Your Name

Mrs. T

1/18/23

Notes

Lesson 4-3

Rational Expressions

Agenda:

1. Lesson with examples/board work/ Workbooks Pg. 100
2. Brain Break
3. Step Sort
4. Practice

Note: Quiz on Exponents and Simplifying Rational Expression this Friday!

Warm Up

$$\frac{(2a^2b^3c)^3}{6a^{-2}bc^{-5}} \rightarrow \frac{8a^6b^9c^3}{6a^{-2}bc^{-5}}$$

$$\frac{8}{6}a^{6-2}b^{9-1}c^{3-5}$$

$$\frac{4}{3}a^8b^8c^8 = \frac{4a^8b^8c^8}{3}$$

$$\frac{-8x^3(y^{-2}z^{-5})^3}{10xz^{-4}} \qquad \frac{-8x^3y^{-6}z^{-15}}{10xz^{-4}}$$

$$-\frac{4}{5}x^{3-1}y^{-6}z^{-15--4} \qquad \frac{-4x^2y^{-6}z^{-11}}{5} \qquad \frac{-4x^2}{5y^6z^{11}}$$

$$\frac{64a^3b^{-5}}{2a^4b^6} \cdot \frac{6a^4b^2}{-4a^5}$$

$$\frac{\cancel{4}\cdot 16\cancel{a^3}}{\cancel{2}\cdot\cancel{a^4}\,b^6b^5} \cdot \frac{\cancel{2}\cdot 3\cancel{a^4}\cancel{b^2}}{\cancel{-4}\cancel{a^5}\,a^2}$$

$$\frac{16\cdot 3}{-1\cdot b^9\cdot a^2}$$

$$-\frac{48}{a^2b^9}$$

$(x+1)(x-4)(x-2)$

Objective: to be able to simplify rational expressions by factoring and canceling. To be able to find excluded values, values that will not work in the function.

Virtue/Skill: Factoring and simplifying rational expressions is necessary to graph them. And you need to find the excluded values and the kind they are so that you can graph them differently.

Rational Expression

An algebraic fraction with numerator and denominator that are polynomials

How to Simplify? Factor and Simplify by canceling/dividing like factors

Excluded Value

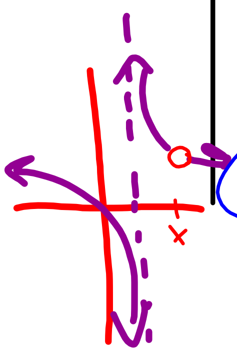
Any value that would make the expression not work. In this case the value of the variable that would make the denominator 0

Because we can't divide by 0, the denominator cannot equal 0

How do we find them? Set the denominator equal to 0, factor and solve for each variable

If a factor in the denominator is canceled out, it is excluded as a HOLE

If a factor in the denominator is not canceled out, it is excluded as an ASYMPTOTE



Simplify the expression and state the excluded values

ex. x = 5 and x = 6 are excluded values

or

$x \neq 5$ and $x \neq 6$

Board Work: What are the Excluded Values!

$$\frac{\overset{4}{\cancel{16}}\,\overset{1}{\cancel{x^3}}\,\overset{1}{\cancel{y^2}}}{\underset{9}{\cancel{36}}\,\underset{x^2}{\cancel{x^5}}\,\underset{y}{\cancel{y^3}}}$$

$$\frac{4}{9x^2y}$$

$x^2=0$ $\quad$ $y=0$

$x\neq 0$ $\quad$ $y\neq 0$

$$\frac{2b}{(b-8)}$$

nothing cancels

$b-8=0$

$b\neq 8$

Asymptote

$$\boxed{\frac{12-a}{32+a}}$$

$\frac{12-4a}{32+4a}$

$\frac{4(3-a)}{4(8+a)}$

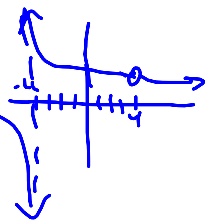
$32+a=0$

$\boxed{a\neq -32}$

Asymptote

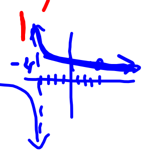
What are the Excluded Values!

