Your Name

Mrs. Theo

12/5/22

Notes

2-3, 3-1 and 3-5

End Behavior and Intercepts of Polynomial Functions

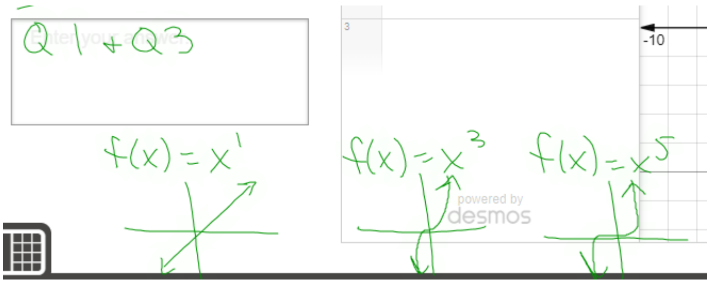
EXPLORE & REASON

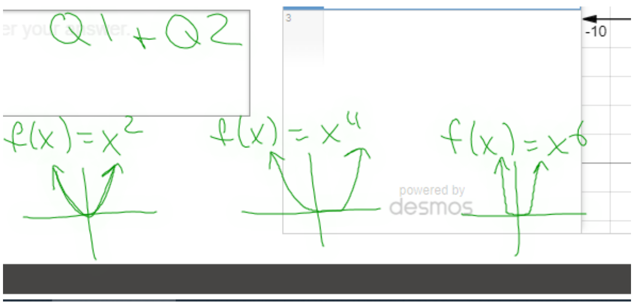
Workbook pg. 61

Consider functions of the form $f(x) = x^n$, where n is a positive integer.

A. Use the tool to graph $f(x) = x^n$ for n = 1, 3 and 5. Look at the graphs in Quadrant I. As the exponent increases, what is happening to the graphs? Which quadrants do the graphs pass through?

B. Look for Relationships Now graph $f(x) = x^n$ for n = 2, 4, and 6. What happens to these graphs in Quadrant I as the exponent increases? Which quadrants do the graphs pass through?





EXAMPLE 1 Classify Polynomials

Standard Form

terms in order from highest to lowest exponent

Lead Coefficient

infront of variable with biggest exponent

Degree

biggest exponent

Terms

separated by + or −

Try It!

1. What is each polynomial in standard form and what are the leading coefficient, the degree, and the number of terms of each?

a. $2x - 3x^4 + 6 - 5x^3$

$-3x^4 - 5x^3 + 2x + 6$

Lead Coeff: −3

Deg: 4

# of Terms: 4

b. $x^5 + 2x^6 - 3x^4 - 8x + 4x^3$

$2x^6 + x^5 - 3x^4 + 4x^3 - 8x$

Lead Coeff: 2

Deg: 6

# of Terms: 5

End behavior

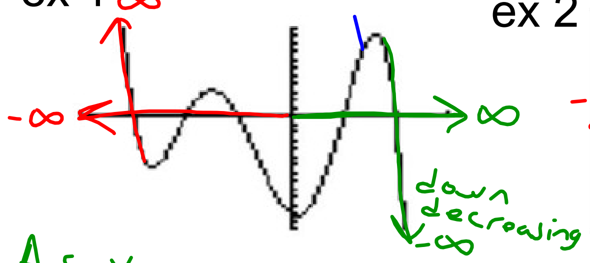
What happens towards the ends of the function

As x approaches ∞, where are the y values headed?

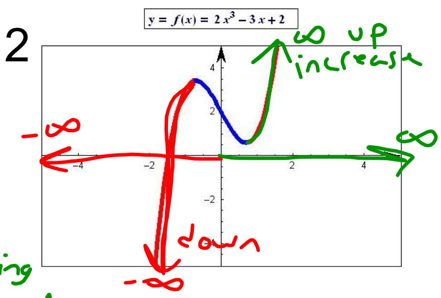
As x → ∞ y → ?

