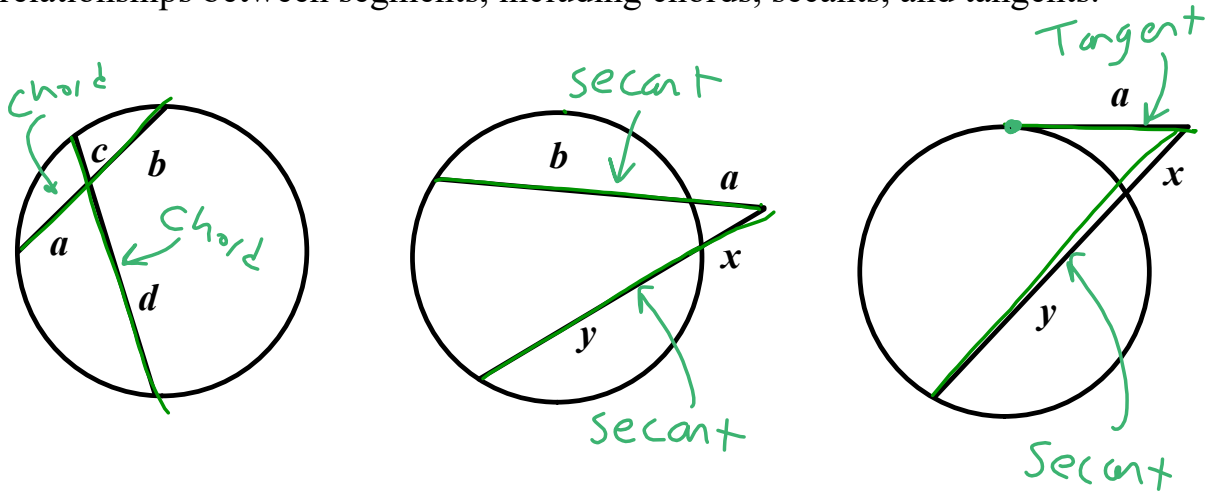


10.6 - Power Theorems

In Chapter 10, most of our previous theorems involved relationships between angles and arcs. These 3 new Power Theorems from 10.6 involve the relationships between segments, including chords, secants, and tangents.



Vocab Check

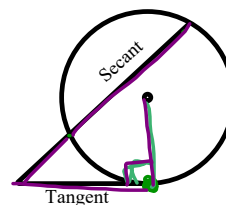
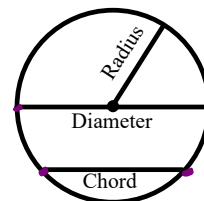
Radius = Segment connecting the center of a circle to any point on the circle

Chord = A segment connecting any two points on a circle

Diameter = A chord that passes through the center of a circle (twice the radius)

Secant = Segment intersecting circle twice (Starts outside circle)

Tangent = Segment touching a circle once (Starts outside circle, hits at point of tangency)



Mental Floss:Write the equations in factored form, then solve for x .

1.) $x^2 - 17x = 60$

2.) $2x^2 - 7x - 15 = 0$

Solve the equations for x .

3.) $6(x+7) = 4(4+8)$

4.) $x(x+6) = 55$

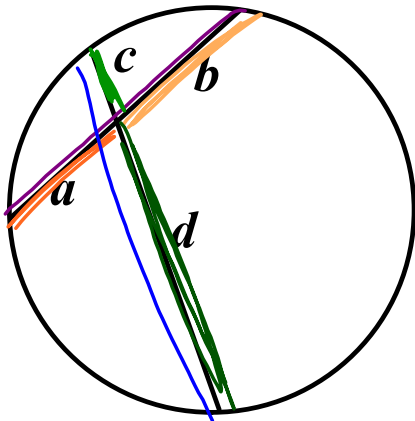
$$\begin{aligned} \textcircled{1} \quad x^2 - 17x &= +60 \\ x^2 - 17x - 60 &= 0 \\ (x-20)(x+3) & \\ x &= 20, -3 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 2x^2 - 7x - 15 &= 0 \\ (2x+3)(x-5) & \\ x &= -\frac{3}{2}, 5 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 6(x+7) &= 4(4+8) \\ 6x+42 &= 48 \\ 6x &= 6 \\ x &= 1 \end{aligned}$$