$$\frac{x-4}{x^2-16}$$



$$\frac{1\,\cancel{(x-4)}}{(x+4)\cancel{(x-4)}}$$

$$\frac{1}{x+4}$$



$x+4=0$ $\quad$ $x-4=0$

$x\neq -4$ $\quad$ $x\neq 4$

Asymptote $\quad$ hole

$$\frac{x^2-4}{x^2+5x+6}$$

$$\frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+3)}$$

6

2 $\quad$ 3

+5

$\boxed{\frac{x-2}{x+3}}$ simplified

$x^2+5x+6=0$

$(x+3)(x+2)=0$

$x+2=0$ $\quad$ $x+3=0$

$\boxed{x\neq -2 \quad x\neq -3}$

hole $\quad$ Asymptote

You CANNOT cancel things that are not multiplied

$$\frac{\cancel{x^2}-\cancel{4}}{5}\cdot\frac{x+2}{\cancel{x}-\cancel{2}}$$

$$\frac{(x+2)(x+2)}{5}$$

$$\frac{(x+2)^2}{5}$$

$$\frac{(x+2)\cancel{(x-2)}(x+2)}{5\cancel{(x-2)}}$$

$$\frac{(x+2)^2}{5}$$

not equal

Workbook pg. 100

Write an expression equivalent to the below.
Give domain for the expression.

1. $\dfrac{3x^5 - 18x^4 - 21x^3}{2x^6 - 98x^4}$

$$\frac{3x^3(x^2-6x-7)}{2x^4(x^2-49)}$$

$$\frac{3\cancel{x^3}\cancel{(x-7)}(x+1)}{2\cancel{x^4}(x+7)\cancel{(x-7)}}$$

$$\frac{3(x+1)}{2x(x+7)}$$

$x^4=0$

$x\neq 0$ Asymptote

$x+7=0$

$x\neq -7$ Asymptote

$x-7=0$

$x\neq 7$ hole

Workbook pg. 100

Simplify each expression below. Give domain for which the identity with the two expressions is valid.

2a. $\dfrac{x^2+2x+1}{x^3-2x^2-3x}$

$\dfrac{\cancel{(x+1)}(x+1)}{x\cancel{(x+1)}(x-3)}$

$\dfrac{x+1}{x(x-3)}$ $x\neq 0$ $x\neq -1$ $x\neq 3$

Asym Hole Asym

2b. $\dfrac{x^3+4x^2-x-4}{x^2+3x-4}$

x^3+4x^2-x-4

$x^2(x+4)-1(x+4)$

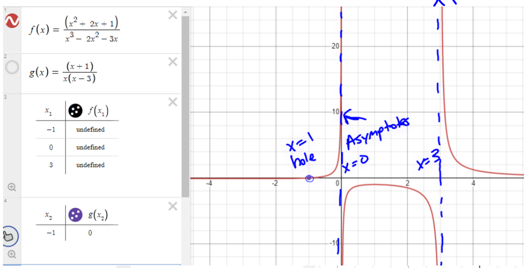
$(x+4)(x^2-1)$

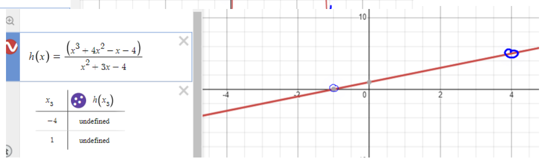
$\dfrac{\cancel{(x+4)}(x+1)\cancel{(x-1)}}{\cancel{(x+4)}\cancel{(x-1)}}$

