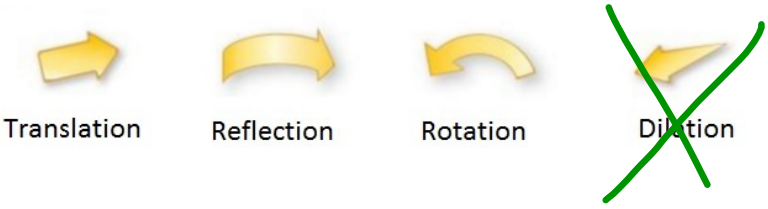
Rigid Motion

Essential Question Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

EXPLORATION 1 **Describing Rigid Motions**

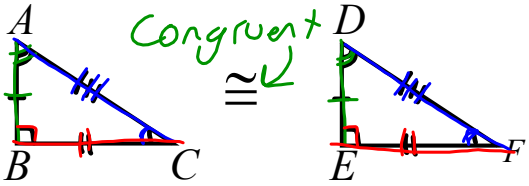
A **rigid motion** is a transformation that preserves length and angle measure. Another name for a rigid motion is an *isometry*. A rigid motion maps lines to lines, rays to rays, and segments to segments.

Work with a partner. Of the four transformations you studied , which are rigid motions?



Congruent Triangles

- Congruent triangles are drawn by applying one or more *transformations* to the original triangle.



Order of Letters Matter

- All corresponding sides are congruent

$\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$ line/bars over segments

- All corresponding angles are congruent

$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$ angle symbols

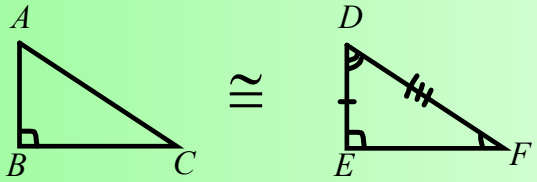
Congruency Transformations include:

- Translations
- Reflections
- Rotations

→ [ ]

Congruent Triangles

$\triangle ABC \cong \triangle DEF$



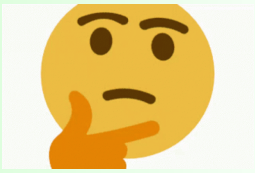
- All corresponding sides are congruent

$\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$

- All corresponding angles are congruent

$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$

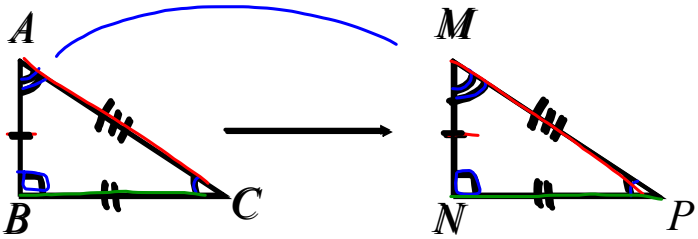
- What happens if you do not have all this information?
- What if we take away some of the tick marks on each triangle?
- What is the fewest number of tick marks we would need to prove the triangles congruent?



Congruency Transformations

Transform or change of appearance

1.) Translations = Move up/down /left/right only



congruence statement → $\triangle ABC \cong \triangle MNP$

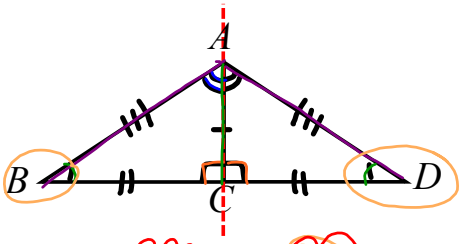
- All corresponding sides are congruent

$\overline{AB} \cong \overline{MN}, \overline{AC} \cong \overline{MP}, \overline{BC} \cong \overline{NP}$

- All corresponding angles are congruent

$\angle A \cong \angle M, \angle B \cong \angle N, \angle C \cong \angle P$

2.) Reflections = Mirror image over a line



$\triangle ABC \cong \triangle ADC$

- All corresponding sides are congruent

$\overline{AB} \cong \overline{AD}$ $\overline{AC} \cong \overline{AC}$ $\overline{BC} \cong \overline{DC}$

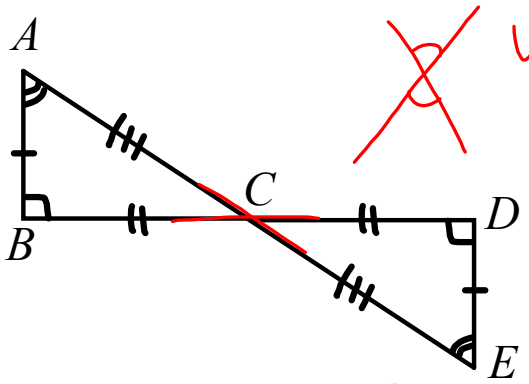
identity

- All corresponding angles are congruent

$\angle BAC \cong \angle DAC$ $\angle ABC \cong \angle ADC$ $\angle ACB \cong \angle ACD$

