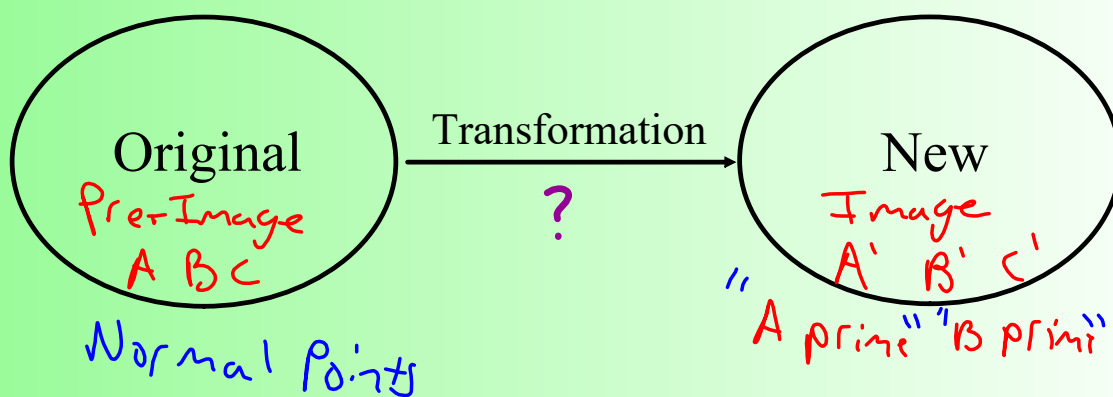## 4.1 - Translations

Lesson Objectives

- Perform <u>translations</u> on points and figures ✔

- Perform <u>composition</u> transformations ✔

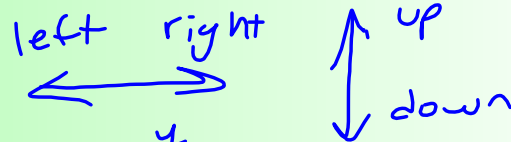- Solve real-life problems involving compositions

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<u>Transformation</u>
- Transform = to change or alter

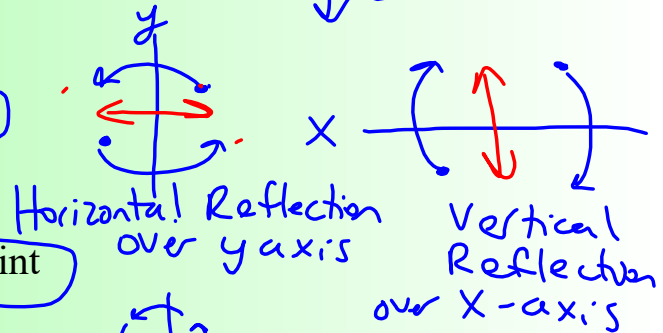- Transformation = A <u>function</u> that moves or changes a figure in some way to <u>produce a new figure.</u>
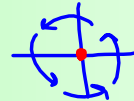
Original
PrerImage
A B C

Normal Points

Transformation
?

New
Image
A' B' C'
"A prime" "B prime"

4 Types of Transformations

1.) (Translation) (Translate)
  – Move or slide

left   right

up

down

2.) Reflection (Reflect)
  – Mirror image over a line

Horizontal Reflection
over y axis

Vertical Reflection
over X-axis

3.) Rotation (Rotate)
  – Turn or spin around a point
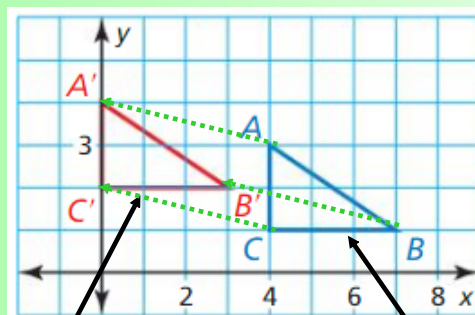
4.) Dilation (Dilate)
  – Increase or decrease scale/size

