Header: Your Name

Mrs. Theo

/ /

Notes

3-4 and 3-5

Zeros of Polynomial Functions Theorems about Roots and Polynomial Equations

Rational Root Theorem

Helps Factor

Using Synthetic Division

determine possible factors/roots by

Dividing

the constant term's factors by the

Polynomial's leading coefficient's factors

The positive and negative versions of every factor and fraction of that quotient are the possible solutions you should start checking for.

You can use a graph to help you pick, or check them in order using synthetic division

Workbook pg. 84

EXAMPLE 1 Identify Possible Rational Solutions

Try It!

1. List all the possible rational solutions for each equation.

a. $4x^4 + 13x^3 - 124x^2 + 212x - 8 = 0$

factors 8: 1, 2, 4, 8

factors of 4: 1, 2, 4

possible roots?

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 4, \pm \frac{8}{1}$

b. $7x^4 + 13x^3 - 124x^2 + 212x - 45 = 0$

factors of 45: 1, 3, 5, 9, 15, 45

factors of 7: 1, 7

$\pm 1, \pm \frac{1}{7}, \pm 3, \pm \frac{3}{7}, \pm 5, \pm \frac{5}{7}, \pm 9, \pm \frac{9}{7}, \pm 15, \pm \frac{15}{7}, \pm 45, \pm \frac{45}{7}$

EXAMPLE 2 Use the Rational Root Theorem

Try It!

2. A jewelry box measures $2x + 1$ in. long, $2x - 6$ in. wide, and x in. tall. The volume of the box is given by the function $v(x) = 4x^3 - 10x^2 - 6x$.

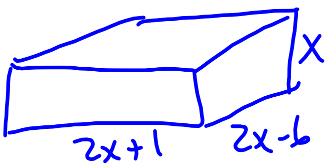
What is the height of the box, in inches, if its volume is 28 in^3.? $= V(x)$

$x = ?$

$28 = 4x^3 - 10x^2 - 6x$
$-28 \qquad\qquad -28$

$0 = 4x^3 - 10x^2 - 6x - 28$

$\pm 1 \ \pm\frac{1}{2} \ \pm\frac{1}{4} \ \pm 2 \ \pm 4 \ \pm 7 \ \pm\frac{7}{2} \ \pm\frac{7}{4} \ \pm 14 \ \pm 28$



check $x = \frac{7}{2}$

$$\begin{array}{r|rrrr} \frac{7}{2} & 4 & -10 & -6 & -28 \\ & \downarrow & 14 & 14 & 28 \\ \hline & 4 & 4 & 8 & \boxed{0} \end{array}$$

The box is 3.5 inch tall

Yes $\frac{7}{2}$ is a solution ✓

Workbook pg. 84

EXAMPLE 3 Find All Complex Roots

Try It!

3. What are all the complex roots of the equation $x^3 - 2x^2 + 5x - 10 = 0$?

Possible Roots: $\pm 1, \pm 2, \pm 5, \pm 10$

3 solutions

$x = 2$	1	-2	5	-10
	↓	2	0	10
	1	0	5	0

$(x-2)(x^2+5)=0$

$x-2=0$

$\boxed{x=2}$

$x^2+5=0$

$x^2=-5$

$x=\pm\sqrt{-5}$

$\boxed{x=\pm 2.236i}$

Integer Root Theorem

Helps Factor

Using Synthetic Division

± counting numbers

The Integer Root Theorem is used to determine possible interger factors/roots of a polynomial

Divide the constant term by the leading coefficient of the polynomial. The positive and negative versions of every factor of that quotient are the possible solutions you should start checking for.

Leading Coefficient

in front of highest degree term

term w/o a variable Constant

Factor: $2x^4 + 7x^3 - 39x^2 + 62x - 56$

$\frac{-56}{2} = -28 \quad \pm1, \pm2, \pm4, \pm7, \pm14, \pm28$

$$
\begin{array}{r|rrrrr}
+2 & 2x^4 & 7x^3 & -39x^2 & 62x & -56 \\
 & \downarrow & 4 & 22 & -34 & 56 \\
\hline
-7 & 2x^3 & 11x^2 & -17x & 28 & \boxed{0} \\
 & \downarrow & -14 & 21 & -28 & \\
\hline
 & 2 & -3 & 4 & \boxed{0} &
\end{array}
$$