Use 3 letters when points are in more than one angle

3.) Rotations = Spin/turn around a point



Vertical Angles are $\cong$

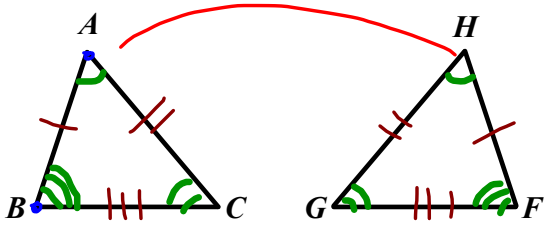
$\triangle ABC \cong \triangle EDC$

- All corresponding sides are congruent

$\overline{AB} \cong \overline{ED}$ $\overline{AC} \cong \overline{EC}$ $\overline{BC} \cong \overline{DC}$

- All corresponding angles are congruent

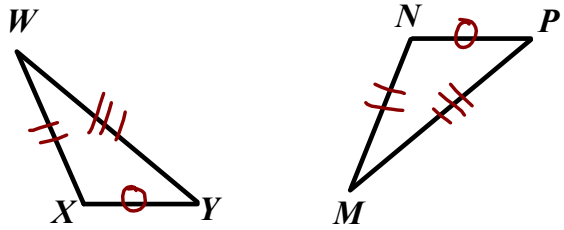
$\angle A \cong \angle E$, $\angle B \cong \angle D$, $\angle ACB \cong \angle ECD$



$\triangle ACB \cong \triangle HGF$

Congruent Parts are:

Sides	Angles
$\overline{AC} \cong \overline{HG}$	$\angle A \cong \angle H$
$\overline{CB} \cong \overline{GF}$	$\angle C \cong \angle G$
$\overline{AB} \cong \overline{HF}$	$\angle B \cong \angle F$



$\triangle \_\_\_\_ \cong \triangle \_\_\_\_$

Congruent Parts are:

$\triangle KAT \cong \triangle NEW$

angles	
	$\angle K \cong \angle N$
	$\angle A \cong \angle E$
	$\angle T \cong \angle W$

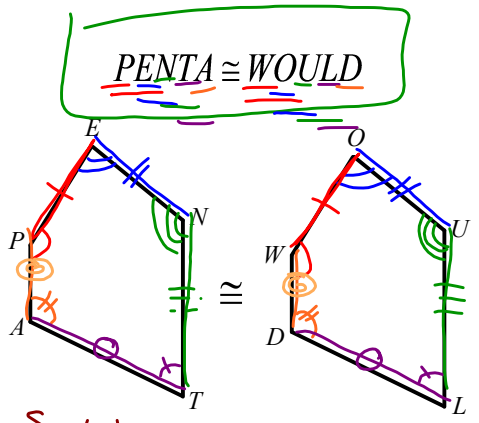
Sides	
	$\overline{KA} \cong \overline{NE}$
	$\overline{AT} \cong \overline{EW}$
	$\overline{KT} \cong \overline{NW}$

$\triangle MEL \cong \triangle BAH$

angles	
	$\angle M \cong \angle B$
	$\angle E \cong \angle A$
	$\angle L \cong \angle H$

Sides	
	$\overline{ME} \cong \overline{BA}$
	$\overline{EL} \cong \overline{AH}$
	$\overline{ML} \cong \overline{BH}$

Congruent Polygons = All pairs of corresponding *parts* are congruent.



Congruent Parts

Angles

$\angle P \cong \angle W$

$\angle E \cong \angle O$

$\angle N \cong \angle U$

$\angle T \cong \angle L$

$\angle A \cong \angle D$

Sides

$\overline{PE} \cong \overline{WO}$

$\overline{EN} \cong \overline{OU}$

$\overline{NT} \cong \overline{UL}$

$\overline{TA} \cong \overline{LD}$

$\overline{PA} \cong \overline{WD}$

4 Things You Can "Assume" From a Diagram

1. Straight Angles

$m\angle 1 + m\angle 2 = 180$

2. Supplementary Angles adds up to be 180°

3. Vertical Angles are equal

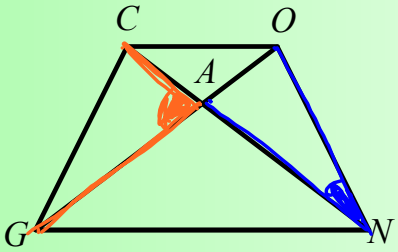
$\angle 3 \cong \angle 4$

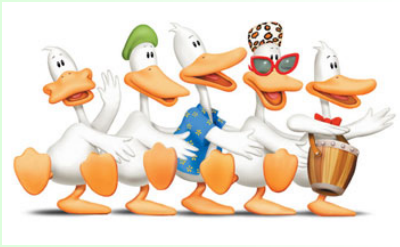
4. Reflexive Property

$\overline{AC} \cong \overline{AC}$

Included angle

= The angle formed between the two sides.



