

Your Name

Mrs. T

4/18/19

Notes

Lesson 4-4

Rational Expressions with Like and Unlike Denominators

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Objective: to be able to add and subtract rational expressions (polynomial fractions).

Virtue/Skill: Why Not! We should always be pushing ourselves to the next step seeing what our prior knowledge applies to. You will use these skills when solving and proving trigonometric expressions.

CRITIQUE & EXPLAIN

Workbook
pg. 103

Teo and Shannon find the following exercise in their homework:

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$$

A. Teo claims that a common denominator of the sum is $2 + 3 + 9 = 14$. Shannon claims that it is $2 \cdot 3 \cdot 9 = 54$. Is either student correct? Explain why or why not.

LCM
18
but Shannon
is correct
54 is a
CD

B. Find the sum, explaining the method you use.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{9} = \frac{9}{18} + \frac{6}{18} + \frac{2}{18} = \frac{17}{18}$$

C. **Construct Arguments** Timothy states that the quickest way to find the sum of any two fractions with unlike denominators is to multiply their denominators to find a common denominator, and then rewrite each fraction with that denominator. Do you agree?

Adding... How do we do this?

$$\frac{28}{42} + \frac{5}{42} = \frac{28+5}{42}$$

Adding... How do we do this?

$$28x + 5 \quad \text{Done}$$

$$28x + 5x \quad 33x$$

Adding and Subtracting with Like Denominators

If the denominators are the same, keep them the same, and only combine like terms from the numerators.

*Remember if you are subtracting a fraction you are subtracting the entire numerator and you must distribute the negative sign.

EXAMPLE 1 Add Rational Expressions With Like Denominators

Try It!

1. Find the sum.

a. $\frac{10x - 5}{2x + 3} + \frac{8 - 4x}{2x + 3}$

$$\frac{10x - 5 + 8 - 4x}{2x + 3}$$

$$\frac{6x + 3}{2x + 3}$$

b. $\frac{x - 5}{x + 5} - \frac{3x - 21}{x + 5}$

$$\frac{x - 5 - (3x - 21)}{x + 5}$$

$$\frac{x - 5 - 3x + 21}{x + 5}$$

$$\frac{-2x + 16}{x + 5}$$

Adding and Subtracting with Like Denominators

If the denominators are the same, keep them the same, and only combine like terms from the numerators.

*Remember if you are subtracting a fraction you are subtracting the entire numerator and you must distribute the negative sign.

$$\frac{3}{n} + \frac{4}{n} = \frac{7}{n}$$



$$\frac{13}{x} - \frac{4+x}{x} = \frac{9-x}{x}$$



$$\frac{3x}{x^2} + \frac{4x}{x^2} = \frac{7x}{x^2} = \frac{7}{x}$$

$$\frac{9}{x+3} - \frac{4-x}{x+3} = \frac{5+x}{x+3}$$

Distribute Subtraction

$$\frac{2}{n} - \frac{4+x}{n} = \frac{2 - (4+x)}{n}$$

$$= \frac{2 - 4 - x}{n}$$

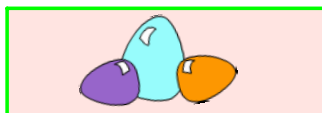
$$\frac{-2 - x}{n}$$

$$\frac{8t}{w-6} - \frac{3t}{6-w}$$

$\frac{8t}{w-6} + \frac{3t}{w-6} = \frac{11t}{w-6}$

$$\frac{x+3}{6} - \frac{x-2}{6}$$

$$\frac{x+3-x+2}{6} = \frac{5}{6}$$



$$\frac{-7x}{x-4} + \frac{4x+x^2}{x-4}$$

$$\frac{-3x+x^2}{x-4} \rightarrow \frac{x(x-3)}{x-4}$$

$$\frac{4x-4y}{4x+4y} - \frac{4x}{4x+4y}$$

$$\frac{4x-4y-4x}{4x+4y} = \frac{-4y}{4(x+y)} = \frac{-y}{x+y}$$

$$\frac{2ab + 8a^2b^2}{2a^2-b} - \frac{10 + 12ab}{2a^2-b}$$

$$\frac{2ab + 8a^2b^2 - 10 - 12ab}{2a^2-b}$$

$$\frac{8a^2b^2 - 10ab - 10}{2a^2-b}$$

Adding... How do we do this?

$$\frac{8}{3} + \frac{7}{5}$$

$1 \cdot 3 \quad 1 \cdot 5$
 $3 \cdot 5 = 15$

$$\frac{9}{4} + \frac{5}{6}$$

$2 \cdot 2 \quad 2 \cdot 3$
 $2 \cdot 2 \cdot 3 = 12$

LCM

1. Factor first
2. Write any common factors once
3. Write the leftover factors needed from each

LCM is the multiplication of both denominators, (common factors are only used once)

ex. $x + 7$ and $x - 7$ are not the same, the LCM would be

$(x+7)(x-7)$ or $x^2 - 49$

ex. $\text{LCM}(x - 8 \text{ and } -8) =$

$(x - 8)(-8) = -8x + 64$

EXAMPLE 2 Identify the Least Common Multiple of Polynomials

Try It!