Stretch   Shrink

---

1.   Translation (Translate)
• Moves every point the <u>same distance</u> in the <u>same direction</u>.

New/Image   Original/Preimage

△A'B'C'   △ABC

Mental Floss: Tue Nov 2nd

Given the translation rule $(x, y) \rightarrow (x+4, y-2)$:
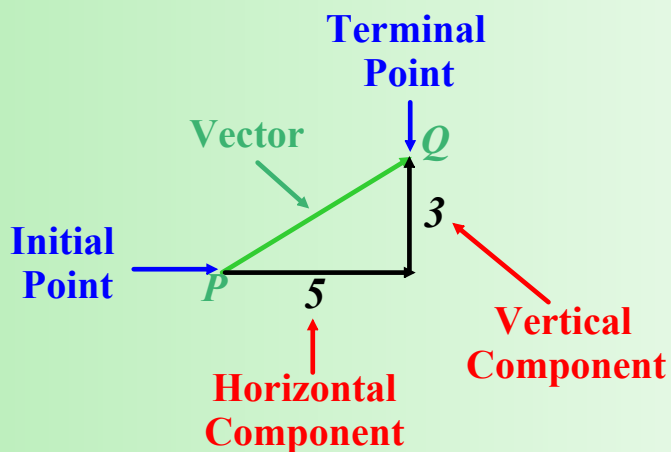
a.) Find the image of $P(4,-5)$.

$$\underline{Rule}$$

$$\boxed{\frac{Pre\ Image}{P(4,-5) \rightarrow}} (4+4, -5-2) \rightarrow \boxed{\frac{Image}{P'(8,-7)}}$$

b.) Find the preimage of $L(-1,7)$.

$$L(?, ?) \xleftarrow{\quad x+4=1 \quad y-2=7 \quad} L'(-1,7)$$
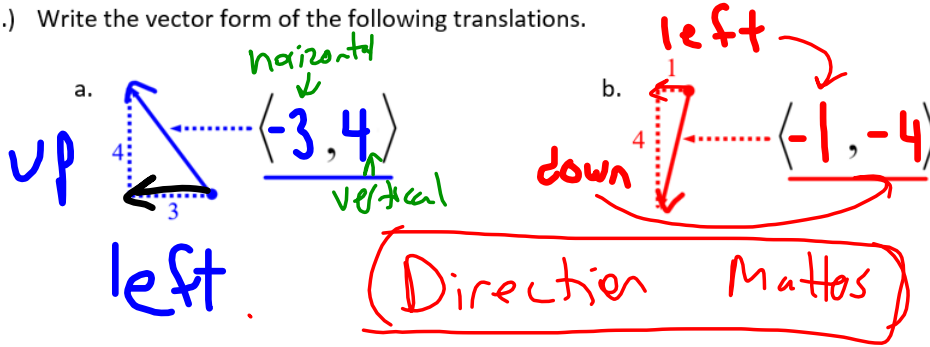

Vector Algebra

Vectors are usually shown in the coordinate plane by an arrow (looks similar to a ray).
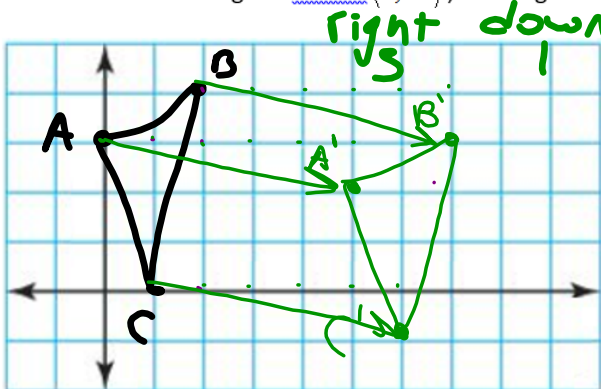

Component Form of a Vector

$$\langle 5,3 \rangle$$

1.) Write the vector form of the following translations.

a. *horizontal* *left*

up *vertical* $\langle -3, 4 \rangle$

left

b. *left* *down* $\langle -1, -4 \rangle$

(Direction Matters)

2.) The vertices of $\triangle ABC$ are $A(0,3)$, $B(2,4)$, and $C(1,0)$.