As x → -∞ y → ?

ex 1



ex 2



As x→∞ y→-∞

As x→-∞ y→∞

As x→∞ y→∞

As x→-∞ y→-∞

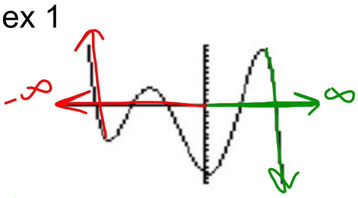
Another way to write it...

The y values are going where, as x approaches ∞?

$f(x) \rightarrow ?$ As $x \rightarrow \infty$

$f(x) \rightarrow ?$ As $x \rightarrow -\infty$

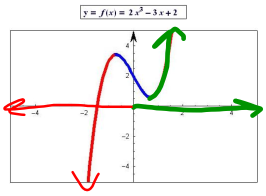
ex 1



$f(x) \rightarrow -\infty$ As $x \rightarrow \infty$

$f(x) \rightarrow \infty$ As $x \rightarrow -\infty$

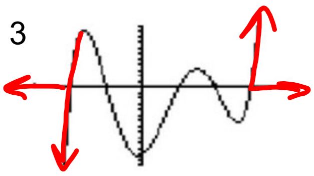
ex 2



$f(x) \rightarrow \infty$ As $x \rightarrow \infty$

$f(x) \rightarrow -\infty$ As $x \rightarrow -\infty$

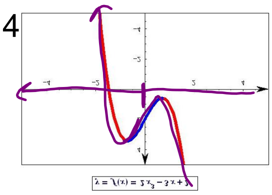
ex 3



$f(x) \rightarrow \infty$ As $x \rightarrow \infty$

$f(x) \rightarrow -\infty$ As $x \rightarrow -\infty$

ex 4



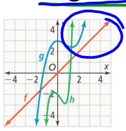
$f(x) \rightarrow -\infty$ As $x \rightarrow \infty$

$f(x) \rightarrow \infty$ As $x \rightarrow -\infty$

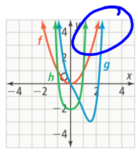
Lead Coefficient of a function: determines right side of end behavior

Positive - right side up

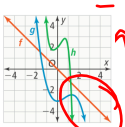
$3x^1$ $0.5x^3$ $2.7x^2$



$\frac{1}{4}x^2$

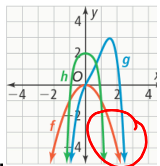


Negative - right side down



$-7x^3$

$-0.8x^4+50x^3$



Degree of a function:

* the highest exponent in standard form
* determines solutions and end behavior

Even degree both ends are the same

x^2 or $-x^2$ $-x^4$ or x^4

Odd Degree opposite

x^1 or $-3x$ x^3 or $-x^3$ $+x^5$

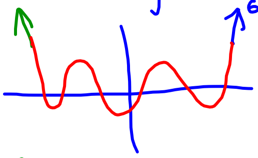
EXAMPLE 2 Understand End Behavior of Polynomial Functions

2. Use the leading coefficient and degree of the polynomial function to determine the end behavior of each graph.

a. $f(x) = 2x^6 - 5x^5 + 6x^4 - x^3 + 4x^2 - x + 1$

Lead: 2 → positive right side up ∞

Deg: 6 → even left side same



as $x \to \infty$ $y \to \infty$
as $x \to -\infty$ $y \to \infty$

b. $g(x) = -5x^3 + 8x + 4$

Lead Coef: -5 → negative right side down

Deg: 3 → odd left side is opposite up

as $x \to \infty$ $y \to -\infty$
as $x \to -\infty$ $y \to \infty$

EXAMPLE 4 Sketch the Graph from a Verbal Description

CO

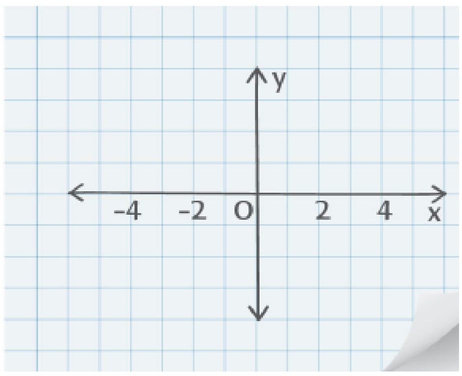
- $f(x)$ is positive on the intervals $(-\infty, -4)$ and $(-1, 4)$.
- $f(x)$ is negative on the intervals $(-4, -1)$ and $(4, \infty)$.
- $f(x)$ is decreasing on the intervals $(-\infty, -2.67)$ and $(2, \infty)$.
- $f(x)$ is increasing on the interval $(-2.67, 2)$.