2. Find the LCM for each set of expressions.

a. $x^3 + 9x^2 + 27x + 27, x^2 - 4x - 21$

$x^2(x+9) + 27(x+3) \quad (x-7)(x+3)$
 Not factorable No common factors

LCM = $(x^3 + 9x^2 + 27x + 27)(x^2 - 4x - 21)$

b. $10x^2 - 10y^2, 15x^2 - 30xy + 15y^2, x^2 + 3xy + 2y^2$

$10(x^2 - y^2) \quad 15(x^2 - 2xy + y^2) \quad (x+1)(x+2)$
 $10(x-y)(x+y) \quad 15(x-y)(x-y) \quad (x+y)(x+2y)$
 $2 \cdot 5 \quad 3 \cdot 5$

$2 \cdot 3 \cdot 5$
 $30(x-y)(x+y)(x-y)(x+2y)$
 $30(x^2 - y^2)(x^2 + xy - 2y^2)$

After you determine the LCM, ask yourself: **what do I need to multiply the denominator by to get that?**

$$\frac{x}{x} \cdot \frac{5}{9x} - \frac{1}{1x^2} \cdot \frac{9}{9}$$

$$\frac{5x - 9}{9x^2}$$

LCM: $9x^2$

$$\frac{3}{3} \cdot \frac{6a}{4a^2} - \frac{2}{3a} \cdot \frac{4a}{4a}$$

$$\frac{18a - 8a}{12a^2} - \frac{8a}{12a^2} \rightarrow \frac{10a}{12a^2} \rightarrow \frac{5}{6a}$$

LCM: $12a^2$

EXAMPLE 3 Add Rational Expressions With Unlike Denominators

Try It!

3. Find the sum.

$$\frac{3}{3} \cdot \frac{3}{4} + \frac{5}{2} \cdot \frac{2}{2}$$

$$\frac{9}{12} + \frac{10}{12}$$

$2 \cdot 2 \cdot 3$

Factor

multiply by what you need

a. $\frac{x+6}{x^2-4} + \frac{2}{x^2-5x+6}$

b. $\frac{2x}{3x+4} + \frac{4x^2-11x-12}{6x^2+5x-4}$

$$\frac{(x-3)(x+6)}{(x-3)(x-2)(x+2)} + \frac{2(x+2)}{(x-2)(x-3)(x+2)}$$

$$\frac{(2x-1)(2x)}{(2x-1)(3x+4)} + \frac{4x^2-11x-12}{(3x+4)(2x-1)}$$

Distribute + combine like terms

$$\frac{x^2+3x-18}{(x-2)(x+2)(x-3)} + \frac{2x+4}{(x-2)(x-3)(x+2)}$$

$$\frac{4x^2-2x}{(3x+4)(2x-1)} + \frac{4x^2-11x-12}{(3x+4)(2x-1)}$$

check if factorable + can cancel

$$\frac{(x+7)\cancel{(x-2)}}{(x-2)(x+2)(x-3)}$$

$$\frac{8x^2-13x-12}{6x^2+5x-4}$$

$$\frac{x+7}{x^2-x-6}$$

EXAMPLE 4 Subtract Rational Expressions

LCM = GCF • leftovers
 $x \cdot 3 \cdot 2 \cdot x$
 $6x^2$

Try It!

4. Simplify.

a. $\frac{1}{3x} + \frac{1}{6x} - \frac{1}{x^2}$
 $3 \cdot x \quad 2 \cdot 3x \quad x \cdot x$

$$\frac{2x}{2x} \cdot \frac{1}{3x} + \frac{x}{x} \cdot \frac{1}{6x} - \frac{6}{6} \cdot \frac{1}{x^2}$$

$$\frac{2x}{6x^2} + \frac{x}{6x^2} - \frac{6}{6x^2}$$

$$\frac{3x-6}{6x^2}$$

$$\frac{3(x-2)}{2 \cdot 3(x^2)}$$

$$\frac{x-2}{2x^2}$$

b. $\frac{3x-5}{x^2-25} - \frac{2}{x+5}$

$$\frac{3x-5}{(x-5)(x+5)} - \frac{2(x-5)}{(x+5)(x-5)}$$

$$\frac{3x-5}{(x-5)(x+5)} - \frac{2x-10}{(x-5)(x+5)}$$

$$\frac{\cancel{(x+5)}}{\cancel{(x-5)}\cancel{(x+5)}} = \frac{1}{x-5}$$

Adding and Subtracting With Unlike Denominators

1. Find the LCM of the Denominators

Remember Factors are expressions that are multiplied

2. Multiply each fraction by the factors needed to have the common denominator. Simplify the numerators

3. Combine the numerators

$$\frac{4}{4} \cdot \frac{1}{a} - \frac{9}{4a}$$

$$\frac{4}{4a} - \frac{9}{4a}$$

$$\frac{-5}{4a}$$

LCM = 4a

If denominator has a + or - LCM is the multiplication of both denominators, (common factors are only used once)

ex. $x + 7$ and $x - 7$ are not the same, the LCM would be $(x+7)(x-7)$ or $x^2 - 49$

