Conditional Probability 12-2 Level 3

Conditional Probability — **Probability of an event happening limited by the situation required.**

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

"given"

kinda like B/A

P(B given A occurred) = $\frac{\text{The ways to get the overlap of A and B}}{\text{The ways to get Total of A}}$

Use when the word "given" is in the problem, or when P(__|__)

Use the Formula

If given the single and overlap, use the formula

EXAMPLE 4 Use Conditional Probability to Make a Decision

Mobile Phone Search and Buying Behavior

Product	Search(S)	Search & Buy (S and B)
W	46%	16%
X	32%	14%
Y	35%	12%
Z	40%	15%

A marketer is looking at mobile phone statistics to help plan an online advertising campaign. She wants to find out which of her company's products is most likely to be purchased after a search for that product on a mobile phone.

Find the probability a mobile phone customer buys, given that they performed a related search.

Use the formula $P(B \mid S) = \frac{P(S \text{ and } B)}{P(S)}$.

Product	$P(B \mid S)$
W	$\frac{P(S \text{ and } B)}{P(S)} = \frac{0.16}{0.46} = 34.8\%$
X	$= \frac{0.14}{0.32} = 43.8\%$
Y	$= \frac{0.12}{0.35} = 34.3\%$
Z	$= \frac{0.15}{0.40} = 37.5\%$

X is most likely to be bought

Try It!

4. The marketer also has data from desktop computers. Which product is most likely to be purchased after a related search?

Computer Search and Buying Behavior (% of computer-based site visitors)

Product	Search	Search & Buy
J	35%	10%
K	28%	9%
L	26%	8%
M	24%	5%

J $\frac{S+B}{S} = \frac{0.10}{0.35} = 28.6\%$

K $\frac{9/100}{28/100} = \frac{.09}{.28} = 32.1\%$

L $= \frac{.08}{.26} = 30.8\%$

M $= \frac{0.05}{.24} = 20.8\%$

K is most likely to be bought given it was searched

Use your brain and logic

When given a table with sections and overlaps, the given is the denominator, the numerator is the desired in that row or column

Monday Club Memberships by Grade

	Drama	Science	Art	Total
Sophomore	3	9	24	36
Junior	6	18	16	40
Senior	8	13	18	39
Total	17	40	58	115

What is the probability that a member of the art club selected at random is a junior?

given

P(Junior | art member picked)

$= \frac{\text{Juniors in Art}}{\text{Total art members}} = \frac{16 \div 2}{58 \div 2} = \boxed{\frac{8}{29}}$

EXAMPLE 1 Understand Conditional Probability

Try It!

1. a. What is the probability that a member of the drama club is a sophomore, P(sophomore | drama)?

$P(S|D) = \frac{\text{Soph. in Drama}}{\text{Total Drama}} = \boxed{\frac{3}{17}}$

b. What is the probability that a sophomore is a member of the drama club, P(drama | sophomore)? Is P(sophomore | drama) the same as P(drama | sophomore)? Explain.

$P(D|S) = \frac{\text{Drama}}{\text{Total Sophomore}} = \boxed{\frac{3}{36}}$

No, the overlap on top is the same, but the denominator given situation is different

Examples:

Diego wonders whether the amount of sleep he gets is related to the amount of coffee he drinks. For about 10 weeks, he records how many cups of coffee he drinks each morning and how many hours he sleeps that night, rounded to the nearest hour.

Coffee Drinking and Amount of Sleep

	0 cups	1 cup	2 cups	Total
6 hours	4	10	13	27
7 hours	6	9	7	22
8 hours	12	7	4	23
Total	22	26	24	72

Based on this data, what is *P*(8 hours | 2 cups)? Round your answer to the nearest whole percent.

P(got 8 hrs of sleep given he drank 2 cups of coffee)

$\frac{4}{24} = \frac{1}{6} = 16.7\% = 17\%$

Examples:

Dr. Jensen is running an experiment on a new drug that cures migraines. The table below breaks down the results from the experiment:

There must be totals add them in

	Number of Subjects		
	Using Drug	Using Placebo	total
Migraines Cured	1600	1200	2800
Migraines Not Cured	800	400	1200
	2400	1600	4000

a) What is the probability that a subject chosen at random was given a placebo?

not an extra condition

$P(\text{placebo}) = \frac{\text{total Placebo}}{\text{total in experiment}} = \frac{1600}{4000} = \frac{16}{40} = \boxed{\frac{2}{5}}$

b) What is the probability that a test subject had their migraine cured, given that he used the drug?

extra condition

$P(MC \mid D) = \frac{\text{Migraine cured}}{\text{Took Drug}} = \frac{1600}{2400} = \frac{16}{24} = \boxed{\frac{2}{3}}$

c) What is the probability that the test subject was NOT cured, given that they took the placebo?

$P(NC \mid P) = \frac{400}{1600} = \frac{4}{16} = \boxed{\frac{1}{4}}$

Make the Sample Space

If you are not given a table, write out the possible ways to create the given situation

a) What is the probability of rolling a 2 on a 6-sided die, given that you rolled an even number?

Sample Space

even #s

2, 4, 6

$P(2 \mid \text{even}) = \boxed{\frac{1}{3}}$

b) Gloria tosses two coins. What is the probability that she has tossed 2 heads, given that at least one of her tosses was a head?

Sample Space

TT HH

TH HT

$P(HH \mid \text{at least 1 is a H}) = \boxed{\frac{1}{3}}$

c) A dodecahedron is a 12-sided, 3-dimensional figure with numbers 1 through 12 on its faces. If you roll it 1 time, what is the probability that you roll a 6 or a multiple of 4 given that it is an even number?