$\boxed{x+1}$

$x\neq -4$ $x\neq 1$

Hole





Multiplying Rational Expressions

Multiply
*straight across

*Factor
*cancel
vertically
or diagonally

ex. 3a $\dfrac{x^2-16}{9-x}\cdot\dfrac{x^2+x-90}{x^2+14x+40}$

$\dfrac{\cancel{(x+4)}(x-4)}{-\cancel{(x-9)}}\cdot\dfrac{\cancel{(x+10)}\cancel{(x-9)}}{\cancel{(x+10)}\cancel{(x+4)}}$

* Factor out -1 to change order

$\dfrac{x-4}{-1}\rightarrow -(x-4)\rightarrow \boxed{-x+4}$

ex. 3b $\dfrac{x+3}{4x}\cdot\dfrac{3x-18}{6x+18}\cdot\dfrac{x^2}{4x+12}$

$\dfrac{\cancel{x+3}}{4\cancel{x}}\cdot\dfrac{\overset{1}{\cancel{3}}(x-6)}{\underset{2}{\cancel{6}}\cancel{(x+3)}}\cdot\dfrac{\cancel{x^2}\,x}{4(x+3)}$

$\dfrac{x(x-6)}{32(x+3)}$

Multiplying Rational Expressions by a Polynomial

$\frac{3}{4} \cdot 5$

Put over 1 to make Fraction

$\frac{3}{4} \cdot \frac{5}{1}$

Multiply straight across

ex. 4a $\frac{x^3 - 4x}{6x^2 - 13x - 5} \cdot \frac{2x^3 - 3x^2 - 5x}{1}$

$x(x^2-4)$

$\frac{x(x-2)(x+2)}{(3x+1)\cancel{(2x-5)}} \cdot \frac{x\cancel{(2x-5)}(x+1)}{1}$

$\frac{x^2(x-2)(x+2)(x+1)}{2x-5}$

ex. 4b $\frac{3x^2 + 6x}{x^2 - 49} \cdot \frac{x^2 + 9x + 14}{1}$

$\frac{3x(x+2)}{\cancel{(x+7)}(x-7)} \cdot \frac{\cancel{(x+7)}(x+2)}{1}$

$\frac{3x(x+2)(x+2)}{x-7}$

Why is it important to identify the domain

of a rational expression

BEFORE you simplify it

rather than after?

So you can catch all the excluded values, the holes will not be seen in the simplified version

Multiplying Fractions: How many ways can you go about this?

$$\frac{14}{7}\cdot\frac{28}{4}$$

Skip Flip + Multiply

Keep Change Flip

$$\frac{\cancel{14}^{\,1}}{7}\cdot\frac{\cancel{4}^{\,2}}{\cancel{28}_{\,2\,1}}$$

$$\frac{2}{7}$$

Dividing Rational Expressions

1. Rewrite with reciprocal of second expression
2. Multiply straight across (numerators with numerators and denominators with denominators), and
3. Cancel common factors that are being divided whether diagonally or vertically and write the leftover factor.

5a. $\underset{\text{Keep}}{\frac{1}{x^2+9x}} \ \underset{\text{change}}{\div} \ \underset{\text{Flip}}{\frac{6-x}{3x^2-18x}}$ Flip

$$\frac{1}{\cancel{x}(x+9)}\cdot\frac{3\cancel{x}\cancel{(x-6)}}{-1\cancel{(x-6)}}$$

$$\frac{3}{-(x+9)} \rightarrow \boxed{\frac{3}{-x-9}}$$

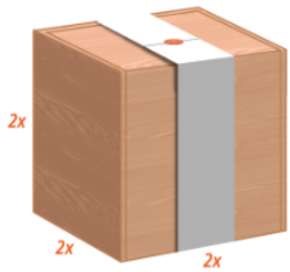
5b. $\frac{2x^2-12x}{x+5} \div \frac{x-6}{x+5}$ Flip

$$\frac{2x\cancel{(x-6)}}{\cancel{x+5}}\cdot\frac{\cancel{x+5}}{\cancel{x-6}}$$

$$2x$$

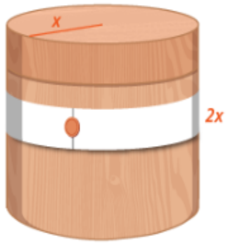
A company is evaluating two packaging options for its product line. The more efficient design will have the lesser ratio of surface area to volume. Should the company use packages that are cylinders or rectangular prisms?

