Your Name

Mrs. Theo

1/25/23

Notes

4-2 Graphing Rational Functions

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Graph using tranformations of

$\boxed{f(x) = \frac{1}{x}}$ parent function has

Vertical asymptote at x = 0

Horizontal asymptote at y = 0

Step 1: long divide

Step 2: Make a xy table around V.A.

$\boxed{g(x) = \frac{a}{x-h} + k}$

Vertical asymptote at x = h

Horizontal asymptote at y = k

Vertical stretch of *a*

X	Y
8	$\frac{8}{8-6} = \frac{8}{2} = 4$
7	$\frac{7}{7-6} = \frac{7}{1} = 7$
5	$\frac{5}{5-6} = \frac{5}{-1} = -5$
4	$\frac{4}{4-6} = \frac{4}{-2} = -2$
0	$\frac{0}{0-6} = 0$

EXAMPLE 1 **Rewrite a Rational Function to Identify Asymptotes**

Try It!

1. Use long division to rewrite each rational function. Find the asymptotes of *f* and sketch the graph.

X	Y
1	$\frac{6(1)}{2(1)+1} = \frac{6}{3} = 2$
0	$\frac{6(0)}{2(0)+1} = 0$
-1	$\frac{6(-1)}{2(-1)+1} = \frac{-6}{-1} = 6$
-2	$\frac{6(-2)}{2(-2)+1} = \frac{-12}{-3} = 4$

a. $f(x) = \frac{6x}{2x+1} = 3 + \frac{-3}{2x+1}$

$$2x+1\overline{)6x+0}\quad 3$$

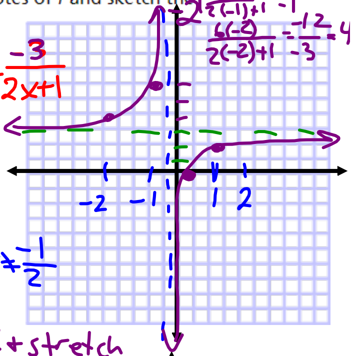
$$-(6x+3)$$

$$-3$$

VA: $2x+1=0 \rightarrow x \neq -\frac{1}{2}$

HA: $y \neq 3$

'a' : Reflected + stretch



b. $f(x) = \frac{x}{x-6} = 1 + \frac{6}{x-6}$

$$x-6\overline{)x+0}\quad 1$$

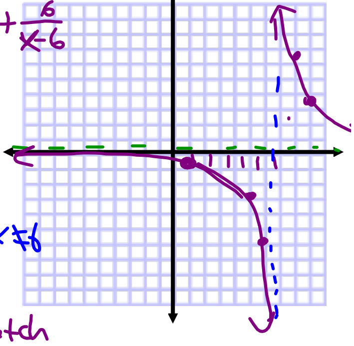
$$-(x-6)$$

$$6$$

VA: $x-6=0 \rightarrow x \neq 6$

HA: $y=1$

'a' = 6 stretch



Vertical Asymptotes

The x values that would make the denominator = 0

Factor, set = 0, solve for x

Asymptotes: leftover factors in denominator

Holes: factors that got canceled out

Horizontal Asymptotes

Case 1: $\frac{x+4}{x^2+1}$ $\boxed{y \neq 0}$

The degree of the numerator is less than the degree of the denominator, there exists a horizontal asymptote at y = 0.

Case 2: $\frac{x^2+1}{x+2}$ $\boxed{y \neq \text{quotient}}$

The degree of the numerator is greater than the degree of the denominator, there exists NO horizontal asymptote BUT there is a SLANT asymptote at the quotient

Case 3: $\frac{\overset{2}{\cancel{2}}x^2+x+1}{1x^2-1}$ y ≠ 2

The degree of the numerator is equal to the degree of the denominator, there exists a horizontal asymptote at

$$\boxed{y \neq \frac{\text{numerator Lead Coeff}}{\text{denominator Lead Coeff.}}}$$

EXAMPLE 2 Find Asymptotes of a Rational Function

Try It!

