Pair of Opposite Angles Congruent
3.) Diagonals are Perpendicular
4.) Non congruent angles bisected by diagonal
Trapezoid
1.) 1 Pair of Opposite sides Parallel
2.) Upper base angle + lower base angle = 180
Isosceles Trapezoid
1.) Non-parallel Sides Congruent
2.) Upper and Lower Base Angles Congruent
3.) Diagonals Congruent
Parallel
2.) Both Pairs of Opposite Sides Congruent
3.) Opposite Angles Congruent
4.) Consecutive Angles Supplementary
5.) Diagonals Bisect Each Other
us
Bisectors of Each Other
Angles
Rectangle
All Right Angles
Square
All the Properties Listed Above!!


Quadrilateral
1.) 4 Sided Polygon
2.) Sum of the Angles is 360 degress
Kite
1.) 2 Pairs Consecutive Sides Congruent
2.) 1 Pair of Opposite Angles Congruent
3.) Diagonals are Perpendicular
4.) Non congruent angles bisected by diagonal
Trapezoid
1.) 1 Pair of Opposite sides Parallel
2.) Upper base angle + lower base angle = 180
Isosceles Trapezoid
1.) Non-parallel Sides Congruent
2.) Upper and Lower Base Angles Congruent
3.) Diagonals Congruent
Parallelogram
1.) Both Pairs of Opposite Sides Parallel
2.) Both Pairs of Opposite Sides Congruent
3.) Opposite Angles Congruent
4.) Consecutive Angles Supplementary
5.) Diagonals Bisect Each Other
Rhombus
1.) All Sides Congruent
2.) Diagonals are Perpendicular Bisectors of Each Other
3.) Diagonals Bisect the Outer Angles
Rectangle
All Right Angles
Square
All the Properties Listed Above!!

5. slope of $\overline{MQ}$ = slope of $\overline{NP}$ and slope of $\overline{MN} \neq$ slope of $\overline{PQ}$; $MN \neq PQ$, so $MNPQ$ is not isosceles.

6. slope of $\overline{HL}$ = slope of $\overline{JK}$ and slope of $\overline{HJ} \neq$ slope of $\overline{LK}$; $HJ = LK$, so $HJKL$ is isosceles.

8. $m\angle Q = m\angle T = 98°$, $m\angle R = m\angle S = 82°$

15. 110°

18. 70°

20. In the kite shown, $\angle B \cong \angle D$. Find $m\angle A$ by subtracting the measures of the other three angles from 360°, $m\angle A = 360° - 50° - 2(120°) = 70°$.

21. rectangle; $JKLM$ is a quadrilateral with 4 right angles.

22. trapezoid; $\overline{PS} \parallel \overline{QR}$ and $\angle QPS$ and $\angle PSR$ are not congruent.

23. square; All four sides are congruent and the angles are 90°.

24. kite; $WXYZ$ has two pairs of consecutive congruent sides and opposite sides are not congruent.

31. $\angle A \cong \angle D$, or $\angle B \cong \angle C$; $\overline{BC} \parallel \overline{AD}$, so base angles need to be congruent.

32. *Sample answer:* $\overline{BC} \cong \overline{DC}$; Then $\triangle ABC \cong \triangle ADC$ and $ABCD$ has two pairs of consecutive congruent sides.

33. *Sample answer:* $\overline{BE} \cong \overline{DE}$; Then the diagonals bisect each other.

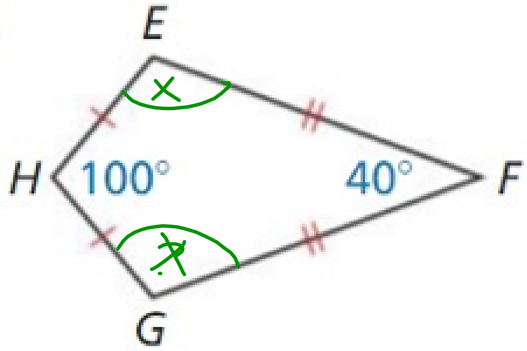
34. *Sample answer:* $\overline{AB} \cong \overline{BC}$; A rectangle with a pair of congruent adjacent sides is a square.

38. yes; $AB = AD = \sqrt{53}$ and $BC = DC = \sqrt{265}$

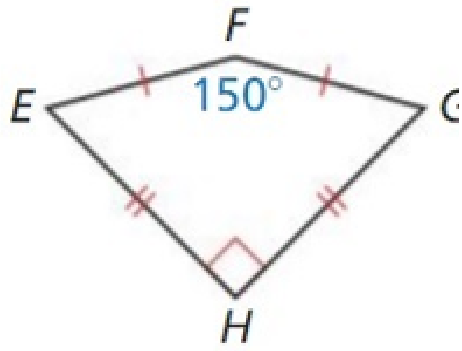
In Exercises 15–18, find $m\angle G$. *(See Example 3.)*

(from book or properties in notes)

15.