Option 1: A rectangular prism with a square base



Surface area: $2(2x)^2 + 4(2x)^2$
Volume: $(2x)^3$

Option 2: A cylinder with the same height as the prism, and diameter equal to the side length of the prism's base



Surface area: $2\pi x^2 + 2\pi x(2x)$
Volume: $\pi x^2(2x)$

The efficiency ratio is $\frac{SA}{V}$, where SA represents surface area and V represents volume.

Option 1:

$$\frac{SA}{V} = \frac{2(4x^2) + 4(4x^2)}{8x^3} = \frac{24x^2}{8x^3} = \frac{3}{x}$$

Option 2:

$$\frac{SA}{V} = \frac{2\pi x^2 + 4\pi x^2}{2\pi x^3} = \frac{6\pi x^2}{2\pi x^3} = \frac{3}{x}$$

The company can now compare the efficiency ratio of the package designs.

Prism: $\frac{3}{x}$ Cylinder: $\frac{3}{x}$

Regardless of what positive value is selected for x, the efficiency ratios for these two package designs will be the same.

In this example, the efficiency ratio of the cylinder is equal to that of the prism. So the company should choose their package design based on other criteria.

SA = 2B + LA
Base area Area of sides

SA = 2πr2 + 2πrH
3D Height

SA = 2bh + PH
perimeter 3D Height

V = BH
Base area 3D Height

V= (bh)H

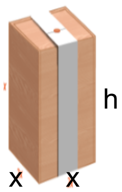
$V = (\pi r^2)H$

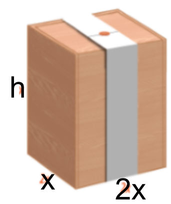
Try It!

$SA : V \rightarrow \frac{SA}{V}$

6. The company compares the ratios of surface area to volume for two more containers. One is a rectangular prism with a square base. The other is a rectangular prism with a rectangular base. One side of the base is equal to the side-length of the first container, and the other side is twice as long. The surface area of this second container is $4x^2 + 6xh$. The heights of the two containers are equal. Which has the smaller surface area-to-volume ratio?

SA = $4x^2$ + 6xh





$SA = 2x^2 + 4xh$
$V = x^2h$

$$\frac{\cancel{x}(2x+4h)}{\cancel{x}^2 h} \rightarrow \frac{2x+4h}{xh}$$

Bigger

$SA = 2(x)(2x) + 6xh$
$SA = 4x^2 + 6xh$
$V = 2x^2h$

$$\frac{\cancel{2x}(2x+3h)}{\cancel{2x}^2 h} \rightarrow \frac{2x+3h}{xh}$$

Dividing Rational Expressions

1. Rewrite with reciprocal of second expression (Factor at same time)

→ Flipped

2. Multiply straight across (numerators with numerators and denominators with denominators), and

Cancel common factors that are being divided whether diagonally or vertically to 1's.