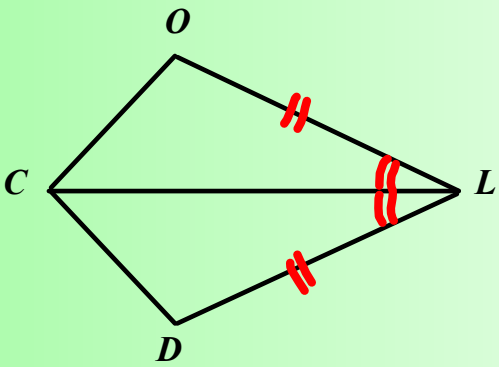


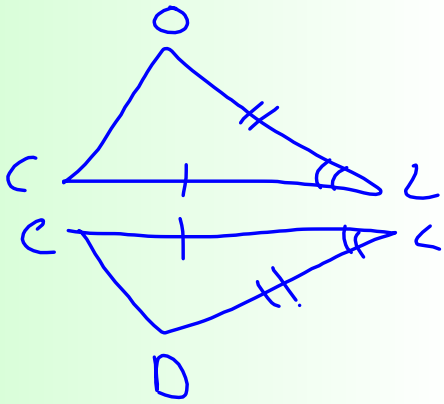
Given the two sides below, identify the *included angle.*

Sides	$\overline{CA}$ and $\overline{AG}$	$\overline{AN}$ and $\overline{ON}$	$\overline{CN}$ and $\overline{GN}$	$\overline{GA}$ and $\overline{AN}$	$\overline{OC}$ and $\overline{GO}$
Included Angle	∠CAG ∠GAC	∠ONA ∠ANO	∠CNG ∠GNC	∠GAN ∠NAG	∠COG ∠GOC

Reflexive Property

- A segment or angle is congruent to itself
  - > Makes a "copy" for you to use in multiple triangles
  - > Hint: Re-draw separately and label





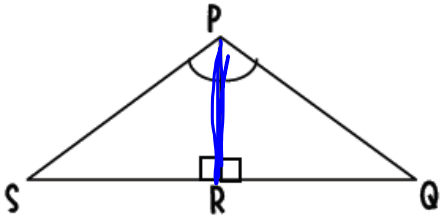
$\triangle LOC \cong \triangle LDC$

b/c $\overline{CL} \cong \overline{CL}$ Refl. Prop

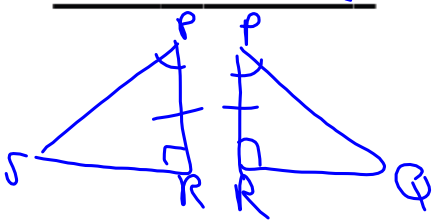
TRIANGLE HINTS PAGE 1

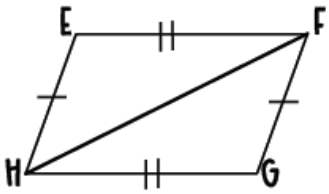
LABELING is so very, very, important!

REFLEXIVE SIDE:

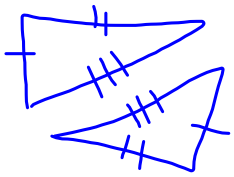


$\overline{PR} \cong \overline{PR}$

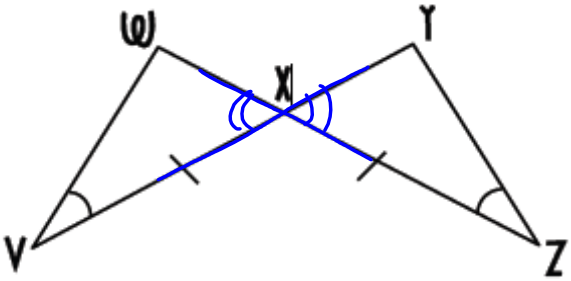




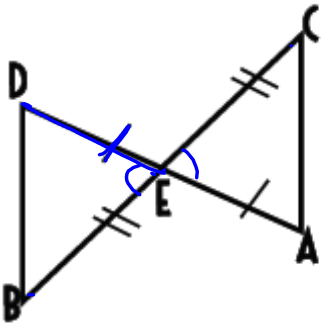
$\overline{HF} \cong \overline{HF}$



VERTICAL ANGLES:



$\angle WXV \cong \angle YXZ$



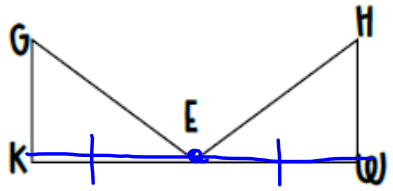
$\angle DEB \cong \angle AEC$

TRIANGLE HINTS PAGE 2

LABELING is so very, very, important!