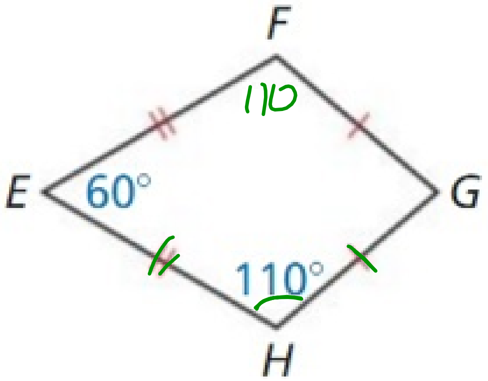


16.

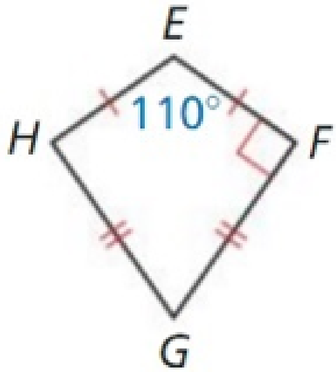


$$x + 40 + x + 100 = 360$$
$$2x + 140 = 360$$
$$2x = 220$$
$$x = 110$$

17.



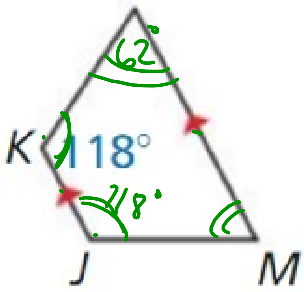
18.



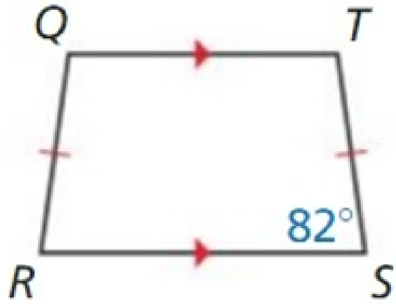
$$360 - 110 - 110 - 60 = m\angle G$$

In Exercises 7 and 8, find the measure of each angle in the isosceles trapezoid. *(See Example 2.)*

7.



8.



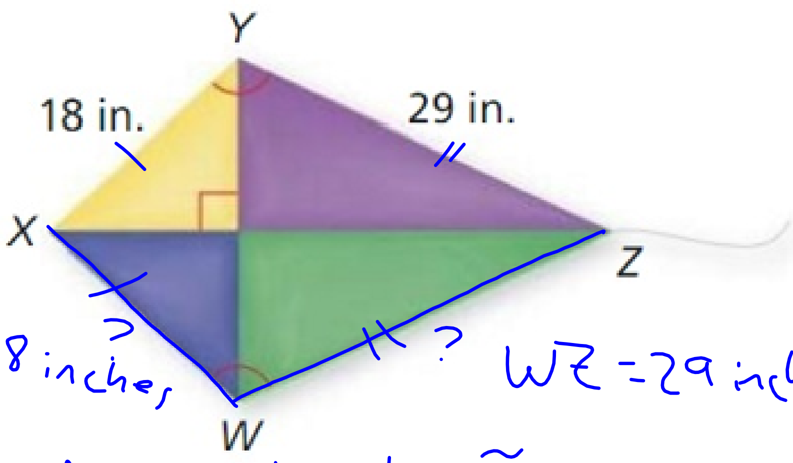
m∠J = 118°

m∠K + m∠L = 180

m∠L = 180 − 118 = 62°

m∠M = 62°

30. PROBLEM SOLVING You and a friend are building a kite. You need a stick to place from *X* to *W* and a stick to place from *W* to *Z* to finish constructing the frame. You want the kite to have the geometric shape of a kite. How long does each stick need to be? Explain your reasoning.(see example 4 and properties)