ex. $\text{LCM}(x - 8 \text{ and } -8) = (x - 8)(-8) = -8x + 64$

$$\frac{-9}{-8(h-8)} - \frac{1}{-8(h-8)}$$

$$\text{LCM: } (-8)(h-8)$$

$$\frac{-8h - h + 8}{-8(h-8)}$$

$$\boxed{\frac{-9h + 8}{-8h + 64}}$$

$$\frac{(y+3)y}{(y+3)(y-3)} - \frac{3}{y+3} \cdot \frac{(y-3)}{(y-3)}$$

$$\text{LCM: } (y-3)(y+3)$$

$$y^2 - 9$$

$$\frac{y^2 + 3y - 3y + 9}{(y+3)(y-3)}$$

$$\boxed{\frac{y^2 + 9}{y^2 - 9}}$$

$$\frac{y+1}{y+1} \cdot \frac{2}{y^2+3y+2} + \frac{4}{y^2+2y+1} \cdot \frac{y+2}{y+2}$$

$$\frac{(y+1)(y+2)}{(y+1)(y+2)} + \frac{4(y+2)}{(y+1)(y+1)}$$

$$\text{LCM: } (y+1)(y+2)(y+1)$$

$$\begin{array}{r}
 4a^2b^2 \\
 -4ab \quad -1ab \\
 \hline
 -5ab \\
 \hline
 \end{array}$$

$$\frac{a-6b}{(2a-b)(a-2b)} - \frac{7}{(2a-b)(a-2b)}$$

$$\frac{a-6b-7}{(2a-b)(a-2b)}$$

$$\frac{a-6b-7}{2a^2-5ab+2b^2}$$

$$\frac{-13a+b}{2a^2-5ab+2b^2}$$

Leah drives her car to the mechanic, then she takes the commuter rail train back to her neighborhood. The average speed for the 10-mile trip is 15 miles per hour faster on the train. Find an expression for Leah's total travel time. If she drove 30 mph, how long did this take?



$$\begin{aligned}
 t \cdot s &= \frac{d}{t} \\
 \frac{t \cdot s}{s} &= \frac{d}{s} \\
 t &= \frac{d}{s}
 \end{aligned}$$

	Distance	Rate	Time
Car	10	r	$\frac{10}{r}$
Commuter Rail	10	$r + 15$	$\frac{10}{r + 15}$

Remember: distance = rate • time,
so time = $\frac{\text{distance}}{\text{rate}}$.

$$\frac{10}{30} + \frac{10}{30+15}$$

EXAMPLE 5 Find a Rate

Try It!

5. On the way to work Juan carools with a fellow co-worker, then takes the city bus back home in the evening. The average speed of the 20-mile trip is 5 miles per hour faster in the carpool. Write an expression that represents Juan's total travel time.

$$\text{LCM: } (r+5)(r)$$

$$t = \frac{r \cdot 20}{r \cdot (r+5)} + \frac{20 \cdot (r+5)}{r \cdot (r+5)}$$

$$\frac{20r}{r^2+5r} + \frac{20r+100}{r^2+5r}$$

$$\frac{40r+100}{r^2+5r}$$

Method 1

Find the LCM of both the numerator and denominator denominators. Multiply the top and bottom by it to cancel ALL denominators at once.

Method 2

Think of the numerator and denominator as individual fractions, simplify/combine if needed first. Then multiply the numerator by the reciprocal of the denominator.

EXAMPLE 6 Simplify a Compound Fraction

A **compound fraction** is in the form of a fraction and has one or more fractions in the numerator and/or the denominator. How can you write a simpler form of a compound fraction?

6. Simplify each compound fraction.

Method 1
 a. $\frac{\frac{1}{x-1}}{\frac{x+1}{x} + \frac{4}{x-1}} \cdot \frac{3(x-1)}{3(x-1)}$

LCM: $3(x-1)$
 $\frac{3}{x^2-1+12}$
 $\frac{3}{x^2+11}$

all denominators cancel!

Method 2
 b. $\frac{\frac{x}{x} \cdot 2 - \frac{1}{x}}{\frac{x}{x} \cdot x + \frac{2}{x}}$

get C.D.
 $\frac{\frac{2x-1}{x}}{\frac{x^2+2}{x}}$
 combine

multiply by reciprocal
 $\frac{2x-1}{x} \cdot \frac{x}{x^2+2}$
 $\frac{2x-1}{x^2+2}$

How are these different?

Would you get the same answer?

$$\frac{2x+16}{x}$$

$$\frac{x}{x+7}$$

Method 1:

$$\frac{2x+16}{x} \cdot \frac{x+7}{x+7}$$

$$\frac{(2x+16)(x+7)}{x}$$

$$\frac{2x^2+30x+112}{x}$$

dividing $2x+16$ by the smaller quotient of x and $x+7$

$$\frac{2x+16}{x}$$

$$\frac{x+7}{1}$$

Method 2:

$$\frac{2x+16}{x} \cdot \frac{1}{x+7}$$

$$\frac{2x+16}{x^2+7x}$$

Dividing $2x+16$ by x and by $x+7$

not the same b/c