Unit 05 - Section 3 - Polygons

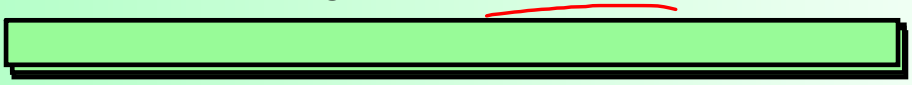
What are Polygons?

Polygons

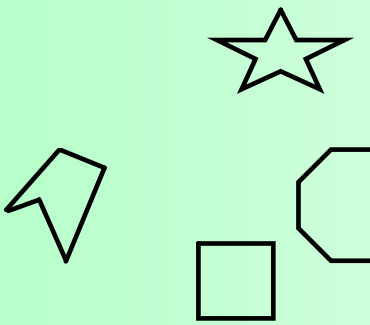
Many Angles

Polygons have/are:

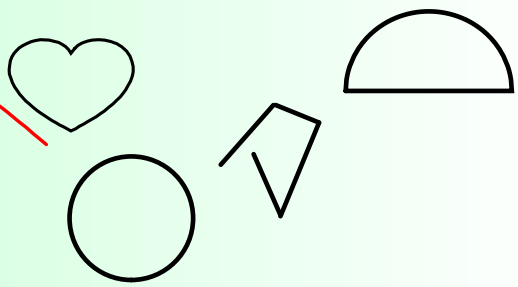
1. Plane figures (2-dimensional)
2. At least 3 sides (triangles, quadrilaterals,...)
3. Closed figures - all sides connected with no gaps
4. All "sides" are segments - no curves!



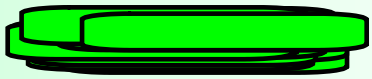
Polygons



NOT Polygons



# of Sides	Polygon Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
15	Pentadecagon
n	*n*-gon



Angles of a Polygon - Work with a partner/group

1.) Each student will need a protractor and a ruler/straightedge.

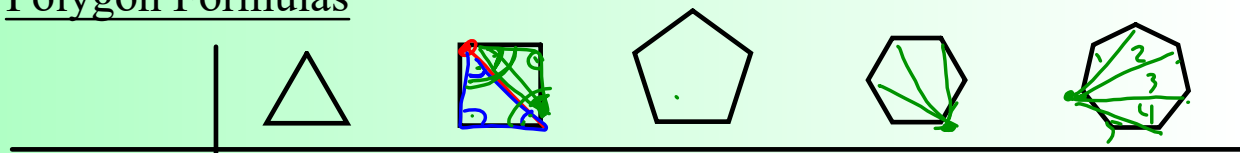
2.) Draw 1 triangle (3 sides), 1 quadrilateral (4 sides), and 1 pentagon (5 sides) (Not too small!)

3.) Using your protractor, find the measure of each angle in all 3 figures. Be as accurate as possible to the nearest degree.

4.) Find the sum of all the angles in each figure.

5.) In your groups, come up with a prediction for what you think the sum of the angles in a hexagon (6 sides) will be. Be able to explain!