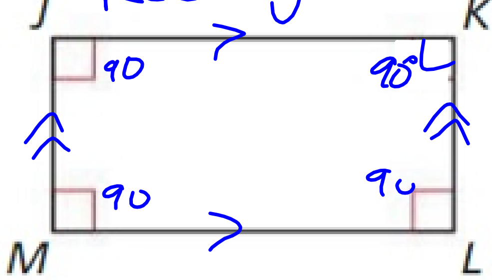
XW = 18 inches

WZ = 29 inches

Adjacent sides ≅

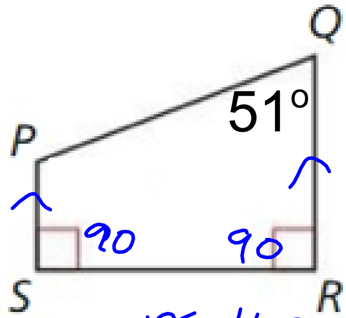
In Exercises 21–24, give the most specific name for the quadrilateral. Explain your reasoning. *(See Example 6.)*

21. Rectangle



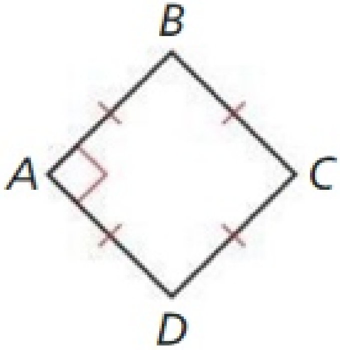
Since adjacent angles added to 180
JK // ML and JM // KL (Parallelogram)
Since // $m\angle K = 180 - 90 = 90°$
Since all angles are 90° (Rectangle)

22.

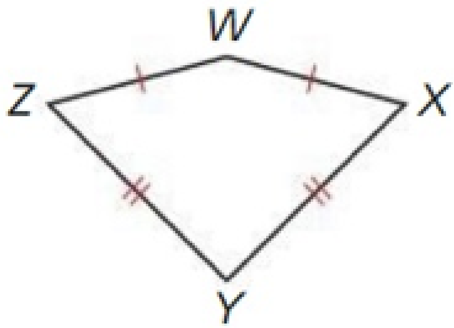


PS // QR b/c
$\angle S + \angle R = 180$
PQ ∦ SR
$51 + 90 \neq 180$
Since $\angle Q \neq \angle R$
not isosceles
Just a Trapezoid.

23.



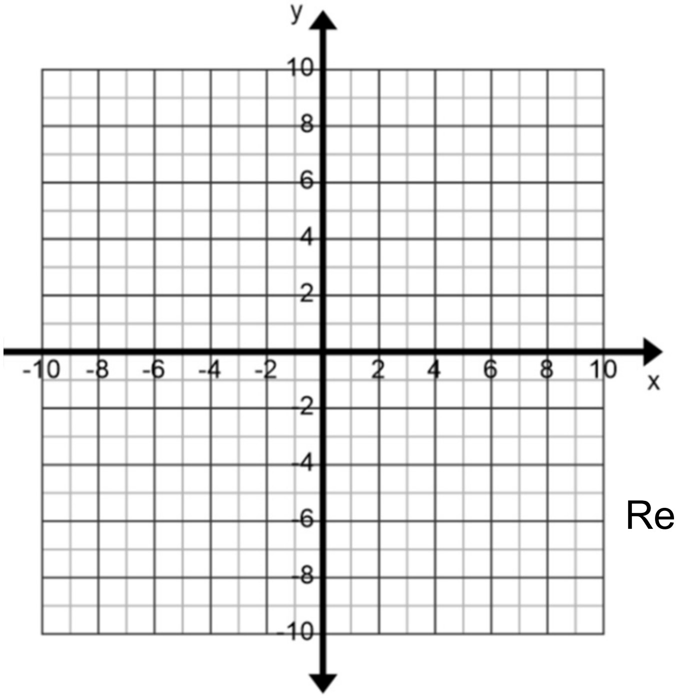
24.



In Exercises 3–6, show that the quadrilateral with the given vertices is a trapezoid. Then decide whether it is isosceles. *(See Example 1.) and 6)*

Work:

