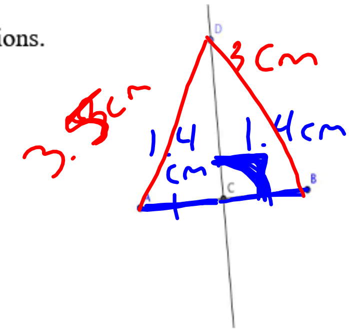
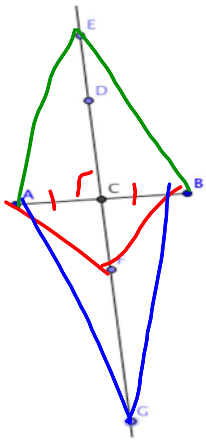
Name: Date: Page 1 of 4

Activity 5.2.1b The Perpendicular Bisector as a Locus of Points

Look at the diagrams to the right. Investigate, discuss, and answer the questions.



1. Measure the distances *CA* and *CB*. What do you notice?
2. *C* is the midpoint of $\overline{AB}$.
3. Measure $\angle DCB$. What do you notice? 90°
4. $\overleftrightarrow{DC}$ is perpendicular to $\overline{AB}$.
5. $\overleftrightarrow{DC}$ is the perpendicular bisector of $\overline{AB}$.
6. Measure the distances *DA* and *DB*. What do you notice?
   they were both 3 cm
   3.5 cm



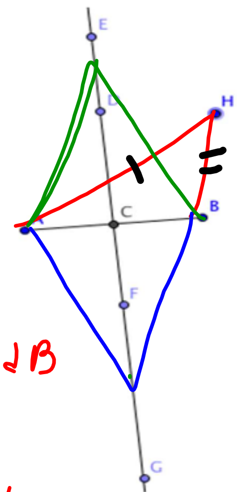
7. Use the points *E*, *F*, and *G* on $\overleftrightarrow{DC}$ shown on the new figure to the right.
8. Measure these distances

   $EA =$ 3 cm $EB =$ 3.1cm

   $FA =$ 2 cm $FB =$ 2 cm

   $GA =$ 3.9 cm $GB =$ 3.9 cm

   What do you notice?
   the points were the same distances away from A and B
9. Make a conjecture about all points that lie on $\overleftrightarrow{DC}$.
   Any + all points on DC will be the same distance from B as A.

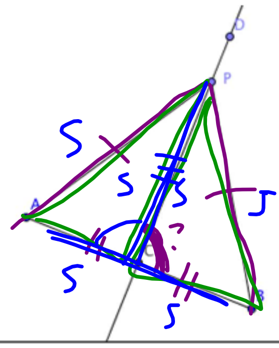


10. Now look at the third figure, Note the point *H* in the plane that is not on $\overleftrightarrow{DC}$. Measure *HA* and *HB*. What do you notice?
    not same length
11. Move point *H* so that $HA = HB$. What do you notice?
    any point not on DC will have different lengths to A and B
12. Make a conjecture about all points that are equidistant from points *A* and *B*.
    all points equidistant from A and B will be on the perpendicular bisector

Name: Date:

Activity 5.2.2b Proof of the Perpendicular Bisector Theorem

The Perpendicular Bisector Theorems says that the locus of points that are equidistant from the endpoints of a segment is the perpendicular bisector of the segment.



To prove this theorem we need to prove two things:

(1) If a point lies on the perpendicular bisector of a line segment, then it is equidistant from the endpoints of the segment, and
(2) If a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

Prove Part (1): 90° cut into 2 = parts

Given: $\overleftrightarrow{DC}$ **is the perpendicular bisector of** $\overline{AB}$
P lies on $\overleftrightarrow{DC}$
Prove: $PA = PB$