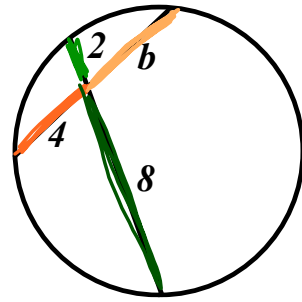
$$\begin{aligned} \textcircled{4} \quad x(x+6) &= 55 \\ x^2+6x-55 &= 0 \\ (x+11)(x-5) &= 0 \\ x &= -11, 5 \end{aligned}$$

Theorem #1 Chord/Chord



$$\underline{a} \cdot \underline{b} = \underline{c} \cdot \underline{d}$$

Example:



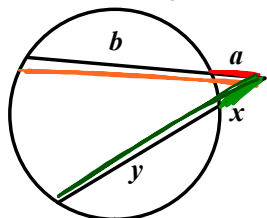
$$4 \cdot b = 2 \cdot 8$$

$$4 \cdot b = 16$$

$$\frac{4 \cdot b}{4} = \frac{16}{4}$$

$$b = 4$$

Theorem #2 Secant/Secant

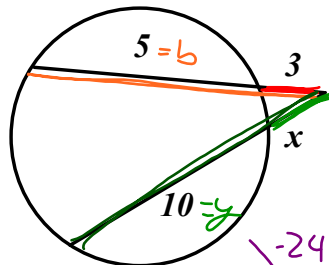


external part *entire secant* *other external part* *other entire secant*

$$\underline{a} \cdot (\underline{a+b}) = \underline{x} \cdot (\underline{x+y})$$

Part (Whole) = Part (Whole)

Example:



~~$$\begin{matrix} -24 & & \\ (12) & \cdot & (-2) \\ \hline & & 10 \end{matrix}$$~~

$$3 \cdot (3+5) = x(x+10)$$

$$3 \cdot 8 = x^2 + 10x$$

$$24 = x^2 + 10x$$

$$-24 \qquad \qquad \qquad -24$$

$$0 = x^2 + 10x - 24$$

$$0 = (x+12)(x-2)$$

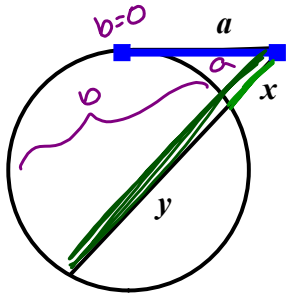
$$x+12=0 \quad x-2=0$$

~~$$-12 \quad -12$$~~
~~$$x = -12$$~~
~~$$+2 \quad +2$$~~

$$x = 2$$

Can't have negative length

Theorem #3. Secant/Tangent

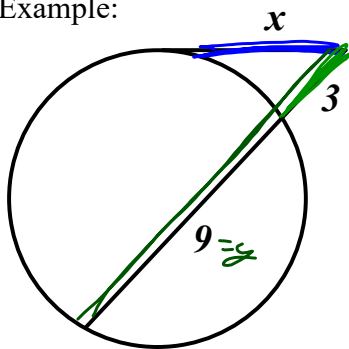


$a(a+b)$
 $a(a+0)$
 $a(a)$

$a^2 = x \cdot (x + y)$

tangent = part(whole)

Example:



$x^2 = 3(3+9)$

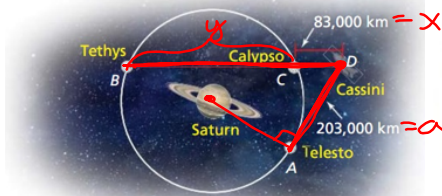
$x^2 = 3(12)$

$\sqrt{x^2} = \sqrt{36}$

$x = \pm 6$

$x = 6$ and $x = -6$

16. **MODELING WITH MATHEMATICS** The Cassini spacecraft is on a mission in orbit around Saturn until September 2017. Three of Saturn's moons, Tethys, Calypso, and Telesto, have nearly circular orbits of radius 295,000 kilometers. The diagram shows the positions of the moons and the spacecraft on one of Cassini's missions. Find the distance DB from Cassini to Tethys when AD is tangent to the circular orbit. (See Example 4.)