2. What are the vertical and horizontal asymptotes of the graph of each function?

a. $g(x) = \frac{2x^2+x-9}{x^2-2x-8}$

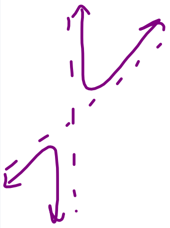
$\frac{2}{1}$

$(x-4)(x+2)$

VA: $x \neq 4$ $x \neq -2$

HA: Case 3 $x^2 = x^2$

$y \neq \frac{2}{1} = 2$



b. $f(x) = \frac{x^2+5x+4}{3x^2-12}$

$(x+4)(x+1)$

$3(x^2-4)$

$3(x+2)(x-2)$

VA: $x \neq 2$ $x \neq -2$

HA: Case 3

$x^2 = x^2$

$y \neq \frac{1}{3}$

EXAMPLE 3 Graph a Function of the Form $\frac{ax+b}{cx+d}$

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x int: set numerator = 0 and solve

y int: plug in 0 for x

Try It!

3. Graph the function.

a. $f(x) = \frac{4x-3}{1x+8}$

VA: $x+8=0$

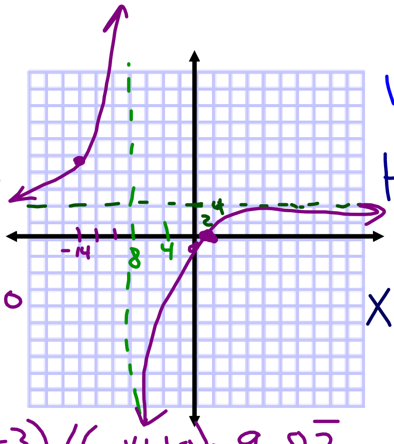
$x = -8$

HA: Case 3

$y = \frac{4}{1} = 4$

X int: $4x-3=0$

$x = \frac{3}{4}$



X	Y
-14	$(4(-14)-3)/(-14+8) = 9.8\overline{3}$
-10	21.5
-8	undefined
-6	-13.5
0	-0.375

as $x \to \infty$ $y \to 4$

as $x \to -\infty$ $y \to 4$

b. $g(x) = \frac{3x+2}{1x-1}$

VA: $x-1=0$

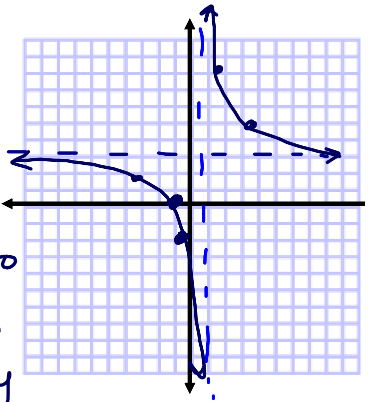
$x \neq 1$

HA: Case 3

$y = \frac{3}{1} = 3$

X int: $3x+2=0$

$x = -2/3$

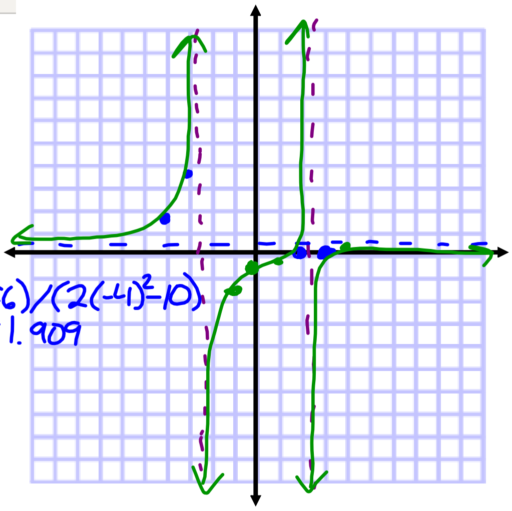


X	Y
-3	$(3(-3)+2)/(-3-1) = \frac{-7}{-4} = 1.75$
0	$(3(0)+2)(0-1) = \frac{2}{-1} = -2$
1	Undef
2	$(3(2)+2)(2-1) = \frac{8}{1} = 8$
4	$(3(4)+2)(4-1) = \frac{14}{3} = 4.6$

EXAMPLE 5 Graph a Rational Function

Try It!