anything can be put over 1

$\frac{6ab}{a^2 b^2} \div \frac{a^2}{b^2}$

$\frac{6ab}{a^2 b^2} \cdot \frac{b^2}{a^2}$

$\frac{6b}{a^3}$

$\frac{x+2}{x-4} \div \frac{3x+6}{1}$

Factor

$\frac{x+2}{x-4} \cdot \frac{1}{3(x+2)}$

$\boxed{\frac{1}{3x-12}}$

$\frac{2n-4}{n+2} \div \frac{2n+4}{n-4}$

$\frac{2(n-2)}{n+2} \cdot \frac{n-4}{2(n+2)}$

$\boxed{\frac{(n-2)(n-4)}{(n+2)^2}}$

$\frac{x^2-5x+6}{x^2+3x} \div \frac{3-x}{4x+12}$

$-1(-3+x)$

switch order by factoring out -1

$\frac{(x-3)(x-2)}{x(x+3)} \cdot \frac{4(x+3)}{-1(x-3)}$

$\frac{4(x-2)}{-x}$

$-\frac{4(x-2)}{x}$

$\frac{p^2-2pq+q^2}{p-q} \div \frac{p^2-q^2}{q-p}$

$\frac{(p-q)(p-q)}{p-q} \cdot \frac{-1(p-q)}{(p-q)(p+q)}$

$\frac{-(p-q)}{p+q}$

$\boxed{\frac{q-p}{p+q}}$

Find the Quotient!

$$\frac{a^2+7a+12}{a^2+3a-10} \div \frac{a^2-9}{a^2-25}$$

$$\frac{(a+3)(a+4)}{(a-2)(a+5)} \cdot \frac{(a-5)(a+5)}{(a-3)(a+3)}$$

$$\boxed{\frac{(a-4)(a-5)}{(a-2)(a-3)}}$$

dont need to distribute further

$$\frac{25-n^2}{n^2-4n-5}$$

Dimensional Analysis

Multiplying fractions that involve units of measure can be simplified by dividing units of measure. This is similar to simplifying rational expressions dividing out common factors.

2 yd = 6 ft $\quad 2\cancel{yd}\cdot\frac{3\,ft}{1\,\cancel{yd}}$

2 yd² = 18 ft² $\quad 2\cancel{yd^2}\cdot\frac{9\,ft^2}{1\,\cancel{yd^2}}$

2 m³ = 2000000 cm³ $\quad 2\cancel{m^3}\cdot\frac{100^3\,cm^3}{1\,\cancel{m^3}}$

32 yd = 96 ft $\quad 32\cancel{yd}\cdot\frac{3\,ft}{1\,\cancel{yd}}$

15 yd² = 135 ft² $\quad 15\cancel{yd^2}\cdot\frac{9\,ft^2}{1\,\cancel{yd^2}}$

0.48 m³ = 480000 cm³ $\quad .48\cancel{m^3}\cdot\frac{100^3\,cm^3}{1\,\cancel{m^3}}$

$$\frac{2000\text{ revolutions}}{1\ \cancel{\text{min}}}\cdot\frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}}\cdot\frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}}\cdot\frac{7\ \cancel{\text{days}}}{1\text{ week}}$$

20,160,000 revolutions per week

In Exercises 1–6, simplify the expression, if possible.

1. $\frac{4x^3}{3x^3+7x}$ $\quad \frac{4x^2\cdot x}{x(3x^2+7)}$ $\quad x \neq 0 \quad x \neq \pm i\sqrt{7/3}$

2. $\frac{x^2+5x+6}{x^2+2x-3}$ $\quad \frac{(x+3)(x+2)}{(x+3)(x-1)}$ $\quad \boxed{\frac{x+2}{x-1}\ \ x\neq -3,\ x \neq 1}$

3. $\frac{2x^2-5x}{x^2+7x+12}$

4. $\frac{x^2-x-20}{x^3+64}$

5. $\frac{x^4-16}{5x^3-3x^2+20x-12}$

6. $\frac{6x^3-6x^2+5x-5}{72x^4-50}$

In Exercises 7–12, find the product.

7. $\frac{x^4(x-4)}{x+3}\bullet\frac{(x+3)(x-2)}{x^5}$

8. $\frac{x^2+6x}{x-4}\bullet\frac{x^2-2x-8}{x}$

9. $\frac{x^2-2x}{x+5}\bullet\frac{x^2+6x+5}{3x}$

10. $\frac{x^2-x-6}{x^2+8x+16}\bullet\frac{3x^2+12x}{x^2-2x-3}$

11. $\frac{x^2+3x-28}{x^2-25}\bullet(x^2-8x+15)$

12. $\frac{x^2+2x-15}{x^2-9}\bullet(x^2-x-12)$

In Exercises 13–16, find the quotient.