Translate $\triangle ABC$ using the vector $\langle 5, -1 \rangle$, drawing both the original and new triangles.
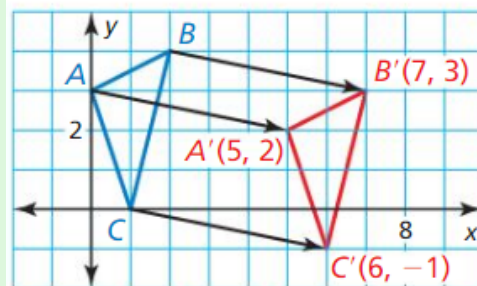
right 5 down 1

Translating a Figure Using a Vector

The vertices of $\triangle ABC$ are $A(0, 3)$, $B(2, 4)$, and $C(1, 0)$. Translate $\triangle ABC$ using the vector $\langle 5, -1 \rangle$.

SOLUTION

First, graph $\triangle ABC$. Use $\langle 5, -1 \rangle$ to move each vertex 5 units right and 1 unit down. Label the image vertices. Draw $\triangle A'B'C'$.

You can also write a ***translation rule*** to describe the changes you are making to the figure.

$$(x, y) \rightarrow (x+5, y-1)$$

1.) Write a translation rule for the following:
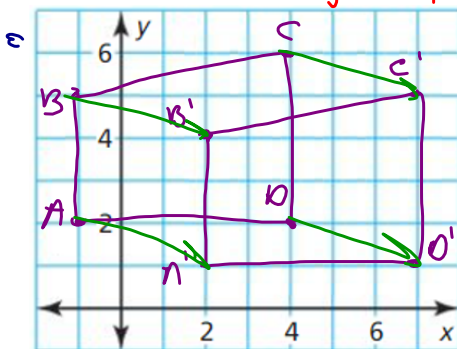
a. Left 3 and down 4

$(x, y) \rightarrow (x-3, y-4)$

b. Right 4

$(x, y) \rightarrow (x+4, y)$

2.) The points $P(1, -3)$ and $R(0, 4)$ are translated using the rule $(x, y) \rightarrow (x-4, y+1)$. What are the coordinates for P' and R'?

Pre image

$P(1, -3) \rightarrow (1-4, -3+1) \rightarrow$

$R(0, 4)$

Image

$P'(-3, -2)$

$R'(-4, 5)$

3.) Graph quadrilateral ABCD with vertices $A(-1, 2)$, $B(-1, 5)$, $C(4, 6)$, and $D(4, 2)$ and its image after the translation $(x, y) \rightarrow (x+3, y-1)$.

right 3 down 1

Rule

Pre Image $(x+3, y-1)$ Image

$A(-1, 2) \rightarrow (-1+3, 2-1) \rightarrow A'(2, 1)$

$B(-1, 5) \longrightarrow B'(2, 4)$

$C(4, 6) \longrightarrow C'(7, 5)$

$D(4, 2) \longrightarrow D'(7, 1)$

1.) Describe the translation below (up/down and left/right). Note: You do not have to use any special notation if you have already done either station 1 or 2. Just describe the translation, such as "left 8 and down 6".

New/Translated Image

Letters Prime

vertical   horizontal

up 2   right 1

$(x, y) \rightarrow (x+1, y+2)$

$\langle 1, 2 \rangle$

Old/original
Pre -Image

Not Prime
Regular Letters

2.) You are asked to translate the points A and B up 5 and left 7.

y change   x change

$\langle -7, 5 \rangle$

$(x-7, y+5)$

a. If point A is located at $(-3, 4)$, what is the location of A'?

$A(-3, 4) \rightarrow A'(-10, 9)$

b. If point B' is located at $(-8, 0)$, what is the location of B?
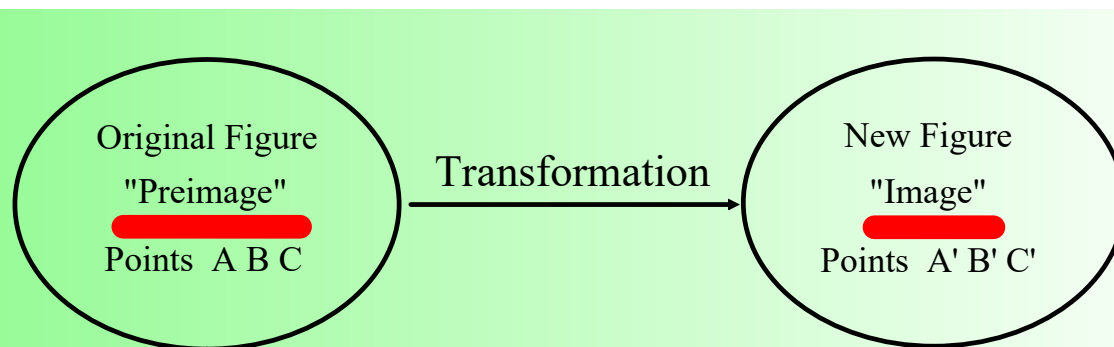
$B(-1, -5) \longleftarrow B'(-8, 0)$