5. Identify the asymptotes and sketch the graph of $g(x) = \frac{x^2 - 5x + 6}{2x^2 - 10} = \frac{(x-3)(x-2)}{(2(x^2-5))}$



VA: $2(x^2-5)=0$

$x^2-5=0$

$x^2=5$

$x=\pm\sqrt{5}$

$x=\pm 2.24$

HA: Case 3

$y=\frac{1}{2}$

Xint: $(x-3)(x-2)=0$

$x-3=0$ $x-2=0$

$x=3$ $x=2$

x	y
-4	$((-4)^2-5(-4)+6)/(2(-4)^2-10)$ =1.909
-3	3.75
$-\sqrt{5}$	undefined
-1	-1.5
0	-0.6
1	-0.25
2	0
$\sqrt{5}$	undefined
3	0
4	0.09
2.3	
2.23	3.27

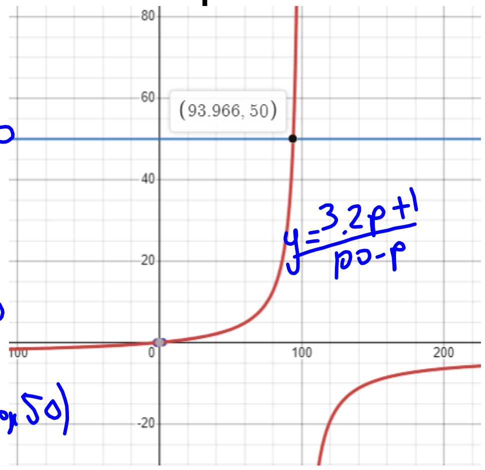
EXAMPLE 4 Use a Rational Function Model

Try It!

4. New techniques have changed the cost function. For the new function $g(p) = \frac{3.2p + 1}{100 - p}$, what percent of the pollutant can be removed for $50 million?

The cost of removing a pollutant is modeled by the given function where $f(p)$ is the cost, in millions of dollars, of removing p percent of the pollutant. What percent of the pollutant can be removed for $78.3 million?

y=50



$y=\frac{3.2p+1}{100-p}$

Can't have over 100% so asymptote at p=100

two functions meet at (93.96,50) so for $50 million, 93.96% of pollutant can be removed

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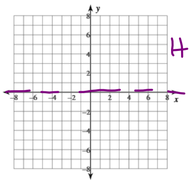
4.4: Graphing Rational Functions Practice

Date________________ Period____

Identify the holes, vertical asymptotes, x-intercepts, horizontal asymptote, and domain of each. Then sketch the graph.

1) $f(x) = \frac{4x^{0}}{x - 3}$

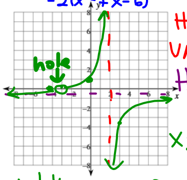


HA: Case1

$y = 0$

2) $f(x) = \frac{1x^2 + 7x + 12}{-2x^2 - 2x + 12} = \frac{\cancel{(x+3)}(x+4)}{-2\cancel{(x+3)}(x-2)}$