▶ STEP 1: Identify or estimate x-intercepts.

▶ STEP 2: Identify or estimate turning points.

▶ STEP 3: Evaluate end behavior.

STEP 4: Sketch the graph.

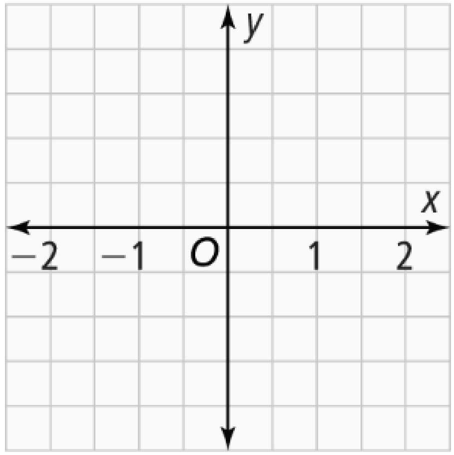


EXAMPLE 4 Sketch the Graph from a Verbal Description ESS

Try It!

4. Use the information below to sketch a graph of the polynomial function $y = f(x)$.

- $f(x)$ is positive on the intervals $(-2, -1)$ and $(1, 2)$.
- $f(x)$ is negative on the intervals $(-\infty, -2)$, $(-1, 1)$, and $(2, \infty)$.
- $f(x)$ is increasing on the intervals $(-\infty, -1.5)$ and $(0, 1.5)$.
- $f(x)$ is decreasing on the intervals $(-1.5, 0)$ and $(1.5, \infty)$.



EXAMPLE 5 Interpret a Polynomial Model

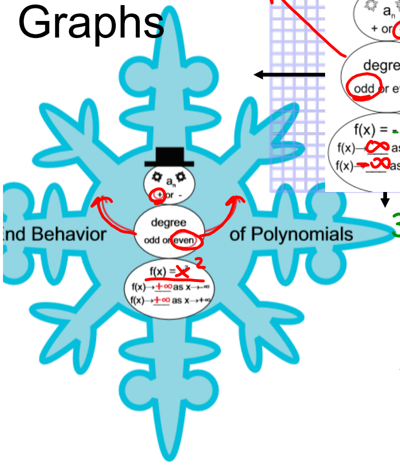
Try It!

5. Danielle is engineering a new brand of shoes. For x shoes sold, in thousands, a profit of $p(x) = -3x^4 + 4x^3 - 2x^2 + 5x + 10$ dollars, in ten thousands, will be earned.

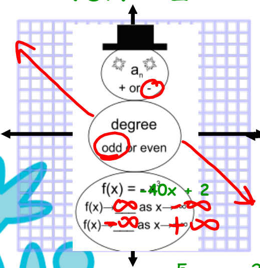
a. How much will be earned in profit for selling 1,000 shoes?

b. What do the x- and y-intercepts of the graph mean in this context? Do those values make sense?