Polygon Formulas



# Sides	3	4	5	6	7
# Triangles Inside	1	2	3	4	5
Sum of Angles	180	360	540	4·180 = 720°	900°

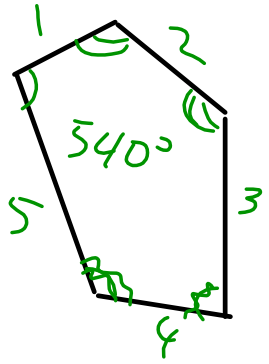
Sum of Angles

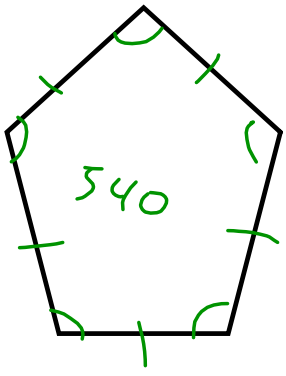
$S = 180(n-2)$ n = number of sides

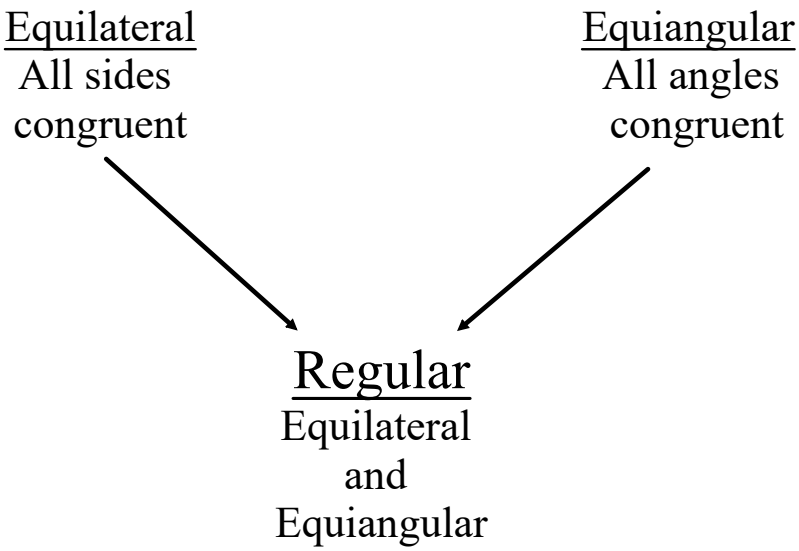
# of Triangles

540

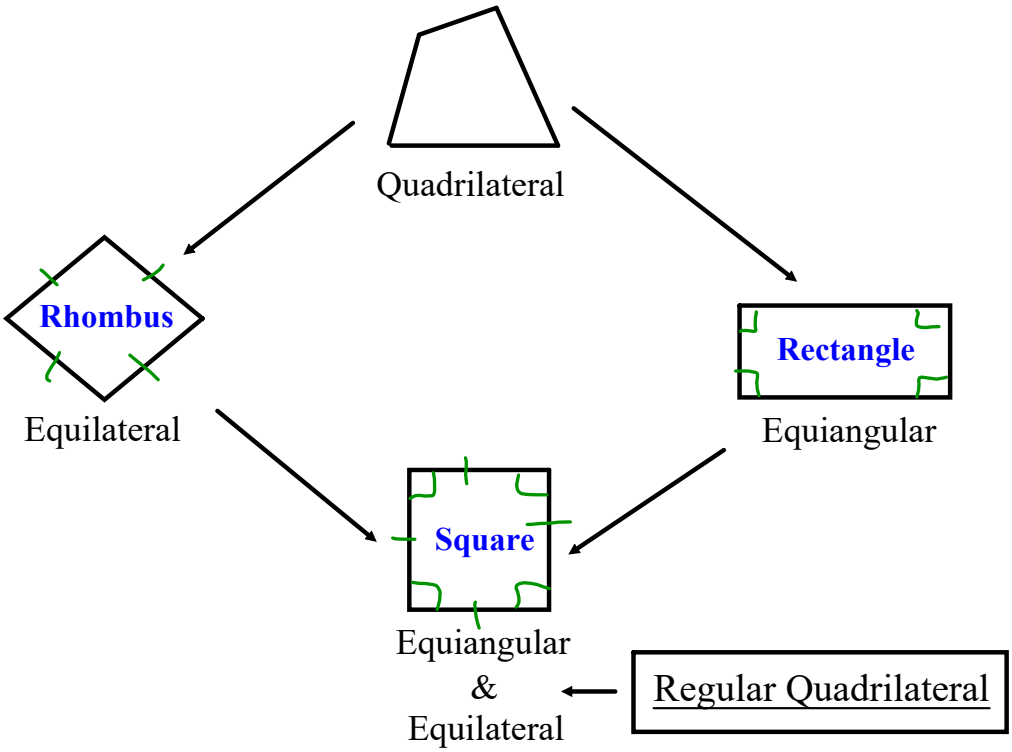
What if we want to find out the measure of one angle?





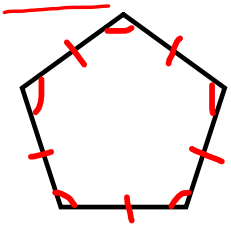


Regular Polygon = Polygon that is both equilateral and equiangular.

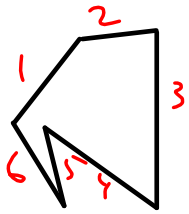


Regular Polygon = Polygon that is both equilateral and equiangular.