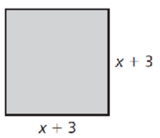
13. $\frac{2x^3+10x^2}{x^2+x-20}\div\frac{2x^2}{x-4}$

14. $\frac{x^2-10x+21}{x+2}\div(x^2-14x+49)$

15. $\frac{x^2-2x-3}{x^2+2x-8}\div\frac{x^2+4x+3}{x^2+6x+8}$

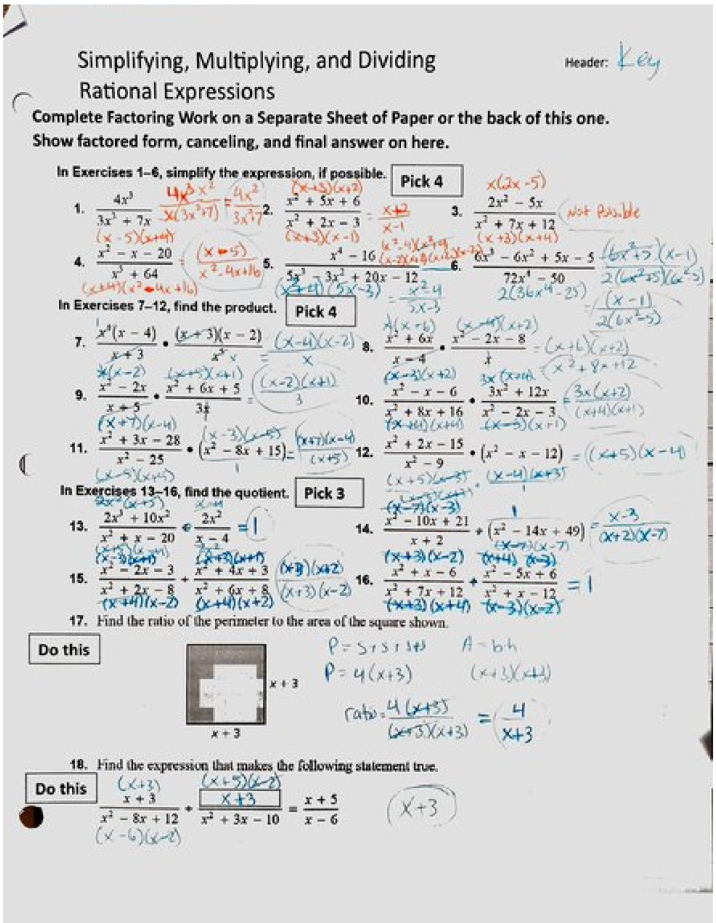
16. $\frac{x^2+x-6}{x^2+7x+12}\div\frac{x^2-5x+6}{x^2+x-12}$

17. Find the ratio of the perimeter to the area of the square shown.



18. Find the expression that makes the following statement true.

$$\frac{x+3}{x^2-8x+12}\div\frac{\boxed{}}{x^2+3x-10}=\frac{x+5}{x-6}$$



Simplifying, Multiplying, and Dividing Rational Expressions

Header: Key

Complete Factoring Work on a Separate Sheet of Paper or the back of this one.
Show factored form, canceling, and final answer on here.

In Exercises 1–6, simplify the expression, if possible. Pick 4

1. $\frac{4x^3}{3x^3 + 7x} = \frac{4x^3 x^2}{x(3x^2+7)} = \frac{4x^2}{3x^2+7}$
2. $\frac{x^2 + 5x + 6}{x^2 + 2x - 3} = \frac{(x+3)(x+2)}{(x+3)(x-1)} = \frac{x+2}{x-1}$
3. $\frac{2x^2 - 5x}{x^2 + 7x + 12} = \frac{x(2x-5)}{(x+3)(x+4)}$ Not Possible
4. $\frac{x^2 - x - 20}{x^3 + 64} = \frac{(x-5)(x+4)}{(x+4)(x^2-4x+16)} = \frac{(x-5)}{x^2-4x+16}$
5. $\frac{x^4 - 16}{5x^3 - 3x^2 + 20x - 12} = \frac{(x^2+4)(x^2-4)}{(x^2+4)(5x-3)} = \frac{x^2-4}{5x-3}$
6. $\frac{6x^3 - 6x^2 + 5x - 5}{72x^4 - 50} = \frac{(6x^2+5)(x-1)}{2(6x^2+5)(6x^2-5)} = \frac{(x-1)}{2(6x^2-5)}$