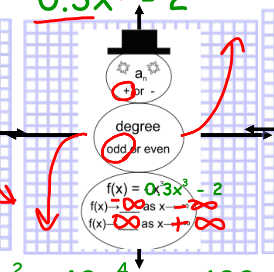
Sketching Graphs



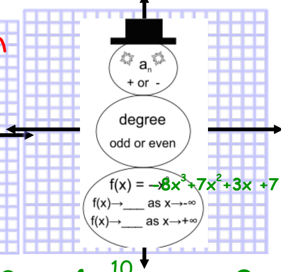
$-40x^{1} + 2$



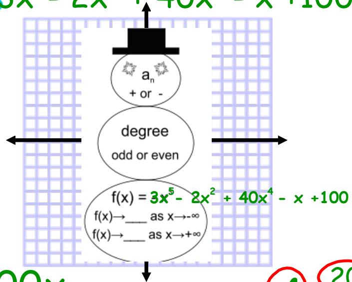
$0.3x^{3} - 2$



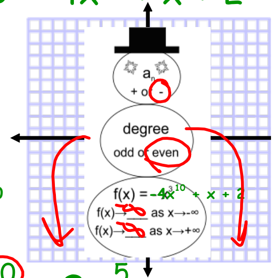
$-8x^3+7x^2 + 3x + 7$



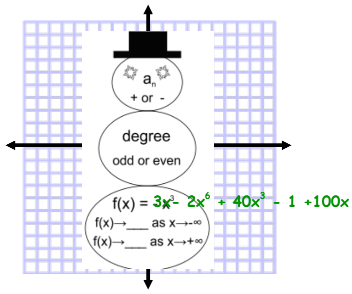
$3x^5 - 2x^2 + 40x^4 - x + 100$



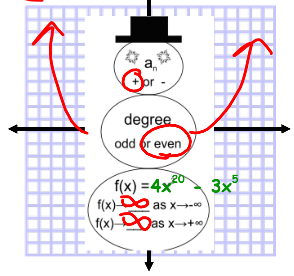
$-4x^{10} + x + 2$



$3x - 2x^6 + 40x^3 - 1 + 100x$

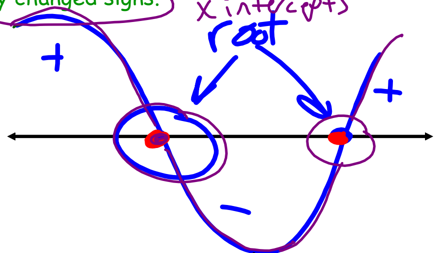


$4x^{20} - 3x^5$



Intermediate Value Theorem Location Principle

If there are positive and negative intervals on the graph, then there is a point where the function P(x) crosses the x axis. Basically, there is a root where y = 0 between when y changed signs.



x	y	
-10	-20	–
-8	-4	–
-6	-1	–
-4	0	
-2	1	+
0	7	+
2	8	+
4	7	+
6	2	+
8	-4	–
10	-13	–

Here is a x intercept/root
X=-4 b/c y=0

Between 6 and 8
there is a root
we see change
in sign of y

x	y	
-10	-30	–
-8	-14	–
-6	-1	–
-4	-6	–
-2	-2	–
0	5	+
2	0	
4	5	+
6	8	+
8	1	+
10	-3	–

root b/w -2 and 0
• changed signs of y

y intercept

x intercept
x=2
y=0

root b/w 8 and 10
• Changed y signs

This P(x) has 2 solutions/roots Zeros

This P(x) has 3 roots/ Zeros/solutions

Solution/ Root/ Zero

The x intercept, x=____, use the factor (x-c) set it =0 and solve for x

Multiplicity

- A solution is repeated if(x-c) is a factor more than once
- the # of times (x-c) it is a factor is its multiplicity m

ex. $(x-4)^3$ 4 is a zero of P with multiplicity 3

$x-4=0$ $(x-4)(x-4)(x-4)$

$x=4$ $m=3$ $x=4$ $x=4$ $x=4$

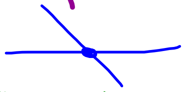
ex. $(x+2)^4$

$x+2=0$

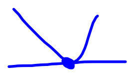
$-2\ \ -2$

$x=-2$ with $m=4$

Pass • If **m is 1** it goes right through

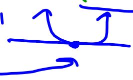


Bounce • If **m is 2** it changes direction (bounces)



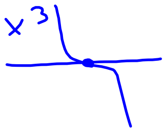
Flattened Bounce • If **m is even** it changes direction (flattens at the bounce)

$m=4,6,8\ldots$ Flatten bounce x^4



Flattened Pass • If **m is odd** it will keep its directions (passes through) but it will flatten at the zero