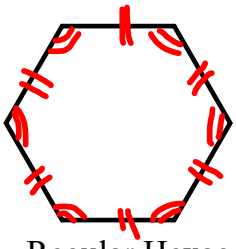
Pentagon



Regular Pentagon



Hexagon



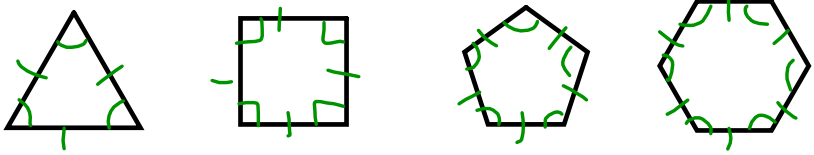
Regular Hexagon

Regular or Irregular?

Polygons can also be classified as either *regular* or *irregular*.

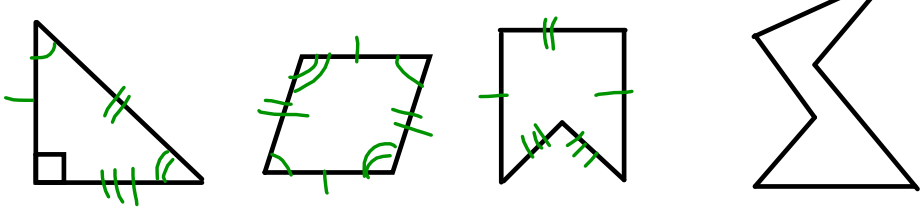
Regular

- All sides are congruent and
- all angles are congruent.



Irregular

- All sides are not congruent or all angles are not congruent.



The Interior Angle **Sum**

To find the total number of degrees in any polygon:

$$(N-2)\cdot 180$$

number of sides

Total Sum

Measure of **Each** Interior Angle

In a regular polygon, the measure of ***each interior angle*** is defined as :

the sum of all the angles divided by the number of sides (which is the same as the number of angles).

Measure of only one

Measure of Each Interior Angle

$$S = \frac{180(n-2)}{n}$$

total sum / # of angles

Find the measure of AN interior angle of the regular polygon

Formula: $\frac{\text{sum of angles}}{\text{number of angles}}$

1. regular pentagon?

$n=5$

$\frac{180(5-2)}{5} = \frac{540}{5}$

each angle is 108°

2. regular nonagon?

$n=9$

$\frac{180(9-2)}{9} = 140°$

3. regular 13-gon?

$n=13$

$\frac{180(13-2)}{13} = 152.307$

$152.31°$

4. regular 100-gon?

$\frac{180(100-2)}{100} = 176.4°$

What are they asking?

5. Find the sum of the Measures of a 38-gon.

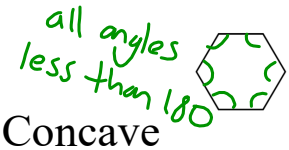
6. Find the measure of one interior angle of a regular 38-gon.

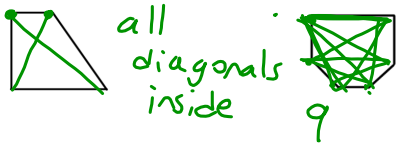
Convex or Concave?

Polygons can also be classified as either *convex* or *concave*.

Convex

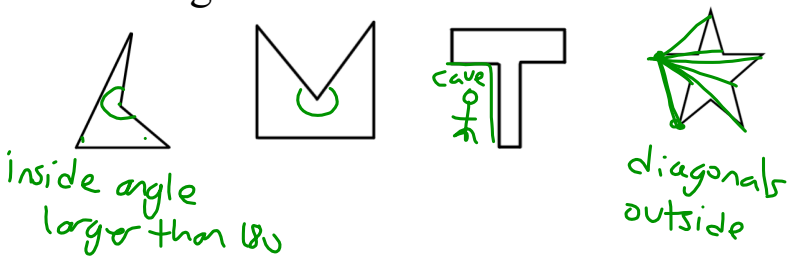
- No interior angles larger than 180^0
- No diagonals pass outside the polygon





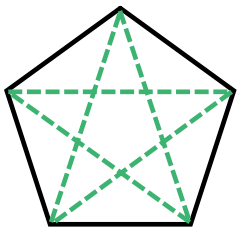
Concave

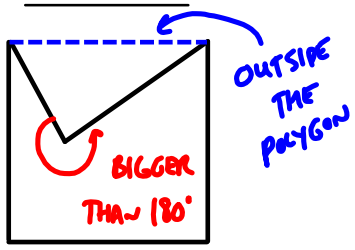
- 1 or more interior angle is larger than 180^0
- 1 or more diagonal passes outside the polygon
- Has a "cave" in it, or one vertex seems to move into the figure