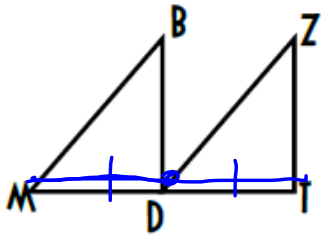
MIDPOINT:

E is the midpoint of KW



$\overline{KE} \cong \overline{EW}$

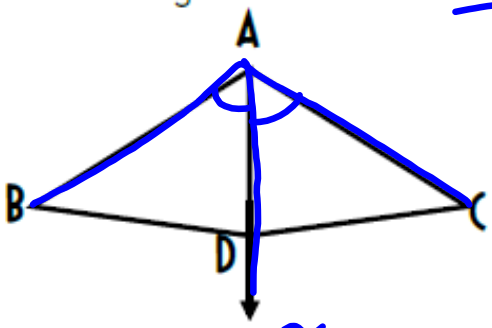
D is the midpoint of MT



$\overline{MD} \cong \overline{DT}$

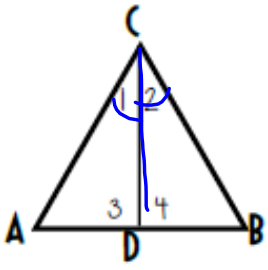
ANGLE BISECTOR:

AD is the angle bisector of <BAC



$\angle BAD \cong \angle CAD$

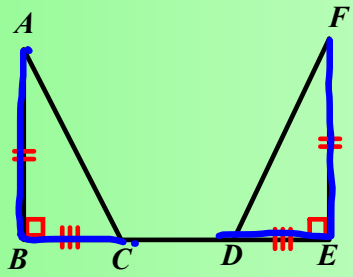
DC bisects <ACB



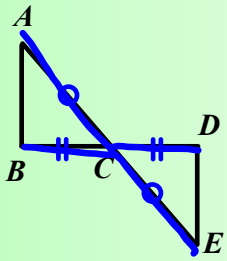
$\angle ACD \cong \angle BCD$

$\angle 1 \cong \angle 2$

List the congruent triangles in each diagram below. If there is not enough to prove congruent triangles, list what additional information would be needed.

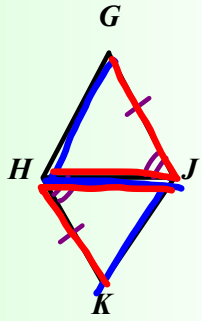


$\triangle ABC \cong \triangle FED$



$\triangle ACB \cong \triangle ECD$

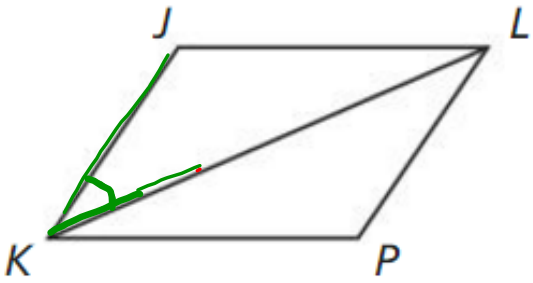
$ABC = EDC$



$\triangle GJH \cong \triangle KJH$

$GJH \quad KHJ$

In Exercises 3–8, name the included angle between the pair of sides given.

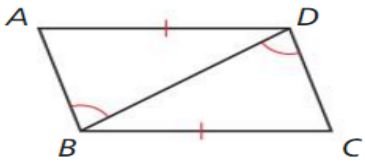


3. $\overline{JK}$ and $\overline{KL}$ $\angle JKL$
4. $\overline{PK}$ and $\overline{LK}$
5. $\overline{LP}$ and $\overline{LK}$
6. $\overline{JL}$ and $\overline{JK}$
7. $\overline{KL}$ and $\overline{JL}$
8. $\overline{KP}$ and $\overline{PL}$

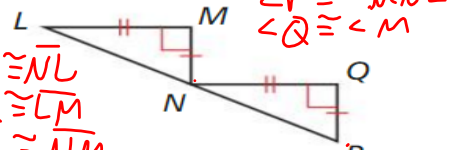
In Exercises 9–14,

Name the Congruent Triangles and Their Congruent Parts

9. △ △

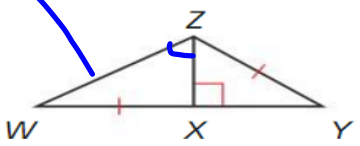


10. △PNQ △NLM

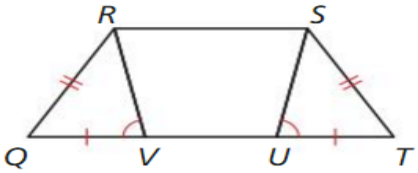


$\angle P \cong \angle MNL$
$\angle Q \cong \angle M$
$\overline{PN} \cong \overline{NL}$
$\overline{NQ} \cong \overline{LM}$
$\overline{PQ} \cong \overline{NM}$
$\angle PNQ \cong \angle L$