X-2 is factor

$\boxed{x = 2 \text{ is a solution}}$

X+7 is a factor

$\boxed{x = -7 \text{ is a solution}}$

$(x-2)(x+7)(2x^2 - 3x + 4)$

Once you get down to a Quadratic Factor or use Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

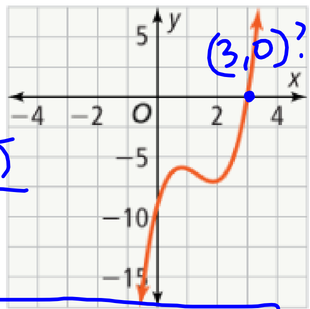
$$\boxed{x = \frac{3 \pm i\sqrt{23}}{4},\ x = -7,\ x = 2}$$

Pg. 78

EXAMPLE 3 Find Real and Complex Zeros

Try It!

3. What are all the real and complex zeros of the polynomial function shown in the graph?



a. $f(x) = 2x^3 - 8x^2 + 9x - 9$

$3 \,|\, 2 \quad -8 \quad 9 \quad -9$

$\downarrow \quad 6 \quad -6 \quad 9$

$2x^2 - 2x + 3 \quad \boxed{0}$

$a=2 \quad b=-2 \quad c=3$

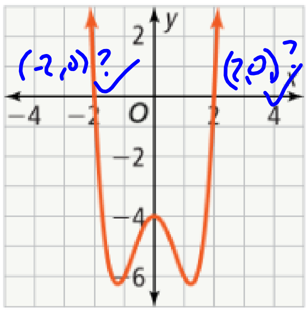
$\boxed{x=3}$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(3)}}{2(2)}$

$x = \dfrac{2 \pm \sqrt{-20}}{4}$

$x = \dfrac{2 \pm 4.472i}{4} \rightarrow \boxed{x = 0.5 \pm 1.118i}$

3. What are all the real and complex zeros of the polynomial function shown in the graph?



b. $f(x) = x^4 - 3x^2 - 4$

$2 \,|\, 1 \quad 0 \quad -3 \quad 0 \quad -4$

$\downarrow \quad 2 \quad 4 \quad 2 \quad 4$

$-2 \,|\, 1 \quad 2 \quad 1 \quad 2 \quad \boxed{0}$

$\downarrow \quad -2 \quad 0 \quad -2$

$1 \quad 0 \quad 1 \quad \boxed{0}$

$x^2 + 1$

$\boxed{\begin{matrix} x = 2 \\ x = -2 \end{matrix}}$

$x^2 + 1$

$(x+i)(x-i)$

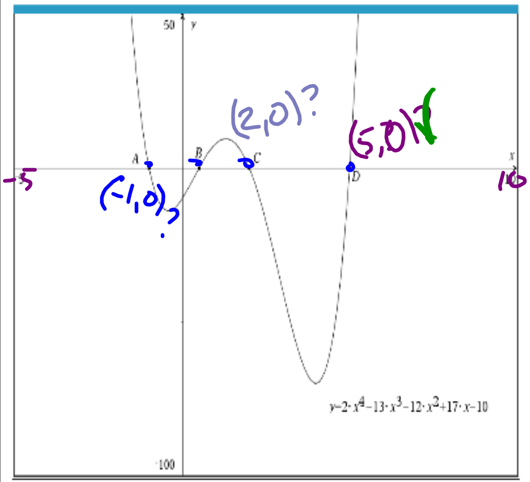
$\boxed{x = \pm i}$

Name____ Rational Root Theorem Practice

factors of 10: 1, 2, 5, 10
factors of 2: 1, 2

Polynomial Function 1

$f(x) = 2x^4 - 13x^3 + 12x^2 + 17x - 10$



List the POSSIBLE RATIONAL ROOTS

$\pm 1, \pm \frac{1}{2}, \pm 2, \pm 5, \pm \frac{5}{2}, \pm 10$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

5 | 2 -13 12 17 -10
↓ 10 -15 -15 10
2 | 2 -3 -3 2 | 0
↓ 4 2 -2
-1 | 2 1 -1 | 0
↓ -2 1
2x-1 | 0

$2x - 1 = 0$
$2x = 1$
$x = \frac{1}{2}$

Are all the roots present?

Yes all 4

Are any of the roots irrational?