Polygons

Convex vs. Concave





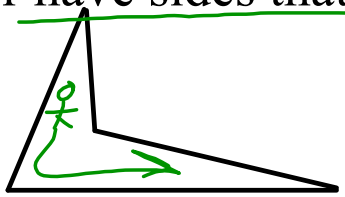
- Concave polygons "go in on themselves."
- You know this if you can draw any of the diagonals of the polygon and they lie outside the figure. (See above)
- Concave polygons also have 1 or more angles that have measures greater than 180 degrees. (See above)

Simple or Complex?

Polygons can also be classified as either *simple* or *complex*.

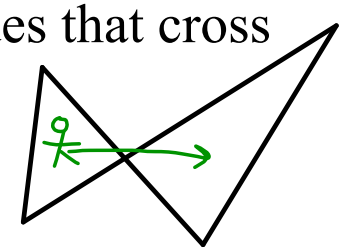
Simple

- Have 1 boundary and do not pass over themselves or have sides that cross.



Complex

- Do not have 1 distinct boundary and/ or have sides that cross



Practice

7) Find the sum of the angles in a decagon.

$180(10-2)$ $n=10$

$1440°$

8) Find the name of the polygon whose angles add up to 1080°.

$\frac{180(n-2)}{180} = \frac{1080}{180}$

$n-2=6$

$+2 \quad +2$

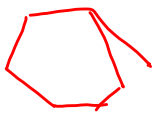
$n=8$

8 sides

Octogon

9) Can a polygon have angles whose sum is 600°?

No



Pentagon 540

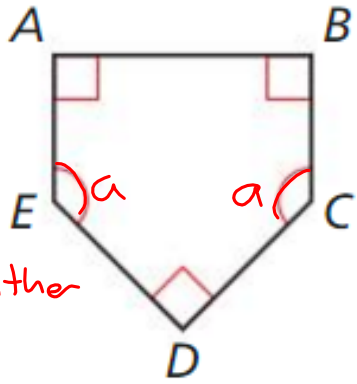
hexagon 720

600 is between

we can't have a fraction of a side

Practice

10.) A home plate for a baseball field is shown to the right.



11.) Is the polygon regular? Explain your reasoning.

No, not all angles are equal
3 sets of 90 and another set of 2 that are equal

12.) Find the measures of $\angle E$ and $\angle C$.

$90 + 90 + 90 + a + a = 540°$

$270 + 2a = 540$

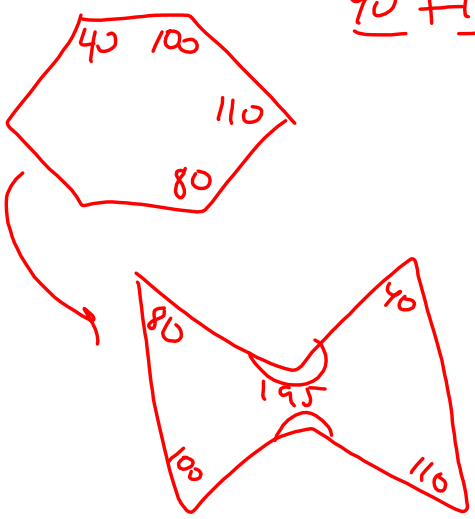
$2a = 270$

$a = 135°$

$m\angle E = 135 = m\angle C$

Examples

13.) A hexagon has 4 angles with measures of 40°, 100°, 110°, and 80°. What is the measure of each of the remaining two angles if they are congruent to each other?



$40 + 100 + 110 + 80 + x + x = 720$

$330 + 2x = 720$

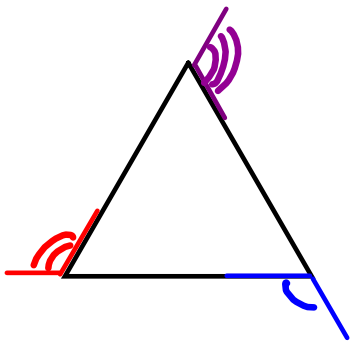
$2x = 390$

$x = 195$

Day 2

Sum of Exterior angles

If one exterior angle is drawn at each of the vertices,
the sum of all the exterior angles is 360°.