In Exercises 7–12, find the product. Pick 4

7. $\frac{x^3(x - 4)}{x + 3} \cdot \frac{(x + 3)(x - 2)}{x^4} = \frac{(x-4)(x-2)}{x}$
8. $\frac{x^2 + 6x}{x - 4} \cdot \frac{x^2 - 2x - 8}{x} = (x+6)(x+2) = x^2+8x+12$
9. $\frac{x^2 - 2x}{x + 5} \cdot \frac{x^2 + 6x + 5}{3x} = \frac{(x-2)(x+1)}{3}$
10. $\frac{x^2 - x - 6}{x^2 + 8x + 16} \cdot \frac{3x^2 + 12x}{x^2 - 2x - 3} = \frac{3x(x+2)}{(x+4)(x+1)}$
11. $\frac{x^2 + 3x - 28}{x^2 - 25} \cdot (x^2 - 8x + 15) = \frac{(x+7)(x-4)}{(x+5)}$
12. $\frac{x^2 + 2x - 15}{x^2 - 9} \cdot (x^2 - x - 12) = (x+5)(x-4)$

In Exercises 13–16, find the quotient. Pick 3

13. $\frac{2x^3 + 10x^2}{x^2 + x - 20} \div \frac{2x^2}{x - 4} = 1$
14. $\frac{x^2 - 10x + 21}{x + 2} \div (x^2 - 14x + 49) = \frac{x-3}{(x+2)(x-7)}$
15. $\frac{x^2 - 2x - 3}{x^2 + 2x - 8} \div \frac{x^2 + 4x + 3}{x^2 + 6x + 8} = \frac{(x-3)(x+2)}{(x+3)(x-2)}$
16. $\frac{x^2 + x - 6}{x^2 + 7x + 12} \div \frac{x^2 - 5x + 6}{x^2 + x - 12} = 1$

17. Find the ratio of the perimeter to the area of the square shown.

Do this

$x + 3$

$x + 3$

$P = s+s+s+s$ $A = b \cdot h$

$P = 4(x+3)$ $(x+3)(x+3)$

$\text{ratio} = \frac{4(x+3)}{(x+3)(x+3)} = \frac{4}{x+3}$

18. Find the expression that makes the following statement true.

Do this

$\frac{x + 3}{x^2 - 8x + 12} \div \frac{\boxed{x+3}}{x^2 + 3x - 10} = \frac{x + 5}{x - 6}$

$(x+3)$

What are the Excluded Values!

$$\frac{x^2 + 4x + 4}{4x^2 + 11x - 3}$$

$4x^2 + 11x - 3 = 0$ $\cdot -12$

$4x^2 + 12x - 1x - 3 = 0$ 12 -1 11

$4x(x+3) - 1(x+3)$

$(4x-1)(x+3) = 0$

$\boxed{x \neq \frac{1}{4} \quad x \neq -3}$

Asymptote Asymptote

$$\frac{(x+2)(x+2)}{(4x-1)(x+3)}$$

Simplified as it gets

$$\frac{25 - n^2}{n^2 - 4n - 5} = \frac{(5-n)(5+n)}{(n-5)(n+1)} = \frac{-1\cancel{(n-5)}(5+n)}{\cancel{(n-5)}(n+1)}$$

$\boxed{n \neq 5 \quad n \neq -1}$

hole Asymptote

$$\boxed{-\frac{5+n}{n+1}}$$

Simplified