No

Are any of the roots complex

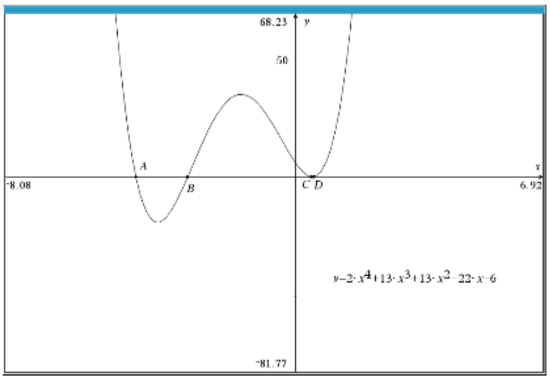
No

List A, B, C, and D as coordinates

D (5,0) B(1/2,0)
C (2,0) A (-1,0)

Polynomial Function 2

$f(x) = 2x^4 + 13x^3 + 13x^2 - 22x + 6$



List the POSSIBLE RATIONAL ROOTS

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

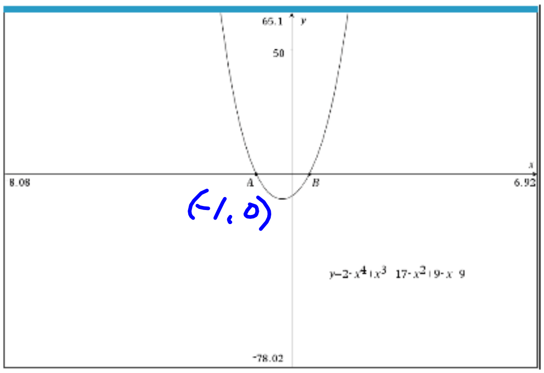
Are all the roots present?

Are any of the roots irrational?

Are any of the roots complex

List A, B, C, and D as coordinates

Polynomial Function 3



$(-1,0)$

Degree 4 Quartic

$f(x) = 2x^4 + x^3 + 17x^2 + 9x - 9$

9: 1, 3, 9

2: 1, 2

List the POSSIBLE RATIONAL ROOTS

$\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

$x = -1$

$x + 1 = 0$

$$\begin{array}{r|rrrrr} -1 & 2 & 1 & 17 & 9 & -9 \\ & \downarrow & -2 & 1 & -18 & 9 \\ \hline & 2x^3 & -x^2 & 18x & -9 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & 18 & -9 \\ & \downarrow & 1 & 0 & +9 \\ \hline & 2 & 0 & 18 & \boxed{0} \end{array}$$

Are all the roots present?

look at Degree

Deg is 4, should be 4 solutions

No, only 2 x intercepts

Are any of the roots irrational?

$x = \frac{1}{2}$

$$\begin{array}{r|l} & x^2 \quad\quad + 9 \\ \hline 2x - 1 & 2x^3 - 1x^2 + 18x - 9 \\ & -(2x^3 - x^2) \\ \hline & 0 \quad\quad 18x - 9 \\ & -(18x - 9) \\ \hline & 0 \end{array}$$

Are any of the roots complex

Yes, 2 are complex

$(x+1)(2x-1)(x^2+9)$

$(x+3i)(x-3i)$

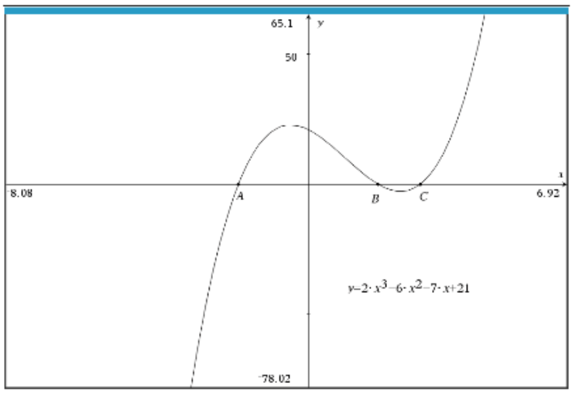
List A and B as coordinates

$A(-1,0)$ $B(\frac{1}{2}, 0)$

$x = 3i$ and

$x = -3i$

Polynomial Function 4



$f(x) = 2x^3 - 6x^2 - 7x + 21$

List the POSSIBLE RATIONAL ROOTS

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

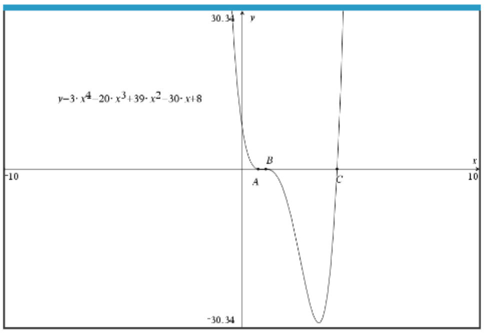
Are all the roots present?