Sum of Exterior Angles

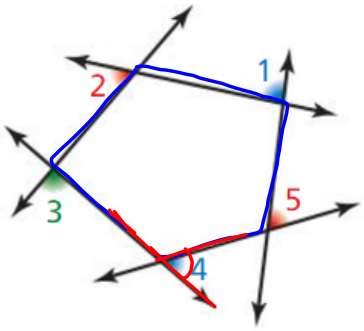
$S =$ [ ]

35

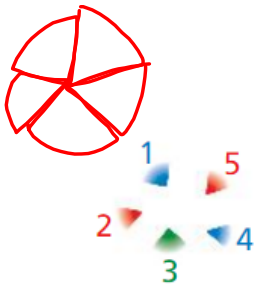
180-35

145

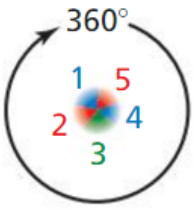
Exterior Angles (Do not copy this slide)



Step 1 Shade one exterior angle at each vertex.



Step 2 Cut out the exterior angles.



Step 3 Arrange the exterior angles to form 360°.

Regular Polygon

In a **regular polygon**, the measure of ***each exterior angle*** is defined as the sum of all the exterior angles divided by the number of sides.

Each Exterior Angle

Measure of Each Exterior Angle

$$S = \frac{360°}{n}$$

Examples

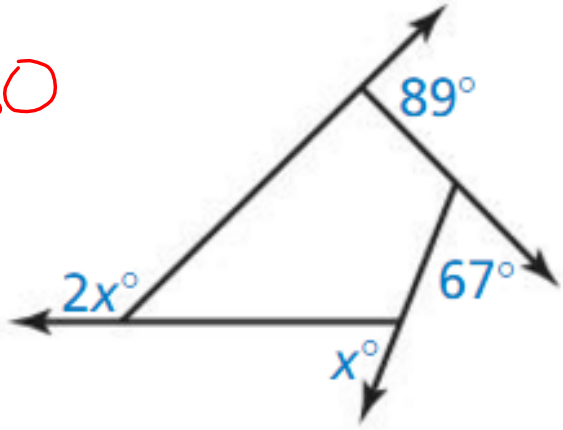
1.) Find the value of x in the diagram.

$2x + 89 + 67 + x = 360$

$3x + 156 = 360$

$\frac{3x}{3} = \frac{204}{3}$

$x = 68$



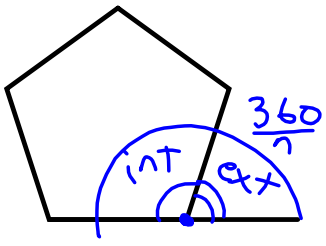
Regular Polygon

In a **regular polygon**, the measure of ***each interior angle*** can be found another way!

Interior and exterior angles at the same vertex are supplementary to each other.

add up to 180°

Each Interior Angle



Measure of Each Interior Angle

$$180 - \frac{360°}{n}$$

Part 2

So, If the polygon is regular,
Find the measure of each exterior angle,
Then, find its supplement to determine the measure of each interior angle of the regular polygon.

Examples

2.) Use two different ways to find the measure of each interior angle in a regular hexagon.

$n = 6$

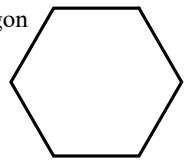
$\frac{180(n-2)}{n}$ | $180 - \frac{360}{n}$

$\frac{180(6-2)}{6} = 120$ | $180 - \frac{360}{6} = 120$

$180 - 60 = 120$

What do regular polygons allow us to do?

Regular Hexagon



3a.) What is the sum of the angles in a regular Hexagon

$180(6-2) = 720$

3b.) What is the measure of ***each angle*** in a regular hexagon?

$\frac{720}{6} = 120°$

3c.) What is the sum of the exterior angles in a regular hexagon?

$360°$

3d.) What is the measure of ***each exterior angle*** in a regular hexagon?