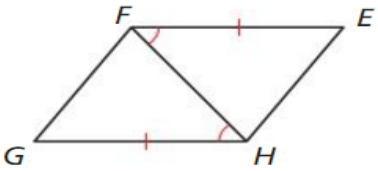
11. △ △



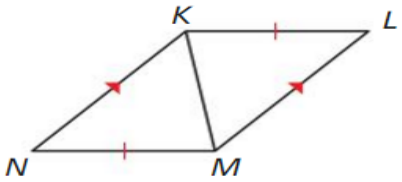
12. △ △



13. △ △

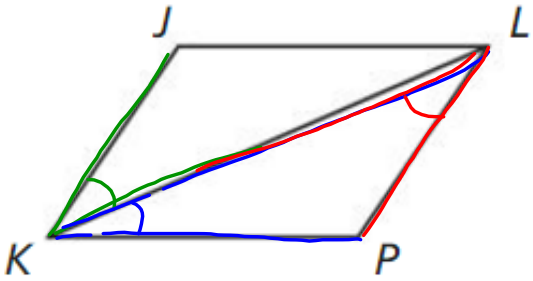


14. △ △



5.2 p.243 #5-10,13,14,17,23,24

In Exercises 3–8, name the included angle between the pair of sides given.



3. $\overline{JK}$ and $\overline{KL}$ ∠JKL

4. $\overline{PK}$ and $\overline{LK}$ ∠LKP

5. $\overline{LP}$ and $\overline{LK}$ ∠PLK

6. $\overline{JL}$ and $\overline{JK}$ ∠LJK

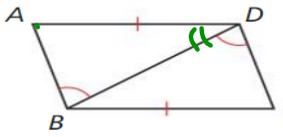
7. $\overline{KL}$ and $\overline{JL}$ ∠KLJ

8. $\overline{KP}$ and $\overline{PL}$ ∠KPL

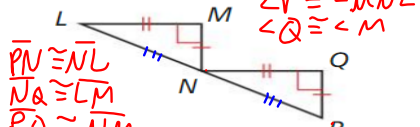
In Exercises 9–14,

Name the Congruent Triangles and Their Congruent Parts

9. △APB△CBD

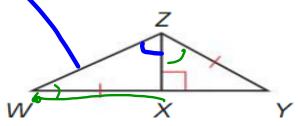


10. △PNQ △NLM

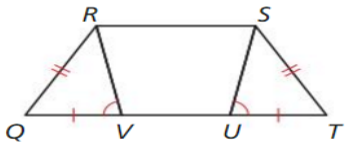


∠P≅∠MNL
∠Q≅∠M
$\overline{PN}\cong\overline{NL}$
$\overline{NQ}\cong\overline{LM}$
$\overline{PQ}\cong\overline{NM}$
∠PNQ≅∠L

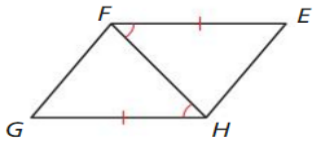
11. △ △



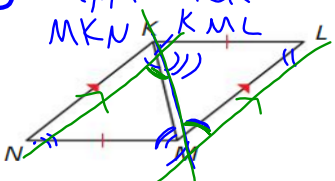
12. △VQR △UTS



13. △GHF△EFH



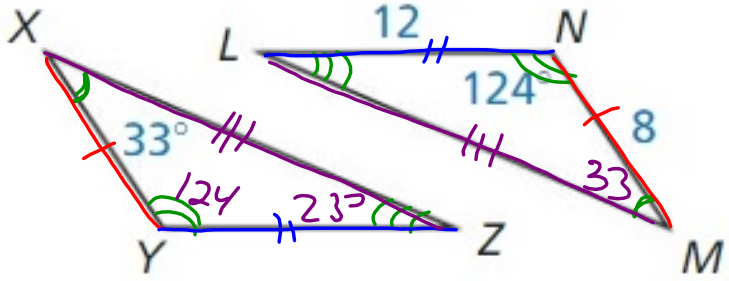
14. △KNM△MLK



$\overline{NM}\cong\overline{LK}$
$\overline{NK}\cong\overline{LM}$
$\overline{KM}\cong\overline{KM}$
∠N≅∠L
∠KML≅∠MKN
∠LKM≅∠NMK