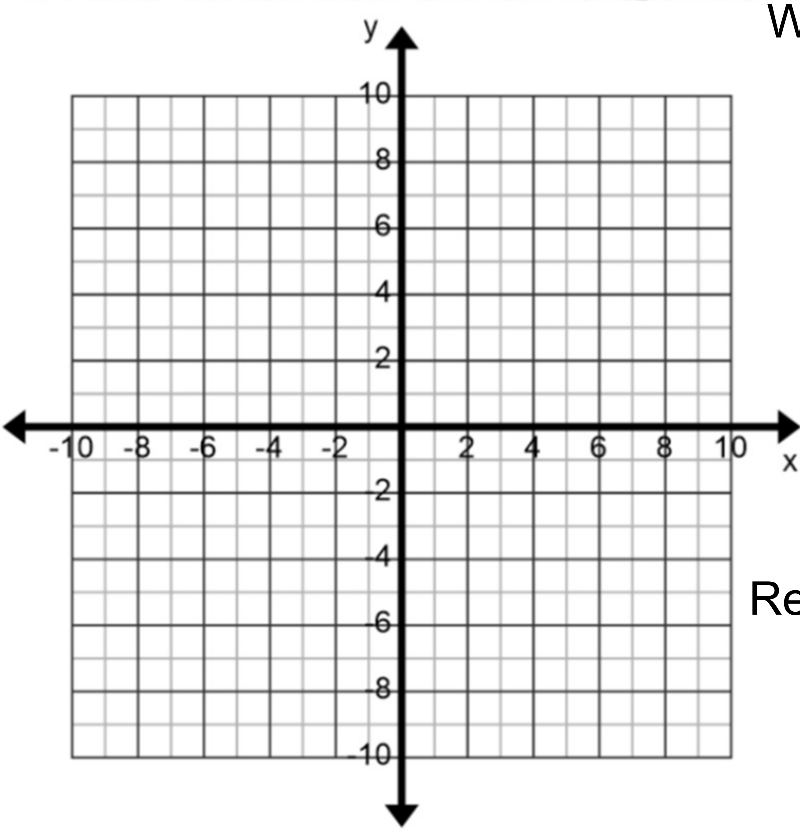
3. $W(1, 4)$, $X(1, 8)$, $Y(-3, 9)$, $Z(-3, 3)$



Reasoning:

5. $M(-2, 0)$, $N(0, 4)$, $P(5, 4)$, $Q(8, 0)$

Work:

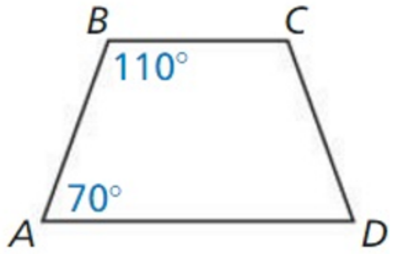


Reasoning:

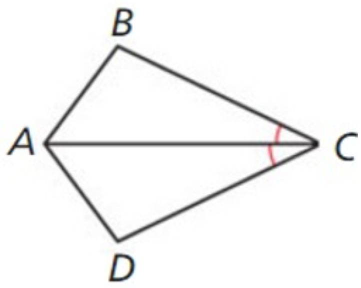
REASONING In Exercises 31–34, determine which pairs of segments or angles must be congruent so that you can prove that *ABCD* is the indicated quadrilateral. Explain your reasoning. (There may be more than one right answer.)

(see example 2 and properties)

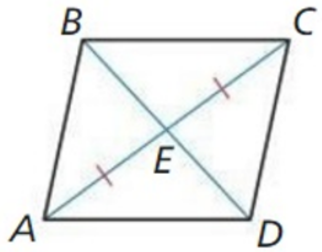
31. isosceles trapezoid



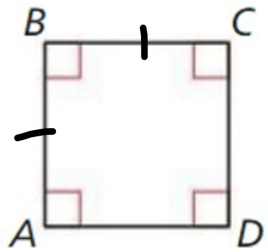
32. kite



33. parallelogram



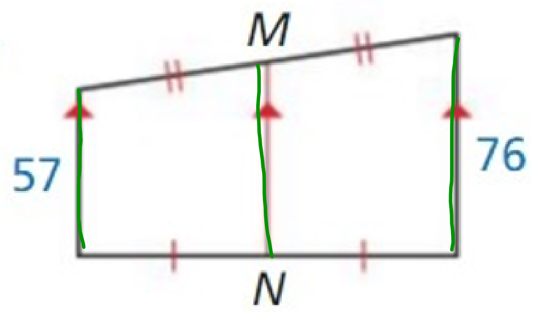
34. square



we need 2 adjacent sides to be ≅ b/c it is a parallelogram and opposite sides will be ≅ so all sides ≅ and ⊥ means square.

In Exercises 9 and 10, find the length of the midsegment of the trapezoid. *(See Example 3.)*

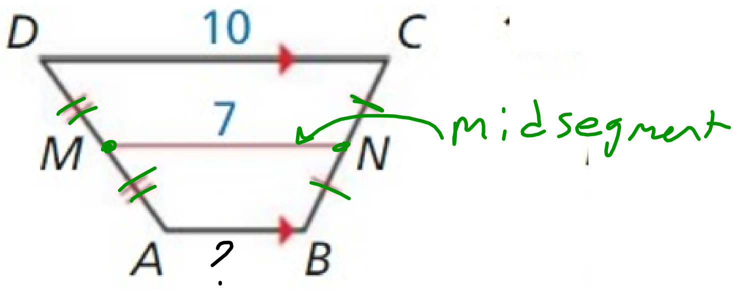
10.