Are any of the roots irrational?

List A, B, and C as coordinates

Are any of the roots complex

Polynomial Function 5



List A, B, and C as coordinates

$f(x) = 3x^4 - 20x^3 + 39x^2 - 30x + 8$

List the POSSIBLE INTEGER ROOTS

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

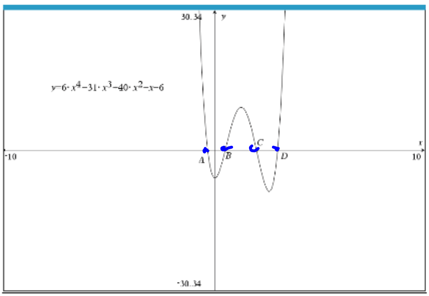
Are all the roots present?

Are any of the roots irrational?

Are any of the roots complex

6: 1, 2, 3, 6
6: 1, 2, 3, 6

Polynomial Function 6



List A, B, C, and D as coordinates

(3, 0)
$\left(-\frac{1}{3}, 0\right)$
$\left(\frac{1}{2}, 0\right)$

$f(x) = 6x^4 - 31x^3 + 40x^2 - x - 6$

List the POSSIBLE INTEGER ROOTS

$\frac{6}{6}: \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 2, \pm\frac{2}{3}, \pm 3, \pm\frac{3}{2}, \pm 6$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

$$\begin{array}{r|rrrrr} 3 & 6 & -31 & 40 & -1 & -6 \\ & \downarrow & 18 & -39 & 3 & 6 \\ \hline 2 & 6 & -13 & 1 & 2 & \boxed{0} \\ & \downarrow & 12 & -2 & -2 & \\ \hline & 6 & -1 & -1 & \boxed{0} & \end{array}$$

Are all the roots present?

Are any of the roots irrational?

Are any of the roots complex

$6x^2 - 1x - 1 = 0$ (a, b, c)

$(3x+1)(2x-1)=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

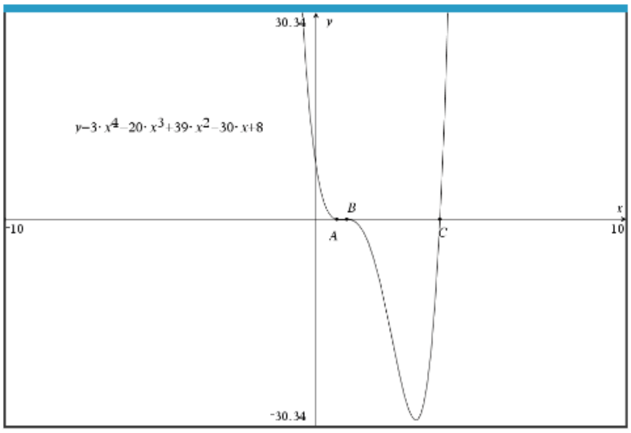
$$x = \frac{-(-1) \pm \sqrt{(-1)^2-4(6)(-1)}}{2(6)}$$

$$x = \frac{1 \pm \sqrt{25}}{12}$$

$$x = \frac{1+5}{12} \qquad x = \frac{1-5}{12}$$

$$x = \frac{1}{2} \qquad x = -\frac{1}{3}$$

Polynomial Function 5



List A, B, and C as coordinates

$$f(x) = 3x^4 - 20x^3 + 39x^2 - 30x + 8$$

List the POSSIBLE INTEGER ROOTS

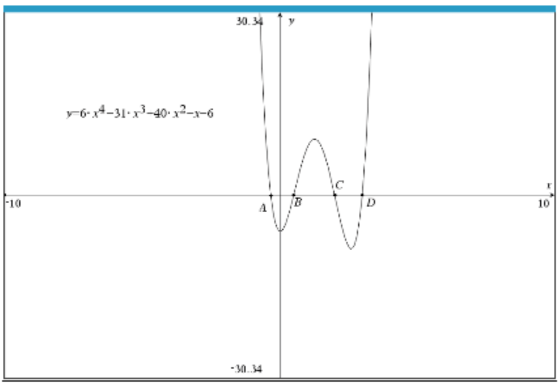
Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

Are all the roots present?