In Exercises 5–8, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

5. $m\angle Y =$ 124°
   $\angle Y \cong \angle N$
6. $m\angle M =$ 33°
   $\angle M \cong \angle X$
7. $m\angle Z =$ 23°
   $\angle Z \cong \angle L$
8. $XY =$ 8
   $\overline{XY} \cong \overline{MN}$



angles in a triangle add up to be 180°

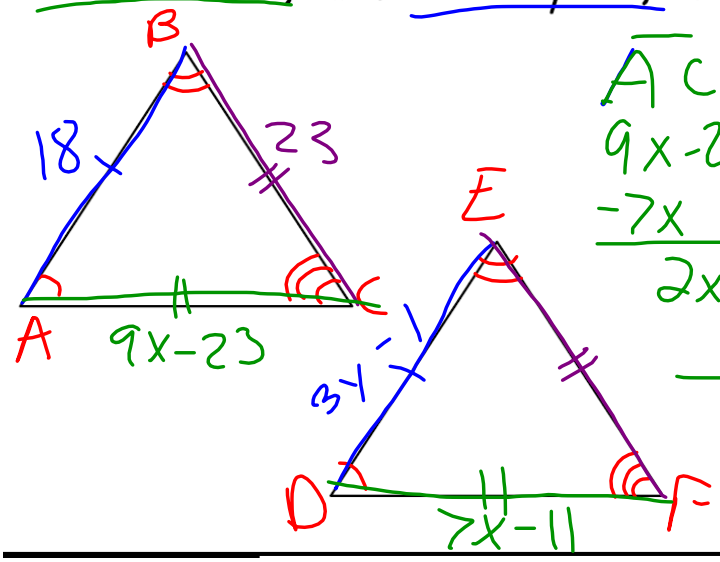
$33 + 124 + m\angle Z = 180$

$157 + m\angle Z = 180$

$m\angle Z = 23°$

example 1

If $\triangle ABC \cong \triangle DEF$, and $AB = 18$, $BC = 23$, $AC = 9x - 23$, $DF = 7x - 11$, and $DE = 3y - 1$, find the values of x and y.



$\overline{AC} \cong \overline{DF}$

$9x - 23 = 7x - 11$

$-7x \quad -7x$

$2x - 23 = -11$

$+23 \quad +23$

$2x = 12$

$\frac{2x}{2} = \frac{12}{2}$

$x = 6$

$\overline{DE} \cong \overline{AB}$

$3y - 1 = 18$

$+1 \quad +1$

$3y = 19$

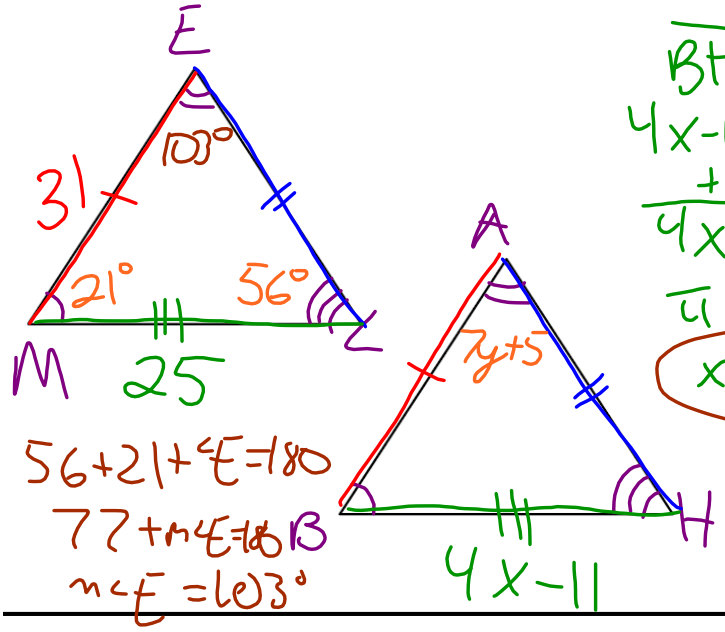
$\frac{3y}{3} = \frac{19}{3}$

$y = 6\frac{1}{3}$

example 2

If $\triangle MEL \cong \triangle BAH$, and $m\angle L = 56°$, $m\angle M = 21°$, and $m\angle A = (7y + 5)°$, $ML = 25$, ~~$IG = 14$~~, $ME = 31$, $BH = 4x - 11$

find the values of x and y.



56+21+mE=180

77+mE=180

mE=103°

$\overline{BH} \cong \overline{ML}$

$4x - 11 = 25$

$+11 \quad +11$

$4x = 36$

$\div 4 \quad \div 4$

$x = 9$

$\angle A \cong \angle E$

$7y + 5 = 103°$

$-5 \quad -5$

$7y = 98$

$\div 7 \quad \div 7$

$y = 14$

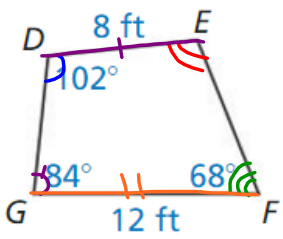
3.