tangent/secant
 $a^2 = x(x+y)$

$203000^2 = 83000(83000+y)$

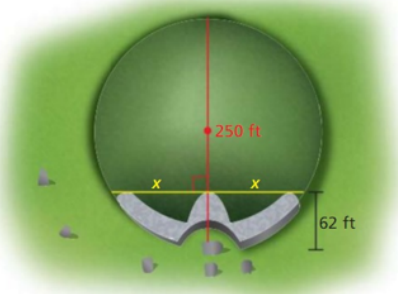
$203000^2 = 83000^2 + 83000y$
 $- 83000^2 \quad - 83000^2$

$\frac{3432000000}{83000} = \frac{83000y}{83000}$

$413493.98 \text{ km} = y$

$AD = x + y = 496493.98 \text{ km}$

17. **MODELING WITH MATHEMATICS** The circular stone mound in Ireland called Newgrange has a diameter of 250 feet. A passage 62 feet long leads toward the center of the mound. Find the perpendicular distance x from the end of the passage to either side of the mound.



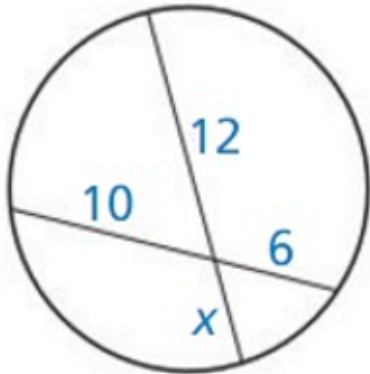
3.432×10^{10}
 3.432×10^{10}

Practice

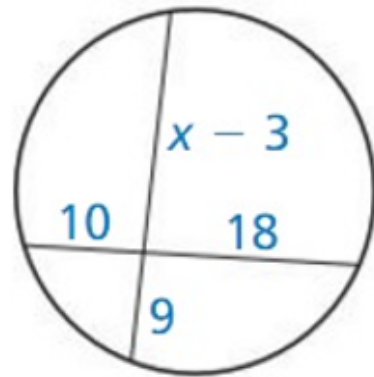
Option A

All from Textbook 10.6 pg. 573

3.

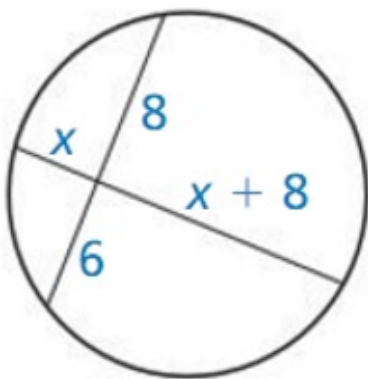


4.

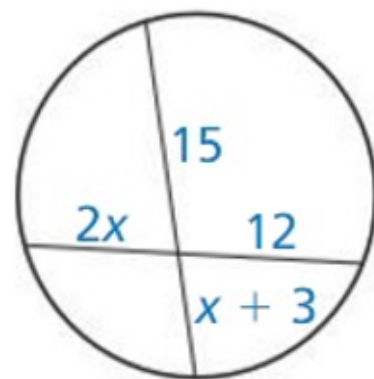


Option B

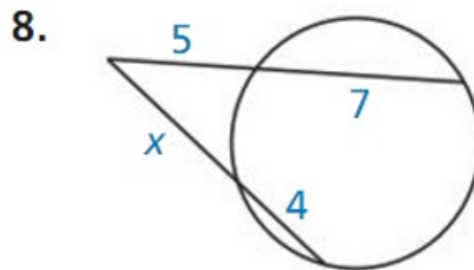
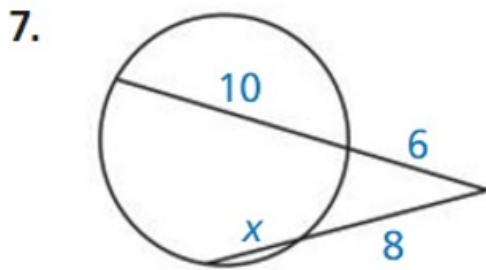
5.



6.