$MN = \frac{1}{2}(57+76)$

In Exercises 11 find AB.

11.



Midsegment Theorem

$MN = \frac{1}{2}(DC + AB)$

$2(7) = \left(\frac{1}{2} \cdot (10 + AB)\right)2$

$14 = 10 + AB$

$-10 \quad -10$

$4 = AB$

Unit 05 - Section 06

1 Give the most descriptive name for

a A quadrilateral whose consecutive sides measure 15, 18, 15, and 18 — PARALLELOGRAM

b A quadrilateral whose consecutive sides measure 15, 18, 18, and 15 — KITE

c A quadrilateral with consecutive angles of 30°, 150°, 110°, and 70° — TRAPEZOID

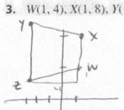
d A quadrilateral whose diagonals are perpendicular and congruent and bisect each other — ~~RHOMBUS~~ SQUARE

e A quadrilateral whose congruent diagonals bisect each other and bisect the angles — SQUARE

RHOMBUS RECTANGLE PARALLELOGRAM

In Exercises 3–6, show that the quadrilateral with the given vertices is a trapezoid. Then decide whether it is isosceles. *(See Example 1.)*

3. $W(1, 4)$, $X(1, 8)$, $Y(-3, 9)$, $Z(-3, 3)$



$\overline{WX} \parallel \overline{YZ}$ (VERTICAL = NO SLOPE)

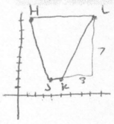
LENGTHS

$\overline{XY} = \sqrt{1^2+4^2} = \sqrt{17}$

$\overline{WZ} = \sqrt{1^2+4^2} = \sqrt{17}$

$\cong$ $\boxed{\text{ISOS.}}$

6. $H(1, 9)$, $J(4, 2)$, $K(5, 2)$, $L(8, 9)$



$\overline{HL} \parallel \overline{JK}$ (HORIZ = $\frac{0}{\text{SLOPE}}$)

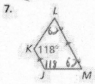
LENGTHS:

$\overline{HJ} = \overline{LK} = \sqrt{3^2+7^2} = \sqrt{58}$

$\boxed{\text{ISOS.}}$

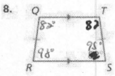
In Exercises 7 and 8, find the measure of each angle in the isosceles trapezoid. *(See Example 2.)*

7.



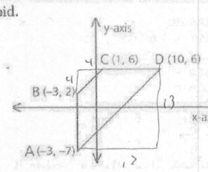
$180 - 118 = 62$

8.



$180 - 82 = 98°$

11 Show that ABCD is an isosceles trapezoid.



SLOPES:

$\overline{BC} = \frac{4}{4} = 1$

$\overline{AD} = \frac{13}{13} = 1$

$\overline{AD} \parallel \overline{BC}$

LENGTHS

$\overline{AB} = 2-(-7) = 9$

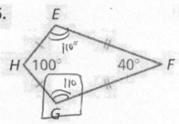
$\overline{CD} = 10 - 1 = 9$

$\overline{AB} \cong \overline{CD}$

ISOSCELES TRAPEZOID

In Exercises 15–18, find $m\angle G$. *(See Example 5.)*

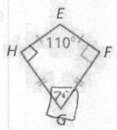
15.



$360 - 100 - 40 = 220$

$\frac{220}{2} = \boxed{110°}$

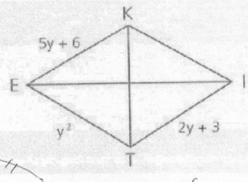
18.



$360 - 110 - 90 - 90 = \boxed{70°}$

30 Given: Kite KITE — 2 PAIRS SIDE $\cong$

Find: The three possible values of the perimeter of KITE



1st POSSIBILITY

$y^2 = 2y + 3$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$\boxed{y = 3, -1}$

y = 3: $5(3) + 6 = 21$; $3^2 \neq 9$; $2(3) + 3 = 9$ — $\boxed{P = 60}$

y = −1: $5(-1) + 6 = 1$; $(-1)^2 = 1$; $2(-1) + 3 = 1$ — $\boxed{P = 4}$

2ND POSSIBILITY

$y^2 = 5y + 6$

$y^2 - 5y - 6 = 0$

$(y-6)(y+1) = 0$

$y = 6, -1$

$y = 6$

$5(6) + 6 = 36$

$6^2 = 36$

$2(6) = 12$

$\boxed{P = 96}$

$y = -1$

$\boxed{\text{PERIMETER} = 60, 4, \text{ or } 96}$