Are any of the roots irrational?

Are any of the roots complex

Polynomial Function 6



List A, B, C, and D as coordinates

$$f(x) = 6x^4 - 31x^3 + 40x^2 - x - 6$$

List the POSSIBLE INTEGER ROOTS

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

Are all the roots present?

Are any of the roots irrational?

Are any of the roots complex

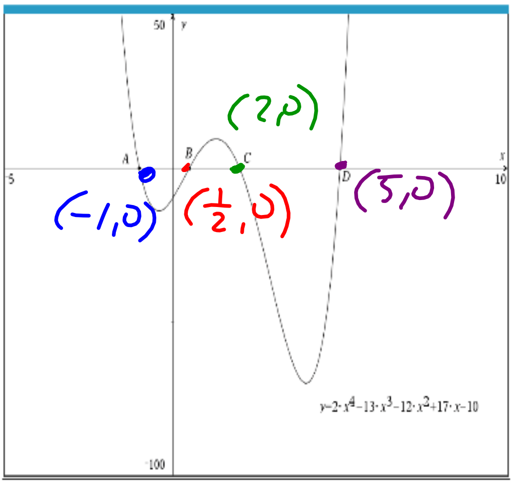
Factors of
p: 1, 2, 5, 10
2: 1, 2

Name____ Rational Root Theorem Practice

Polynomial Function 1

Lead coefficient   Constant

$f(x) = 2x^4 - 13x^3 + 12x^2 + 17x - 10$



List the POSSIBLE RATIONAL ROOTS

$\dfrac{\pm \text{each factor of constant}}{\text{each factor of Lead coefficient}}$

$\pm \frac{1}{1}, \frac{2}{1}, \frac{5}{1}, \frac{10}{1}, \frac{1}{2}, \frac{2}{2}, \frac{5}{2}, \frac{10}{2}$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

$-1 \,|\, 2x^4 \; -13x^3 \; 12x^2 \; 17x \; -10$
$\quad\;\; -2 \quad 15 \quad -27 \quad 10$
$5 \,|\, 2x^3 \; -15x^2 \; 27x \; -10 \,|\, 0$
$\quad\;\; 10 \quad -25 \quad 10$
$2 \,|\, 2 \quad -5 \quad 2 \,|\, 0$
$\quad\;\; 4 \quad -2$
$\quad 2 \quad -1 \,|\, 0$

$2x - 1 = 0$
$2x = 1$
$x = \frac{1}{2}$

Are all the roots present?

Yes

Are any of the roots irrational?

No

Are any of the roots complex

No

List A, B, C, and D as coordinates

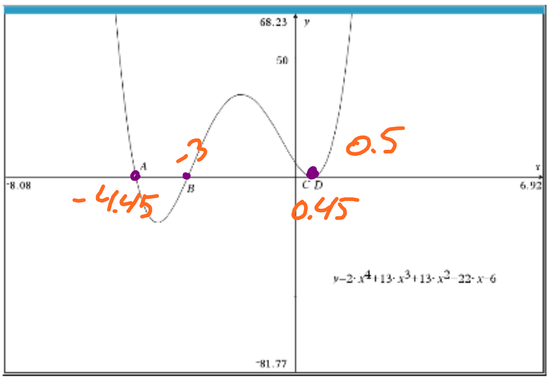
Polynomial Function 2

$\dfrac{\pm \text{constant factors}}{\text{lead coefficient factors}}$

$f(x) = 2x^4 + 13x^3 + 13x^2 - 22x + 6$

$(x+3)(2x-1)(x^2+4x-2)$

6: 1, 2, 3, 6
2: 1, 2



List the POSSIBLE RATIONAL ROOTS

$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2},$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

$-3 \,|\, 2x^4 \; 13 \; 13 \; -22 \; 6$
$\quad\;\; -6 \quad -21 \quad 24 \quad -6$
$\frac{1}{2} \,|\, 2x^3 + 7x^2 - 8x + 2 \,|\, 0$
$\quad\;\; 1 \quad 4 \quad -2$
$\quad 2 \quad 8 \quad -4 \,|\, 0$ ✓

Are all the roots present?