3.) You are given two points, $M(-2, 3)$ and $M'(-1, 5)$, that have undergone a translation. Using this same translation, find:
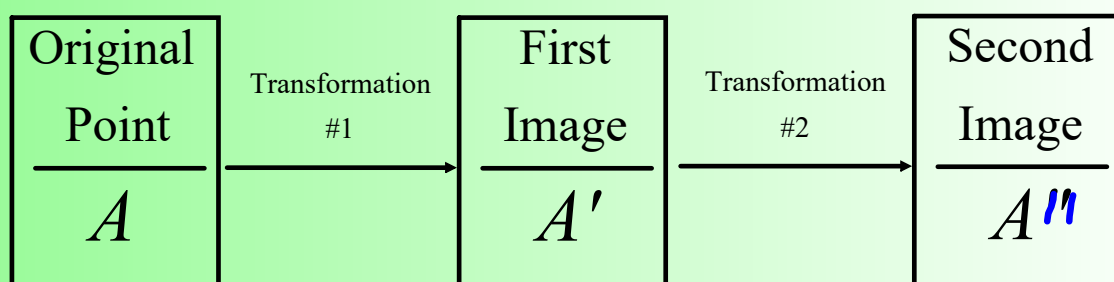
a. N' if $N(0, 3)$

b. P if $P'(4, 10)$

Original Figure
"Preimage"
━━━━━━━
Points A B C

Transformation →

New Figure
"Image"
━━━━━━━
Points A' B' C'

Composition of Transformations

Two or more transformations combined into one transformation are called a *composition*.

Original Point		First Image		Second Image
A	Transformation #1 →	A'	Transformation #2 →	A''

A double prime

EXAMPLE   **Performing a Composition**

Graph $\overline{RS}$ with endpoints $R(-8, 5)$ and $S(-6, 8)$ and its image after the composition.

   Translation: $(x, y) \rightarrow (x + 5, y - 2)$
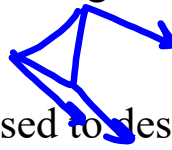   **Translation:** $(x, y) \rightarrow (x - 4, y - 2)$

$$\underline{\text{Pre Image}} \qquad \underset{\text{Transl. Image}}{\overset{\text{First}}{\underline{\text{Transl. Image}}}} \qquad \underset{\text{Trans. Image}}{\overset{\text{Second}}{\underline{\text{Trans. Image}}}}$$

$$R(-8,5) \xrightarrow{x+5,y-2} R'(-3, 3) \xrightarrow{x-4, y-2} R''(-7, 1)$$

$$S(-6,8) \rightarrow S'(-1, 6) \longrightarrow S''(-5, 4)$$

Mental Floss: Thursday Nov 4$^{\text{th}}$

Without graphing, determine the coordinates for both the first and second translations of the triangle below.

$\boxed{\Delta XYZ \text{ with vertices } X(2,4), Y(6,0), \text{ and } Z(7,2)}$

Translation: $(x, y) \rightarrow (x + 12, y + 4)$ right 12 up 4
Translation: $\langle -5, -9 \rangle$

left down
  5   9

$X(2,4) \rightarrow X'(14, 8) \longrightarrow X''(9, -1)$

$Y(6,0) \rightarrow Y'(18, 4) \longrightarrow Y''(13, -5)$

$Z(7,2) \rightarrow Z'(19, 6) \longrightarrow Z''(14, -3)$

0:00:00

Summary

- Translations move every point in a figure the same distance in the same direction.

- Vectors in the form $\langle a,b \rangle$ are used to describe the horizontal *(a)* and vertical *(b)* changes of a translation.