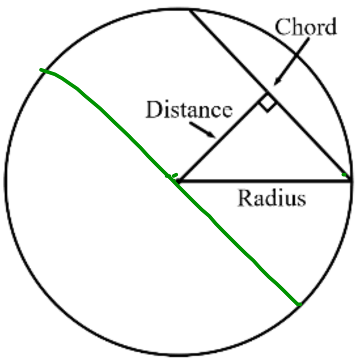
Statements	Reasons
$\overleftrightarrow{DC}$ **is the perpendicular bisector of** $\overline{AB}$ **and** P **lies on** $\overleftrightarrow{DC}$	Given
S $AC = CB$	(a) Def. of bisector
$\angle ACP$ and $\angle BCP$ are both right angles	(b) Def. of perpendicular
A (c) $\angle ACP \cong \angle BCP$	All right angles are congruent
S $PC = PC$	(d) Reflexive Prop.
(e) $\Delta ACP \cong \Delta BCP$	(f) SAS $\cong$
(g) $PA = PB$	Corresponding parts of congruent triangles are congruent

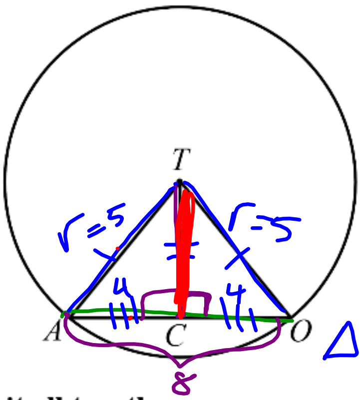
Prove Part (2)
Given: $PA = PB$ middle
C is the midpoint of $\overline{AB}$
Prove: $\overleftrightarrow{PC} \perp \overline{AB}$ 90°

Statements	Reasons
S $PA = PB$	Given
C is the midpoint of $\overline{AB}$	Given
S $AC = CB$	(a) Def. Midpoint
S (b) $PC = PC$	Reflexive Property
(c) $\Delta PAC \cong \Delta PBC$	(d) SSS $\cong$
$m\angle ACP = m\angle BCP$	(e) CPCTC
(f) $m\angle ACP + m\angle BCP = 180°$	(g) Def Linear Pair
$m\angle ACP + m\angle ACP = 180°$	Substitution (substitute $m\angle ACP$ for $m\angle BCP$)
$2\cdot(m\angle ACP)$ $\frac{2m\angle ACP}{2} = \frac{180°}{2}$	(h) combine like terms
(i) $m\angle ACP = 90°$	Division property of equality
(j) $\overline{PC} \perp \overline{AB}$	Definition of perpendicular lines

Definitions

- A **chord** of a circle is a geometric line segment whose endpoints both lie on the circle. *Not all chords are the same length*
- A **diameter** of a circle is any chord that passes through the center of the circle. *All diameters in the same circle are equal in length.*
- The **radius** (plural is **radii**) of the circle is a line segment in which one endpoint is the center of the circle and the other endpoint is on the circle. *All radii of the same circle are equal in length.*
- The **distance** from the center of a circle to any chord is always the shortest distance, which happens to be perpendicular to the chord.





HL
Hypotenuse leg ≅
$\triangle TAC \cong \triangle TOC$

PART 1: Bringing it all together.

Use the diagrams and information above to answer the following questions.

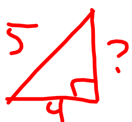
1.) In the circle above, $\overline{AO}$ is a chord, $\overline{TA}$ and $\overline{TO}$ are radii, and $\overline{TC}$ is the distance from the center of the circle to the chord.

a.) What do you know about the measure of $\angle ACT$? 90°

b.) If $\overline{AO}$ has a length of 8, what is the measure of $\overline{AC}$?

$\frac{}{2} = 4$ $AC = 4 = CO$

c.) If the radius of the circle is 5, find the distance from the center of the circle to chord $\overline{AO}$.



$a^2+b^2=c^2$
$4^2+d^2=5^2$

Pythagorean Triple 3-4-5 ✓✓
distance = 3

d.) What is the relationship between the radius, the chord, and the distance from the center of the circle to the chord?

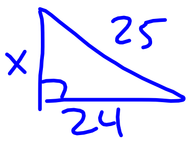
the radius is the hypotenuse, the distance and half the chord are the legs of a right triangle

$$\left(\frac{chord}{2}\right)^2 + (distance)^2 = (radius)^2$$

Name: Date:

PART 2: Bringing it all together.
Trying them on your own.

2.) A circle with radius 25 has a chord that is 24 units from the center of the circle. Find the length of the chord $\overline{VW}$.