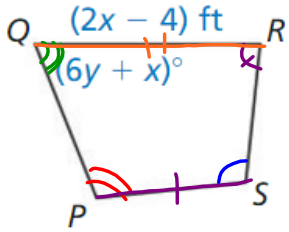
EXAMPLE **Using Properties of Congruent Figures**

In the diagram, $DEFG \cong SPQR$.

a. Find the value of x.

b. Find the value of y.





$68 = 6y + x$

$68 = 6y + (8)$

$60 = 6y$

$10 = y$

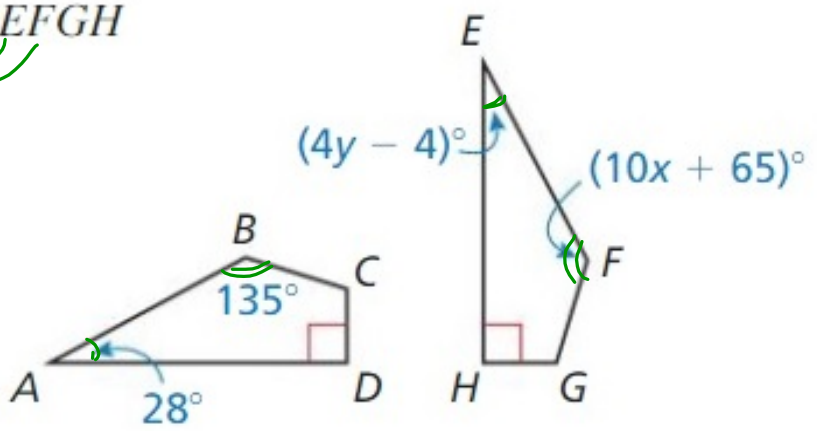
$12 = 2x - 4$

$16 = 2x$

$8 = x$

In Exercises 9 and 10, find the values of x and y.
(See Example 2.)

9. $ABCD \cong EFGH$



$\angle A \cong \angle E$

$28 = 4y - 4$

$32 = 4y$

$8 = y$

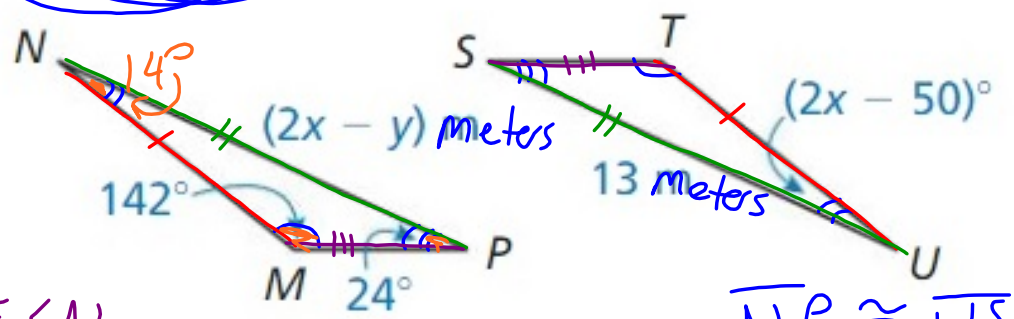
$\angle B \cong \angle F$

$135 = 10x + 65$

$70 = 10x$

$7 = x$

10. $\triangle MNP \cong \triangle TUS$



$\angle U \cong \angle N$

$2x - 50 = 14$

$+50 \quad +50$

$2x = 64$

$x = 32$

Angles in a triangle add up to be 180

$m\angle N + 142 + 24 = 180$

$m\angle N + 166 = 180$

$m\angle N = 14°$

$\overline{NP} \cong \overline{US}$

$2x - y = 13$

$2(32) - y = 13$

$64 - y = 13$

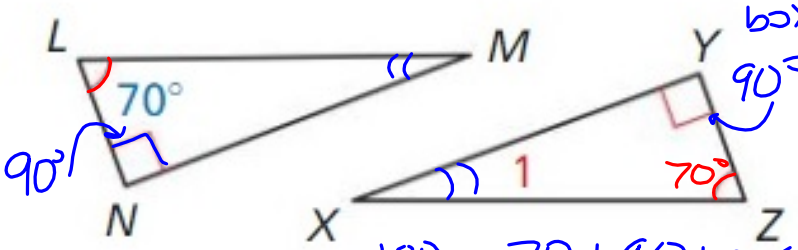
$-64 \quad -64$

$-y = -51$

$y = 51$

In Exercises 13 and 14, find $m\angle 1$. *(See Example 4.)*

13.



box means 90°

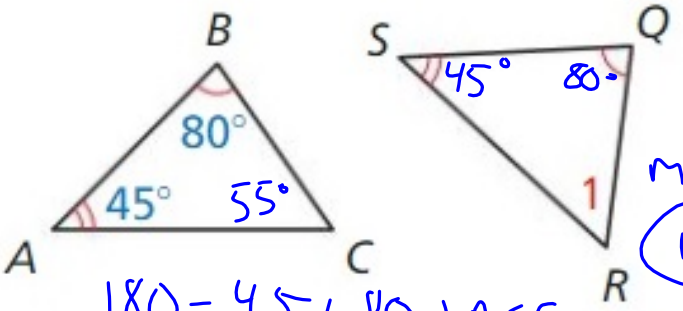
90°

$180 = 70 + 90 + m\angle 1$

$180 = 160 + m\angle 1$

$m\angle 1 = 20°$

14.