$m=3,5,7\ldots$ x^3



Sketching Polynomial Graphs

1. Plot x intercept
2. Plot end behavior
3. Plot y intercept
4. Sketch passes and bounces

$f(x) = x(x-3)(x+3)$

$x=0$ $x-3=0$ $x+3=0$

$+3\ \ +3$ $-3\ \ -3$

$x=3$ $x=-3$

Degree:

Zeros and Multiplicities

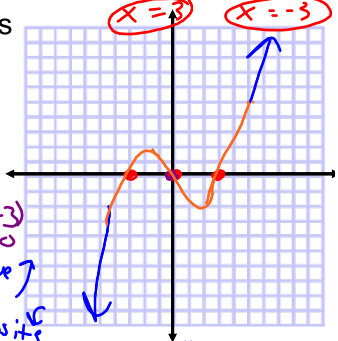
$x=3$	$x=0$	$x=3$
$m=1$	$m=1$	$m=1$
pass	pass	pass

y intercept: $(0,0)$ $y=0(0-3)(0+3)$ $0\cdot -3\cdot 3=0$

End behavior Lead C: $a=1$ positive

as x->∞ y->∞ Deg: 3 x^3 odd

as x->$-\infty$ y->$-\infty$ opposite



$f(x) = 0.4x^2(x-5)(x+2)$

$x^2=0$ $x-5=0$ $x+2=0$

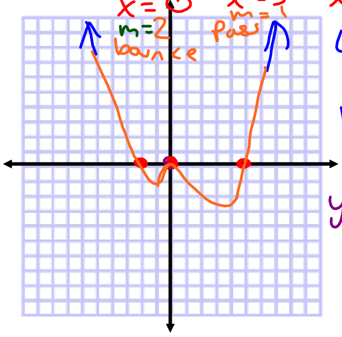
$x=0$ $x=5$ $x=-2$

$m=2$ bounce $m=1$ pass $m=1$ pass

Lead Coef: $a=0.4$ positive

Deg: 4 x^4 even same as right

y int $(0,0)$



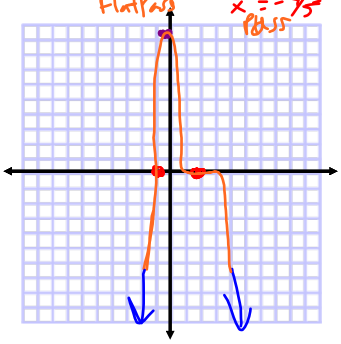
$f(x) = -3(x-2)^3(5x+4)$ Deg: 4 match

y int: $(0,96)$ $-3(0-2)^3(5(0)+4)$

$x-2=0$ $5x+4=0$

$x=2$ $5x=-4$

Flat Pass $x=-4/5$ Pass



Homework

8. $y = -3(x-2)^3(5x+4)$

Lead Coef:

Degree:

End behavior

as x-> __ y->__

as x-> __ y->__

Zeros and Multiplicities

y intercept:

9. $y = -x(x+5)^2(x-3)$

Lead Coef:

Degree:

End behavior

as x-> __ y->__

as x-> __ y->__

Zeros and Multiplicities

y intercept:

10. $y = -2(x+2)^4(x-3)^3$

Lead Coef:

Degree:

End behavior

as x-> __ y->__

as x-> __ y->__

Zeros and Multiplicities

y intercept:

11. $y=0.4x^2(x-5)(x+2)$

Lead Coef:

Degree:

End behavior

as x-> __ y->__

as x-> __ y->__

Zeros and Multiplicities

y intercept:

In Exercises 16-24, graph the function. *(See Example 1.)*

16. $f(x) = (x-2)^2(x+1)^1$

LC: 1 D: 3 x=2 m=2 x=-1 m=1 (0, 4)