$-2(x^2 + x - 6)$



Hole: $x = -3$

VA: $x = 2$

HA: case3

$y = \frac{1}{-2} =$

X-int: Numerator = 0

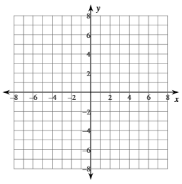
$x = -3$ $x = -4$

Domain: ℝ, $x \neq -3, 2$

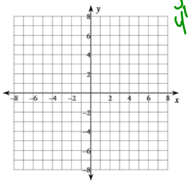
x	y
-6	
-5	
-4	0
-3	undef

x	y
-1	
0	
1	
2	undefined
3	
4	

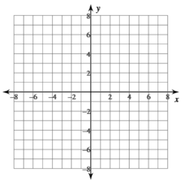
3) $f(x) = \frac{1}{-x + 4}$



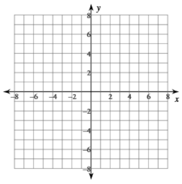
4) $f(x) = \frac{-3x + 12}{x^2 - 3x - 4}$



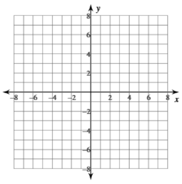
5) $f(x) = \frac{-2x^2 + 4x + 16}{x^2 - 5x + 4}$



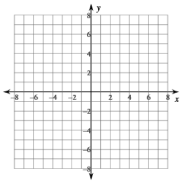
6) $f(x) = \frac{x^2 - 3x}{2x^2 + 2x - 12}$



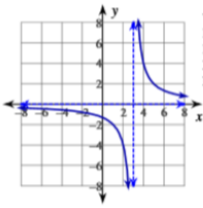
7) $f(x) = \frac{3x + 6}{x + 3}$



8) $f(x) = \frac{x^2 + 5x + 4}{x^2 - 1}$

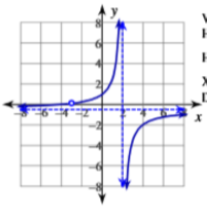


1)



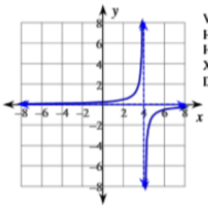
Vertical Asym.: $x = 3$
Holes: None
Horz. Asym.: $y = 0$
X-intercepts: None
Domain:
All reals except 3

2)



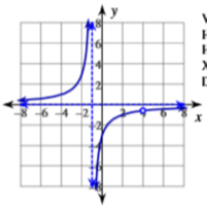
Vertical Asym.: $x = 2$
Holes: $x = -3$
Horz. Asym.: $y = -\frac{1}{2}$
X-intercepts: −4
Domain:
All reals except 2, −3

3)



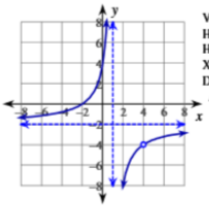
Vertical Asym.: $x = 4$
Holes: None
Horz. Asym.: $y = 0$
X-intercepts: None
Domain:
All reals except 4

4)



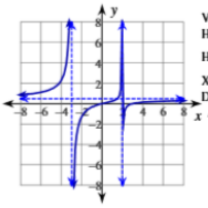
Vertical Asym.: $x = -1$
Holes: $x = 4$
Horz. Asym.: $y = 0$
X-intercepts: None
Domain:
All reals except −1, 4

5)



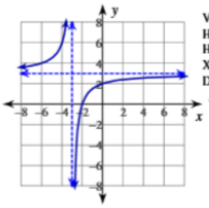
Vertical Asym.: $x = 1$
Holes: $x = 4$
Horz. Asym.: $y = -2$
X-intercepts: −2
Domain:
All reals except 1, 4

6)



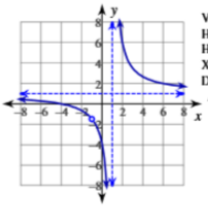
Vertical Asym.: $x = -3$, $x = 2$
Holes: None
Horz. Asym.: $y = \frac{1}{2}$
X-intercepts: 0, 3
Domain:
All reals except −3, 2

7)



Vertical Asym.: $x = -3$
Holes: None
Horz. Asym.: $y = 3$
X-intercepts: −2
Domain:
All reals except −3

8)



Vertical Asym.: $x = 1$
Holes: $x = -1$
Horz. Asym.: $y = 1$
X-intercepts: −4
Domain:
All reals except 1, −1