$\angle 1 \cong \angle C$

$m\angle 1 = m\angle C$

$m\angle 1 = 55°$

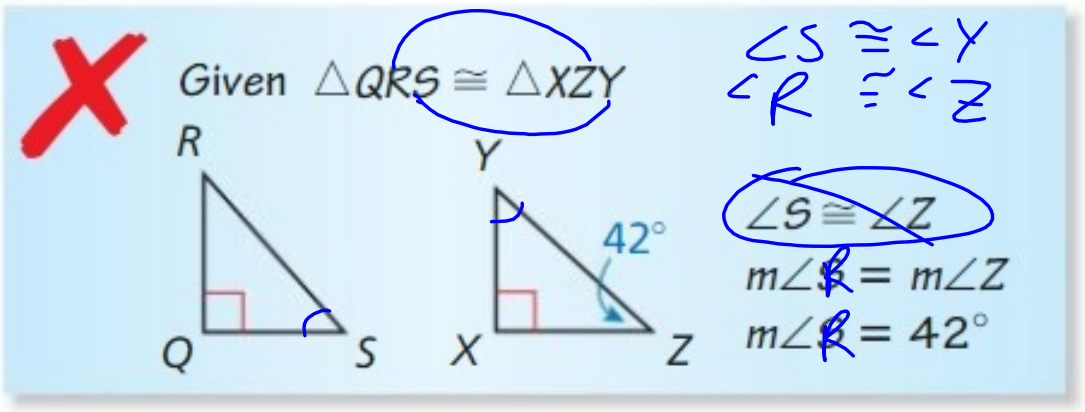
$180 = 45 + 80 + m\angle C$

$180 = 125 + m\angle C$

$m\angle C = 55°$

ERROR ANALYSIS **In Exercises 17 and 18, describe and correct the error.**

17.



$\angle S \cong \angle Y$

$\angle R \cong \angle Z$

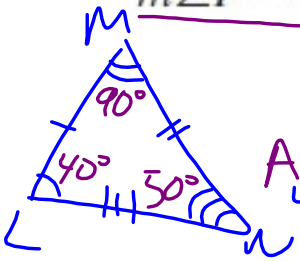
~~$\angle S \cong \angle Z$~~

~~$m\angle R = m\angle Z$~~

~~$m\angle R = 42°$~~

MATHEMATICAL CONNECTIONS In Exercises 23 and 24, use the given information to write and solve a system of linear equations to find the values of x and y.

23. $\triangle LMN \cong \triangle PQR$, $m\angle L = 40°$, $m\angle M = 90°$, $m\angle P = (17x - y)°$, $m\angle R = (2x + 4y)°$



$\angle L \cong \angle P$

A) $40 = 17x - y$

$4(40 = 17x - y)$

$160 = 68x - 4y$

$\angle R \cong \angle N$

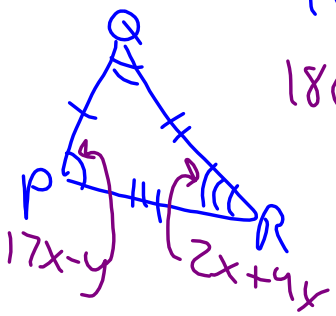
B) $2x + 4y = 50$

A) $68x - 4y = 160$

$70x = 210$

$x = 3$

A) $40 = 17(3) - y$

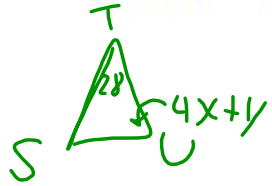


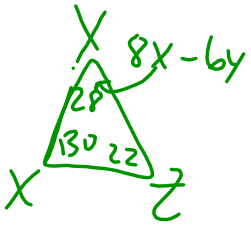
$180 = 90 + 40 + m\angle N$

$m\angle N = 50°$

MATHEMATICAL CONNECTIONS In Exercises 23 and 24, use the given information to write and solve a system of linear equations to find the values of x and y.

24. $\triangle STU \cong \triangle XYZ$, $m\angle T = 28°$, $m\angle U = (4x + y)°$, $m\angle X = 130°$, $m\angle Y = (8x - 6y)°$





$8x - 6y = 28 \rightarrow 8x - 6y = 28$

$6(4x + y = 22) \rightarrow 24x + 6y = 132$

$32x = 160$

$\frac{32}{32} \quad \frac{32}{32}$

$x = 5$

$4(5) + y = 22$