17. $f(x) = (x+2)^2(x+4)^2$

18. $h(x) = (x+1)^2(x-1)(x-3)$

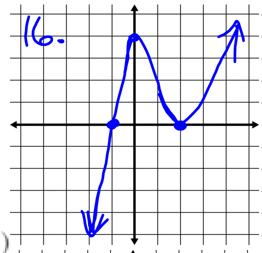
19. $g(x) = 4(x+1)(x+2)(x-1)$

20. $h(x) = \frac{1}{3}(x-5)(x+2)(x-3)$

21. $g(x) = \frac{1}{12}(x+4)(x+8)(x-1)$

22. $h(x) = (x-3)(x^2+x+1)$

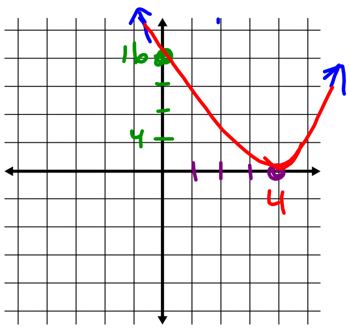
23. $f(x) = (x-4)(2x^2-2x+1)$



Homework Key

$y = (x-4)^2$

$f(0) = (0-4)^2 = 16$



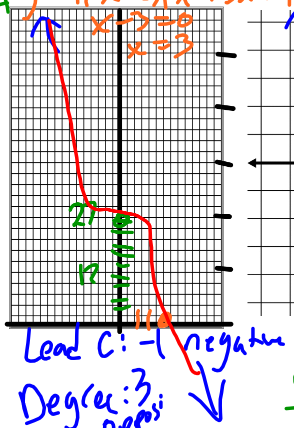
$y = -x^3+27$

$y = -1(x^3-27)$

$y = -1(x-3)(x^2+3x+9)$

$x-3=0$

$x=3$



Lead C: -1 negative

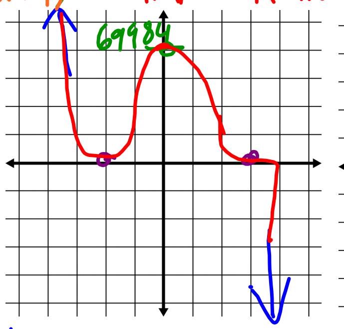
Degree: 3 opposi

$f(0) = -(0)^3+27 = 27$

x^{11}

$y = -2(x+2)^4(x-3)^7$

bounce flatter pass flatten

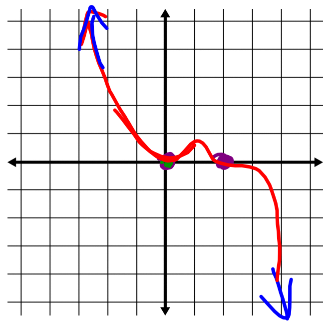


$f(0) = -2(0+2)^4(0-3)^7$

$= -2(2)^4(-3)^7$

$= 69984$

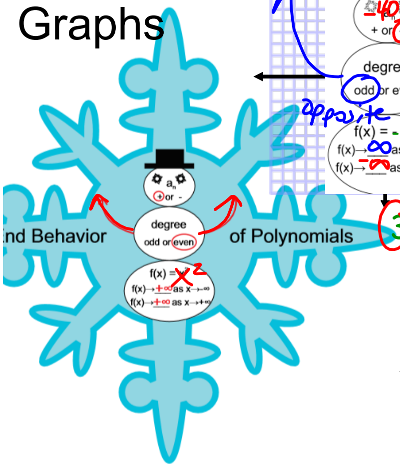
$y = -1/2x^2(x-2)^5$



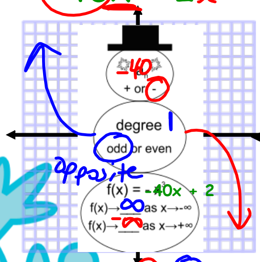
$f(0) = -\frac{1}{2}(0)^2(0-2)^5$

$= 0$

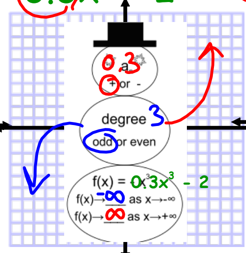
Sketching Graphs



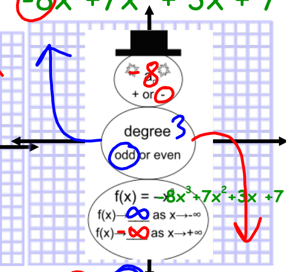
$-40x^1 + 2x^0$



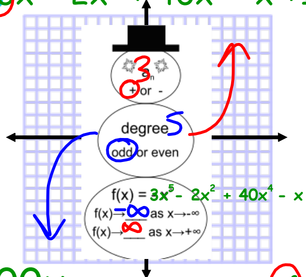
$0.3x^3 - 2$



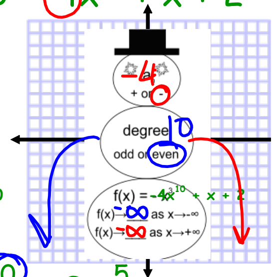
$-8x^3+7x^2+3x+7$