$x^2+24^2=25^2$

$x^2+576=625$

$-576 \quad -576$

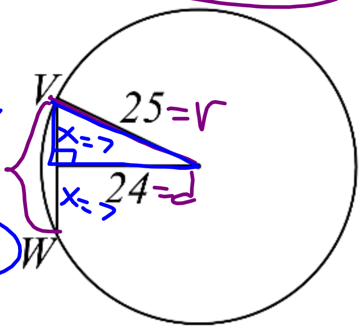
$\sqrt{x^2}=\sqrt{49}$

$x=\pm 7$

$VW=x+x$

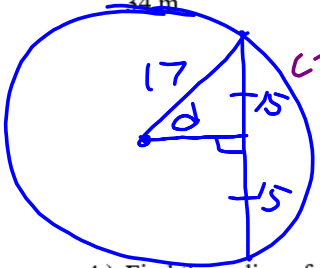
$\overline{VW}=7+7$

$\overline{VW}=14$



3.) Find the distance from the center of a circle to a chord 30 m long if the diameter of the circle is 34 m. 34 m

$30 \div 2 = 15$



$d=8m$

$d^2+15^2=17^2$

$d^2+225=289$

$d^2=64$

$d=8$

diam = 34

radius = $\frac{34}{2}$ = 17

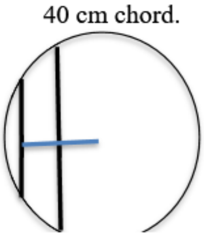
Pythag Triple

8-15-17

4.) Find the radius of a circle if a 24 cm chord is 9 cm from the center.

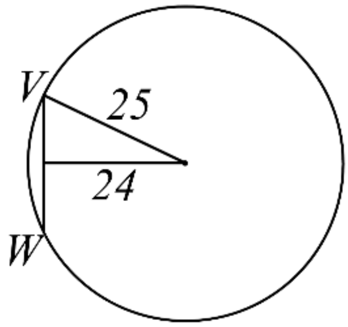
5.) Find the length of a chord that is 15 cm from the center of a circle with a radius of 17 cm.

6.) Challenge! Find the radius of a circle in which a 48 cm chord is 8 cm closer to the center than a 40 cm chord.



PART 2: Bringing it all together.
Trying them on your own.

2.) A circle with radius 25 has a chord that is 24 units from the center of the circle. Find the length of the chord $\overline{VW}$.



VW = 14

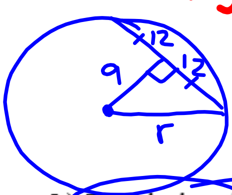
3.) Find the distance from the center of a circle to a chord 30 m long if the diameter of the circle is 34 m.

d = 8 m

4.) Find the radius of a circle if a 24 cm chord is 9 cm from the center.

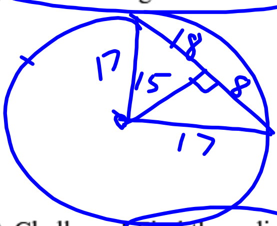
r = 15

$3\times\begin{cases} 9-12-15 \\ 3-4-5 \end{cases}$



5) chord = 16 6) r = 25

5.) Find the length of a chord that is 15 cm from the center of a circle with a radius of 17 cm.



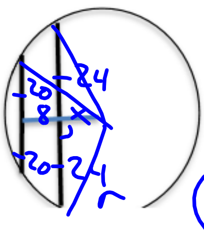
8 - 15 - 17

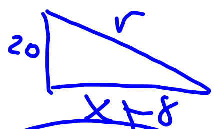
chord = 8 + 8 = 16

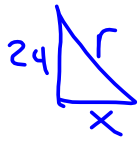
$17^2 = 15^2 + x^2$

$x = 8$

6.) Challenge! Find the radius of a circle in which a 48 cm chord is 8 cm closer to the center than a 40 cm chord.







$20^2 + (x+8)^2 = 24^2 + x^2$

$400 + (\cancel{x^2} + 16x + 64) = 576 + \cancel{x^2}$

$-x^2 \qquad -x^2$

$20^2 + (x+8)^2 = r^2 \qquad 24^2 + x^2 = r^2$