Sample Space

1 2 3 4 5 6

7 8 9 10 11 12

$P(6 \text{ or multiple } 4 \mid \text{even}) = \frac{4}{6} = \frac{2}{3}$

Make the Tree Diagram

When given probabilities as percentages, fill out the missing information using the branches. Multiply down the branch.

Concert Survey Results

Students who plan to attend concert

- 70% of students plan to attend.
- 80% of students who plan to attend are fans of the group.

Students who do not plan to attend

- 30% of students do not plan to attend,
- 25% are fans of the band.

A band's marketing agent conducted a survey to determine how many high school fans the band has. What is the probability that a surveyed student plans to attend the band's concert and is a fan of the group?

.70 .30

Attend Not Attend

.80 .20 .25 .75

Fan Not Fan Fan Not Fan

Tree diagram Fill in % with given info

Fill in missing % should add to 100%

P(A and F)

0.7 • 0.8

0.56

= 56%

Try It!

3. What is the probability that a surveyed student plans to attend but is not a fan of the group?

P(Attend and Not Fan)

multiply

0.7 • .20

.14 → 14%

are not fans but went anyway

What is the probability that a surveyed students who doesn't plan to attend is a fan?

P(Not Attend and is Fan)

0.3 • 0.25

.075 = 7.5% is a fan but didn't attend

Events are Independent: If $P(B|A) = P(B)$

Events are Dependent: If $P(B|A) \neq P(B)$

	Car	Van	Pickup	Totals
Red	5	0	2	7
White	0	0	2	2
Black	6	3	4	13
Totals	11	3	8	22

The table below shows the vehicles in a parking garage one afternoon. A vehicle in the garage will be selected at random. Let *B* represent "the vehicle is black" and *V* represent "the vehicle is a van." Are the events *B* and *V* independent or dependent?

is $P(B) = P(B|V)$

$\frac{13}{22} \stackrel{?}{=} \frac{3}{3}$

Not equal, so dependent

Try It!

2. Let *R* represent "the vehicle is red" and *C* represent "the vehicle is a car." Are the events *R* and *C* independent or dependent? Explain.

$P(R) = P(R|C)$

$\frac{7}{22} \stackrel{?}{=} \frac{5}{11}$

Not equal, so dependent

Name: Key Date: ______________

Level 3 Conditional Probability

1. Two dice are tossed. Find the probability that the numbers showing on the dice match given that their sum is greater than five. Make a Sample Space

$\frac{4}{15}$

2. A card is chosen at random from a standard deck of cards. Find each probability.

a. P(ace| card is black) $\frac{1}{13}$

b. P(4| card is red) $\frac{1}{13}$

c. P(face card| card is black) $\frac{3}{13}$ or $\frac{4}{13}$ if ace is a face

d. P(queen of hearts| card is black) 0

e. P(6 of diamonds| card is red) $\frac{1}{26}$

f. P(jack or ten| card is black) $\frac{2}{13}$

3. Three coins are tossed. Find the probability, given the last coin shows a head.

Make a Sample Space:

a. The first coin shows a head $\frac{1}{8}$

b. at least one coin shows a head
1 – No coins show H
$\frac{7}{8}$

c. at least two coins show heads
2 show H + 3 show H
$\frac{1}{8} + \frac{1}{8}$
$\frac{1}{4}$

4. A pair of number dice are thrown. Find each probability given that their sum is greater than or equal to 9.

Make a Sample Space:

a. P(numbers match) $\frac{1}{3}$

b. P(sum is even) $\frac{1}{2}$

c. P(numbers match or sum is even) $\frac{5}{6}$

Name:_______________________ Date: ______________

5. To test the effectiveness of a new vaccine, researchers gave 100 volunteers the conventional treatment and gave 100 other volunteers the new vaccine. The results are shown in the table below. Finish the Table with the totals.

Treatment	Disease Prevented	Disease Not Prevented	
New Vaccine	68	32	100
Conventional Treatment	62	38	100
	130	70	200

a. What is the probability that the disease is prevented in a volunteer chosen at random?

$\frac{13}{20}$

b. What is the probability that the disease is prevented by a volunteer who was given the new vaccine?

$\frac{17}{25}$

c. What is the probability that the disease is prevented in a volunteer who was not given the new vaccine?

$\frac{31}{50}$

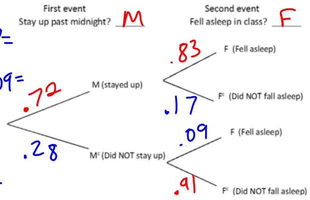
d. Given the disease was prevented, how likely was it that they got the new vaccine?

$\frac{34}{75}$

6. There is a 72% probability that a randomly chosen high school student stayed up past midnight. If they stayed up past midnight, there is a 83% chance that they fell asleep during class the next day. If they didn't stay up past midnight there is a 91% chance that they did not fall asleep during class the next day. Complete the Tree diagram and answer the following.

First event
Stay up past midnight? M

Second event
Fell asleep in class? F



a. P(stayed up and did NOT fall asleep)

$P(M) \cdot P(\text{Not}F) = .72 \cdot .17 =$

b. b. P(went to bed and did fall asleep)

$P(\text{Not}M) \cdot P(F) = 0.28 \cdot 0.09 =$

c. P(fell asleep | stayed up)

$0.83 = 83\%$ or $\frac{0.83 \cdot \cancel{0.72}}{\cancel{0.72}}$

d. P(fell asleep | went to bed)

$0.09 = 9\%$ or $\frac{\cancel{0.28} \cdot .09}{\cancel{0.28}}$