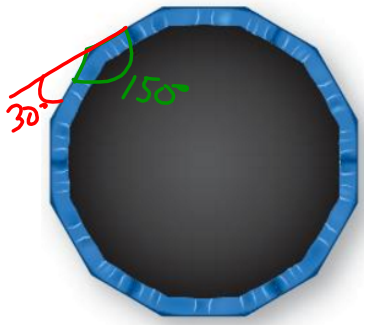
$\frac{360}{6} = 60$ or $180 - 120$

Examples

4.) The trampoline to the right is a regular dodecagon. $n=12$

a.) Find the measure of each interior angle.

$\frac{180(12-2)}{12} = 150°$



b.) Find the measure of each exterior angle.

$\frac{360}{12} = 30°$ or $180 - 150 = 30$

Summary of all Formulas

#1 and 2 apply to all polygons

1.) Sum of Interior Angles ..... $S_I = 180(n-2)$

2.) Sum of Exterior Angles ..... $S_E = 360$

#3 and 4 apply to only regular (or equiangular) polygons

3.) Measure of Each Interior Angle ..... $A_I = \frac{180(n-2)}{n}$ or $180 - \frac{360}{n}$

4.) Measure of Each Exterior Angle ..... $A_E = \frac{360}{n}$

Unit 05 - Section 03

Key

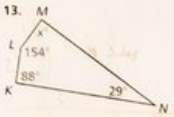
In Exercises 3–6, find the sum of the measures of the interior angles of the indicated convex polygon. *(See Example 1.)*

5. 16-gon: $180(16-2)$; sum = 2520°
6. 20-gon: $180(20-2)$; sum = 3240

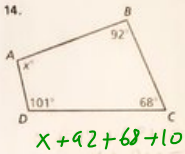
In Exercises 7–10, the sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides. *(See Example 2.)*

9. 2520°: $180(n-2)=2520$; $n=16$; 16-gon
10. 3240°: $\frac{180(n-2)}{180}=\frac{3240}{180}$; $n-2=18$; $+2\ +2$; $n=20$; 20-gon

In Exercises 11–14, find the value of x. *(See Example 3.)*

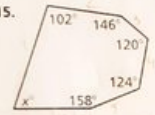
13. 

$x+154+88+29=360$; $x=89$

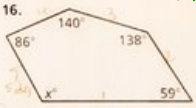
14. 

$x+92+68+101=360$; $x=99°$

In Exercises 15–18, find the value of x.

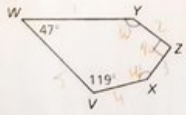
15. 

$x+102+146+120+124+158=720$; $x=70°$

16. 

$x+86+140+138+59=540$; $x=117°$

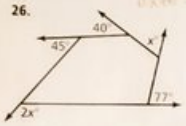
In Exercises 19–22, find the measures of $\angle X$ and $\angle Y$.

20. 

$w+w+90+119+47=540$

$m\angle X = 142° = m\angle Y$

In Exercises 23–26, find the value of x. *(See Example 5.)*

26. 

$x+40+45+2x+77=360$; $x=66$

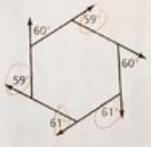
In Exercises 27–30, find the measure of each interior angle and each exterior angle of the indicated regular polygon. *(See Example 6.)*

27. Heptagon: each Ext 51.43°; each Int 128.57°
28. 18-gon: 20°; 160

34. **MODELING WITH MATHEMATICS** The floor of the gazebo shown is shaped like a regular decagon. Find the measure of each interior angle of the regular decagon. Then find the measure of each exterior angle.

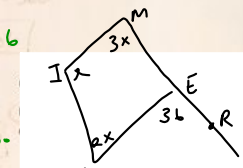
36°; 144°

50. **HOW DO YOU SEE IT?** Is the hexagon a regular hexagon? Explain your reasoning.



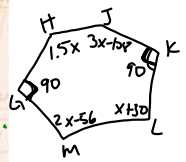
All Interior Angles measure all Exterior angles

Not a regular hexagon, not all ext. angles are = 60



20. $x+3x+2x+(180-36)=360$; $x=36$

21. $m\angle T=72°$ $m\angle M=108°$ $m\angle I=36°$ $m\angle TEM=144°$



24. $1.5x+3x-108+x+30+2x-54+90+90$; $7.5x+46=720$; $=720$; $x=89.87°$

$\angle G=90$ $\angle H=134.88°$ $\angle J=161.61$ $\angle K=90°$ $\angle L=119.87°$ $\angle M=123.74$

$\frac{360}{6}=60°$