$3x^5 - 2x^2 + 40x^4 - x + 100$

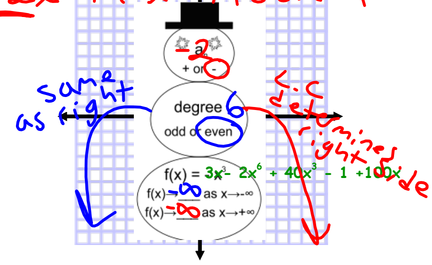


$-4x^{10} + x + 2$

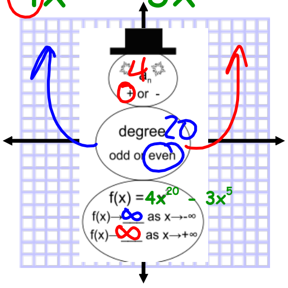


$3x - 2x^6 + 40x^3 - 1 + 100x$

$-2x^6 + 40x^3 + 103x - 1$



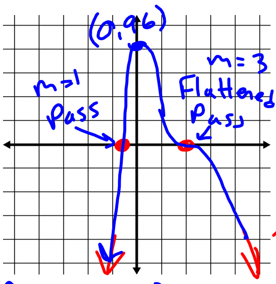
$4x^{20} - 3x^5$



Homework

or 12 $(x-2)(x-2)(x-2)(5x-4)$

8. $y = -3(x-2)^3(5x+4)^1$



Lead Coef: $-3 \cdot 1 \cdot 5$ -15

Degree: 4

End behavior

as x->∞ y->$-\infty$

as x->$-\infty$ y->$-\infty$

Zeros and Multiplicities

$x-2=0$ $5x+4=0$

$x=2$ m=3 $x=\frac{-4}{5}$ m=1

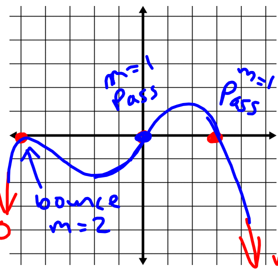
y intercept:

$f(0)=-3(0-2)^3(5(0)+4)$ (0,96)

$-3(-8)(4)$

9. $y = -1x(x+5)^2(x-3)^1$

$-1 \cdot 1 \cdot 1$



Lead Coef: -1

Degree: 4

neg

same

End behavior

as x->∞ y->$-\infty$

as x->$-\infty$ y->$-\infty$

Zeros and Multiplicities

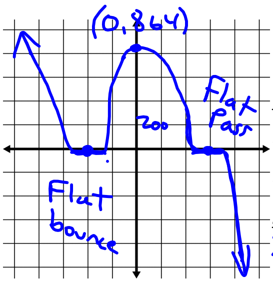
$x=0$ $x+5=0$ $x-3=0$

m=1 $x=-5$ m=2 $x=3$ m=1

y intercept:

(0,0)

10. $y = -2(x+2)^4(x-3)^3$



Lead Coef: -2

Degree: 7

End behavior

opposite

as x->∞ y->$-\infty$

as x->$-\infty$ y->∞

Zeros and Multiplicities

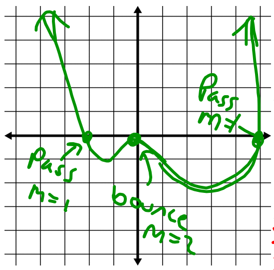
$x=3$ $x=-2$

m=3 m=4

y intercept:

(0,864)

11. $y=0.4x^2(x-5)^1(x+2)^1$



Lead Coef: 0.4

Degree: 4

End behavior

as x->∞ y->∞

as x->$-\infty$ y->∞

Zeros and Multiplicities

$x^2=0$ $x-5=0$ $x+2=0$

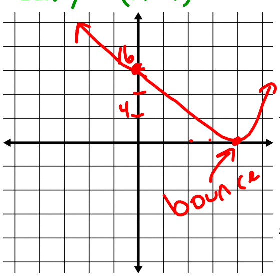
$x=0$ m=2 $x=5$ m=1 $x=-2$ m=1

y intercept:

(0,0)

Homework continued...

12. $y = (x-4)^2$



Lead Coef: 1

Degree: 2

End behavior

as x->∞ y->∞

as x->$-\infty$ y->∞

Zeros and Multiplicities

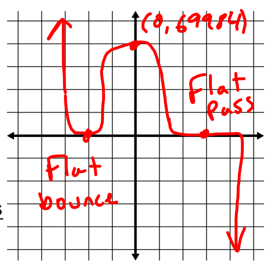
$x=4$

w/m=2

y intercept:

(0,16)

13. $y = -2(x+2)^4(x-3)^7$



Lead Coef: -2

Degree: 13

End behavior

as x->∞ y->$-\infty$

as x->$-\infty$ y->∞

Zeros and Multiplicities

$x=-2$ $x=3$

w/m=4 w/m=7

y intercept:

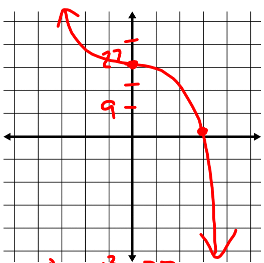
(0, 69984)

$y=-2(0+2)^4(0-3)^7$

$y=-2(16)(-2187)$

$y=69984$

14. $y = -x^3+27$



Lead Coef: -1

Degree: 3

End behavior

as x->∞ y->$-\infty$

as x->$-\infty$ y->∞

Zeros and Multiplicities

$x=3$ and 2

w/m=1 imaginary

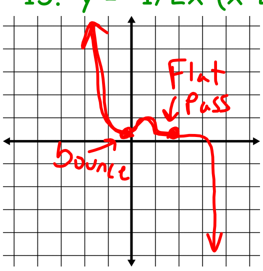
y intercept: (0,27)

$0=-x^3+27$

$x^3=27$

$x=3$

15. $y = -1/2x^2(x-2)^5$



Lead Coef: $-\frac{1}{2}$

Degree: 7

End behavior

as x->∞ y->$-\infty$

as x->$-\infty$ y->∞

Zeros and Multiplicities

$x=0$ $x=2$

m=2 m=5

y intercept:

(0,0)

In Exercises 16-24, graph the function. *(See Example 1.)*

16. $f(x) = (x-2)^2(x+1)$
(0,4) x = 2 bounce x = −1 pass

17. $f(x) = (x+2)^2(x+4)^2$
x = −2 B x = −4 B
(0, 64)

18. $h(x) = (x+1)^2(x-1)(x-3)$
(0, 3) x = −1 B x = 1 P x = 3 P

19. $g(x) = 4(x+1)(x+2)(x-1)$
(0, −8) x = −1 P x = −2 P x = 1 P

20. $h(x) = \frac{1}{3}(x-5)(x+2)(x-3)$
0,10) x = 5 P x = −2 P x = 3 P

21. $g(x) = \frac{1}{12}(x+4)(x+8)(x-1)$
(0, −2.66) x = −4 P x = −8 P x = 1 P

22. $h(x) = (x-3)(x^2+x+1)$
(0, −3) x = 3 P Imaginary

23. $f(x) = (x-4)(2x^2-2x+1)$
(0, −4) x = 4 Imaginary