- A translation rule in the form $(x,y) \longrightarrow (x+a, y+b)$ can also be used to describe a translation.

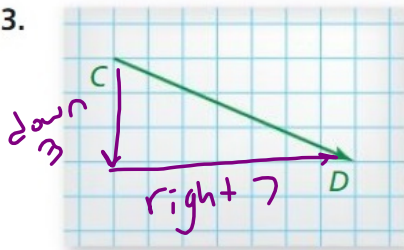- Composition transformations combine two or more transformations into one.
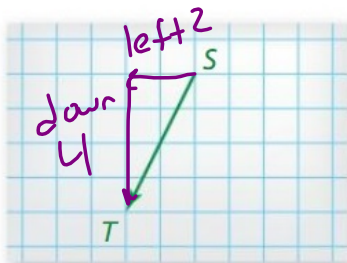
4.1 p.178
#3,4,7-12,14,15
#19,20,23,32,34

In Exercises 3 and 4, name the vector and write its component form. *(See Example 1.)*

3.

down 3

right 7

$\overrightarrow{CD}$ $\langle 7,-3 \rangle$

4.

left 2

down 4

$\overrightarrow{ST}$ $\langle -2,-4)$

In Exercises 5–8, the vertices of $\triangle DEF$ are $D(2, 5)$,
$E(6, 3)$, and $F(4, 0)$. Translate $\triangle DEF$ using the given
vector. Graph $\triangle DEF$ and its image. *(See Example 2.)*

7. $\langle -3, -7 \rangle$

$(x-3, y-7)$

$D(2,5) \rightarrow D'(-1,-2)$

$E(6,3) \rightarrow E'(3,-4)$

$F(4,0) \rightarrow F'(1,-7)$

8. $\langle -2, -4 \rangle$

$(x-2, y-4)$

$D(2,5) \rightarrow D'(0,1)$

$E(6,3) \rightarrow E'(4,-1)$

$F(4,0) \rightarrow F'(2,-4)$

In Exercises 9 and 10, find the component form of the
vector that translates $P(-3, 6)$ to P'.

9. $P'(0, 1)$

$P(-3,6) \rightarrow P'(0,1)$

$x+3 \qquad y-5$
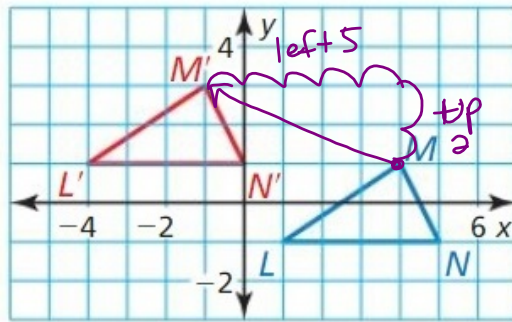
$\boxed{\langle 3, -5 \rangle}$

10. $P'(-4, 8)$

$P(-3, 6) \rightarrow P(-4,8)$

$x-1 \qquad y+2$
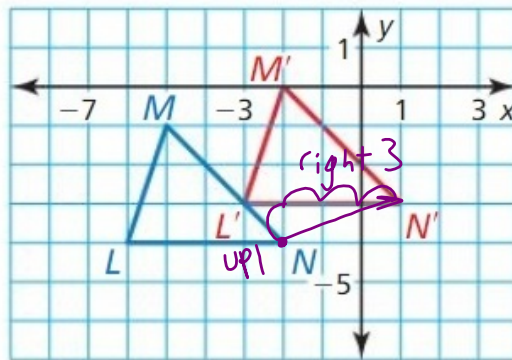
$\boxed{\langle -1, 2 \rangle}$

In Exercises 11 and 12, write a rule for the translation of △LMN to △L'M'N'. *(See Example 3.)*

11.

left 5
$(x,y) \rightarrow (x-5, y+2)$

up 2

12.

right 3
up 1
$(x,y) \rightarrow (x+3, y+1)$

In Exercises 13–16, use the translation.

$(x, y) \rightarrow (x - 8, y + 4)$

14. What is the image of $B(-1, 5)$? $(-1-8, 5+4) \rightarrow \boxed{B'(-9, 9)}$

15. What is the preimage of $C'(-3, -10)$?

$(x+8, y-4)$

$\boxed{C(5, -14)}$