Degree is 4

We see all 4 x intercepts

Are any of the roots irrational?

a= 1, b= 4, c= -2: $1x^2 + 4x - 2$

$2x-1 \,|\, 2x^3 + 7x^2 - 8x + 2$
$-(2x^3 + x^2)$
$6x^2 - 8x + 2$
$-(6x^2 + 4x)$

Are any of the roots complex

No, we see all 4 xint.

List A, B, C, and D as coordinates

B(-3,0) $x=-3 \rightarrow x+3=0$

D($\frac{1}{2}$,0) $x = \frac{1}{2} \rightarrow 2(x - \frac{1}{2}) = 0 \cdot 2$

$2x - 1 = 0$

A(-4.45,0)

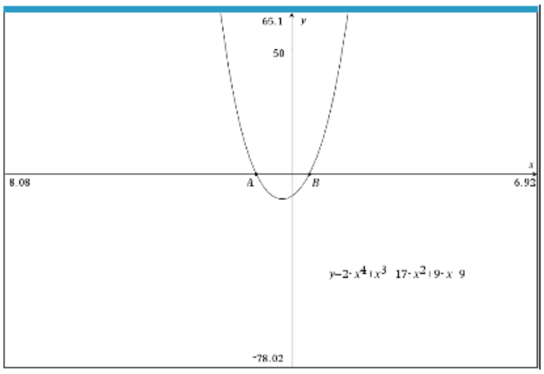
C(0.45,0)

$x = \dfrac{-(4) \pm \sqrt{(4)^2 - 4(1)(-2)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{24}}{2}$

$x = \dfrac{(-4 + \sqrt{24})}{2} =$ $\quad x = \dfrac{(-4 - \sqrt{24})}{2}$

Polynomial Function 3



$f(x) = 2x^4 + x^3 + 17x^2 + 9x - 9$

List the POSSIBLE RATIONAL ROOTS

$\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

$-1 \,|\, 2 \quad 1 \quad 17 \quad 9 \quad -9$

$\downarrow \quad -2 \quad 1 \quad -18 \quad 9$

$\frac{1}{2} \,|\, 2 \quad -1 \quad 18 \quad -9 \quad \boxed{0}$

$\downarrow \quad 1 \quad 0 \quad 9$

$2 \quad 0 \quad 18 \quad \boxed{0}$

$x^2 + 9$

$2x-1 \,|\, 2x^3 - 1x^2 + 18x - 9$

$-(2x^3 - x^2)$

$0 + 18x - 9$

$-(18x - 9)$

0

Are all the roots present?

No

Are any of the roots irrational?

Are any of the roots complex

2

$x = 3i$ and $x = -3i$

List A and B as coordinates

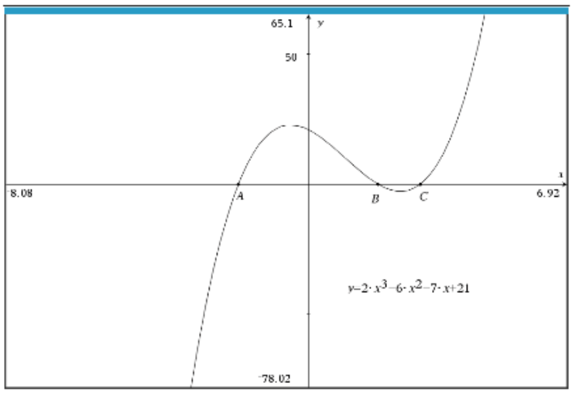
$A(-1, 0)$

$B(\frac{1}{2}, 0)$

$(x+1)(2x-1)(x^2+9)$

$(x+3i)(x-3i)$

Polynomial Function 4



$f(x) = 2x^3 - 6x^2 - 7x + 21$

List the POSSIBLE RATIONAL ROOTS

$\pm 1, \pm 3, \pm 7, \pm 21, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2}$

Show how either synthetic division or the rational root theorem can be used to look for roots of f(x)

$3 \,|\, 2 \quad -6 \quad -7 \quad 21$

$\downarrow \quad 6 \quad 0 \quad -21$

$2 \quad 0 \quad -7 \quad \boxed{0}$

Are all the roots present?

Yes

$(x-3)(2x^2-7)$

Are any of the roots irrational?

Yes

$2x^2 - 7 = 0$

$2x^2 = 7$

$x^2 = 7/2$

$x = \pm\sqrt{3.5}$

Are any of the roots complex

No

List A, B, and C as coordinates

$C(3, 0)$

$B(1.871, 0)$

$A(-1.871, 0)$