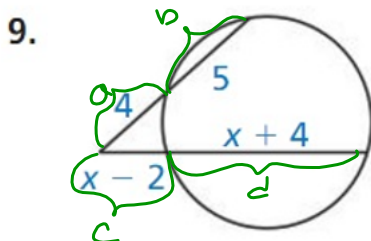


Option A



$$5(5+7) = x(x+4)$$

Option B



$$a(a+b) = c(c+d)$$

$$4(4+5) = (x-2)(x-2+x+4)$$

$$4 \cdot 9 = (x-2)(2x+2)$$

$$36 = 2x^2 + 2x - 4x - 4$$

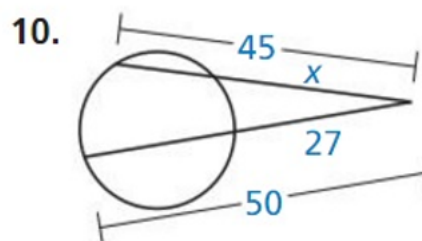
$$0 = \frac{2x^2 - 2x - 40}{2}$$

$$0 = x^2 - x - 20$$

$$0 = (x-5)(x+4)$$

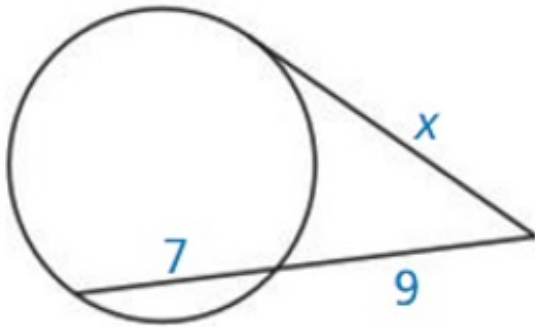
$$x-5=0 \quad x+4=0$$

$$\boxed{x=5} \quad x \neq -4$$

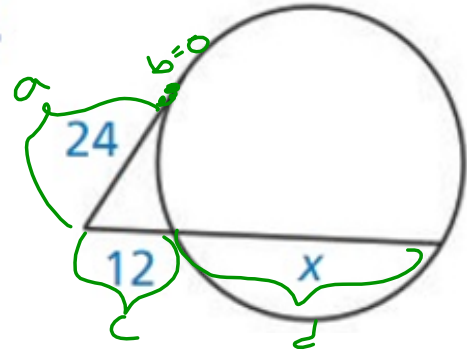


Option A

11.



12.



$$a^2 = c(c+d)$$

$$24^2 = 12(12+x)$$

$$576 = 144 + 12x$$

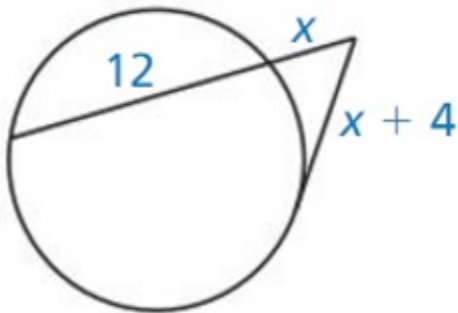
$$\begin{array}{r} -144 \\ -144 \end{array}$$

$$432 = 12x$$

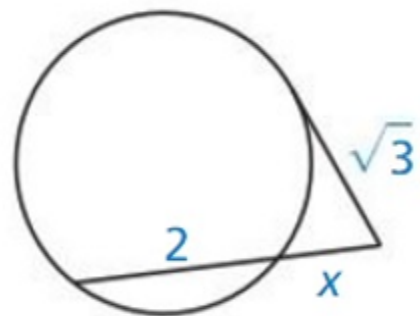
$$36 = x$$

Option B

13.



14.



Key

Odd answers can be found in the back of the textbook or using the online answer key link on my website.

3. $x \cdot 12 = 10 \cdot 6$
 $12x = 60$
 $x = 5$

6. 5	5. $x = 4$ $x = -12$ A negative value for x does not make sense. solution is $x = 4$.	8. 6	7. $x = 4$
10. 30	9. $x = -4$ $x = 5$ A negative value for x does not make sense. solution is $x = 5$.	11. $x^2 = 9 \cdot 16$ $x = \sqrt{144} = \pm 12$	12. 36
14. 1	13. $x = 4$	16. about 496,494 km	A negative value for x does not make sense. solution is $x = 12$.
17. $x \cdot x = 188 \cdot 62$ $x^2 = 11,656$ $x \approx \pm 107.96$ A negative distance does not make sense. So, the distance from the end of the passage to either side of the mound is approximately 108 feet.		18. It takes the sparkles 3 seconds to move from point C to point D . Because $CN = 12$ cm and the sparkles have 3 seconds to move from point C to point N , the sparkles need to move at a speed of 4 centimeters per second from point C to point N .	