Level 2

Complement: The probability of failure.

Use when: you see the word NOT or when you want AT LEAST 1 to happen, because you can subtract the probability of not at all.

The formula is: The probability of Not an event S happening, is equal to 100% minus the probability of the event happening. This gets you the probability of everything else happening.

Complement Formula: P(Not S) = 1 - P(s)

Examples

a) What is the probability of NOT drawing a king from a standard 52-card deck?

$P(\text{Not King}) = 1 - P(\text{Pick King})$

Note: 4 kings in a deck

$= 1 - \frac{4}{52}$

$= \frac{52}{52} - \frac{4}{52} = \frac{48 \div 4}{52 \div 4} = \boxed{\frac{12}{13}}$

b) What is the probability of NOT rolling a 2 or a 3 on a 6-sided die?

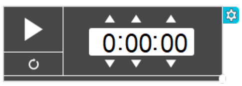
$P(\text{Not } 2 \text{ or } 3) = 1 - P(\text{roll } 2 \text{ or } 3)$

$1 - \left(\underset{\text{roll a 2}}{\frac{1}{6}} + \underset{\text{roll 3}}{\frac{1}{6}}\right)$

$\frac{6}{6} - \frac{2}{6}$

$\frac{4}{6} = \boxed{\frac{2}{3}}$

Examples



A bag contains 5 yellow, 6 blue, and 4 white chips.

a) What is the probability that a chip selected at random will NOT be yellow?

$P(\text{Not } Y) = 1 - \frac{5}{15}$

$\frac{15}{15} - \frac{5}{15}$

$\frac{10}{15} = \frac{2}{3}$

$(6+4)$

$\frac{10}{15}$

$\frac{2}{3}$

$P(\text{Not } Y) = 1 - \frac{5}{15}$

$1 - \frac{1}{3}$

$\frac{3}{3} - \frac{1}{3}$

$\frac{2}{3}$

b) What is the probability that a chip selected at random will NOT be white?

$P(\text{Not } W) = 1 - \frac{4}{15}$

$\frac{15}{15} - \frac{4}{15}$

$\boxed{\frac{11}{15}}$

c) What is the probability that if you select 3 chips, AT LEAST 1 will be blue?

$P\left(\begin{matrix}\text{3 chips} \\ \text{at least 1} \\ \text{Blue}\end{matrix}\right) = 1 - P\left(\begin{matrix}\text{3 chips} \\ \text{None are} \\ \text{blue}\end{matrix}\right)$

$= 1 - \left(\underset{\text{1st not blue}}{\frac{\cancel{9}\,3}{\cancel{15}\,5}} \cdot \underset{\text{2nd}}{\frac{\cancel{8}\,4}{\cancel{14}}} \cdot \underset{\text{3rd}}{\frac{\cancel{7}}{13}}\right)$

$= 1 - \frac{504}{2730}$

$\frac{65}{65} - \frac{12}{65}$

$\frac{2730}{2730} - \frac{504}{2730}$

$\frac{2226 \div 14}{2730 \div 14} = \frac{159 \div 3}{195 \div 3} = \boxed{\frac{53}{65}}$

Composite Events

Mutually Inclusive Events

Two or more events that can happen together because some characteristics are shared there is "overlap"

And

If you want both events to happen together at the same time

Count the options considered together.

Or

If you are happy with at least one event out of the two happening.

Add options, subtract any overlap

pulling a red card AND a seven VS pulling a red card OR a seven

$A \cap B = \frac{2}{52}$ means "and" only 2 cards that are both 7 and red

$A \cup B$ means "or" $= A + B - A \cap B$ means "and"

$\frac{26}{52} + \frac{4}{52} - \frac{2}{52}$ red 7s overlap red 7s

$A \cup B$ means "or" $= \frac{26}{52} + \frac{2}{52}$ red cards with red 7s, 2 other black 7s

Non-Mutually Exclusive Events

Events that share characteristics and could happen together.

P(A or B) = P(A) + P(B) - P(A and B)

Examples

Total: 15

A bag contains 5 yellow, 6 blue, and 4 white chips.

a) What is the probability that a chip will be yellow or not blue?

$P(Y \text{ or Not } B) = P(Y) + P(\text{Not } B) - (\text{overlap } Y \text{ and Not } B)$

$\frac{5}{15} + \frac{9}{15} - \frac{5}{15} = \frac{9}{15} \div 3 = \frac{3}{5}$

b) What is the probability that a chip will be yellow and not blue?

$P(Y \text{ and Not } B) = \frac{5}{15}$ overlap

c. P(rolling a 4 and an even number on a die)

$P(4 \text{ and even}) = \frac{1}{6}$

d. P(rolling a 4 or an even number on a die)

$P(4 \text{ or even}) = P(4) + P(\text{even}) - \text{overlap}$

$= \frac{1}{6} + \frac{3}{6} - \frac{1}{6}$

$= \frac{3}{6} = \frac{1}{2}$

M D MD

$\frac{8}{20} + \frac{5}{20} - \frac{1}{20}$

$\frac{12}{20} \div 2 = \frac{6}{10} \div 2 = \frac{3}{5}$

Refresher: Independent events are two or more events in which the previous event has no impact on the remaining events

Probability of two (or more) Independent Events

If two events, A and B, are independent, then the
probability of BOTH events occurring is the product of
each probability.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

52 cards total
2 colors Red/Black
26 26
4 suits
♣ ♡ ◇ ♤
13 13 13 13
4 of every Number and face
2-10, Ace, Jack, Queen King

Example

a.) For a standard 52-card deck, what is the probability of drawing the three cards: a 5, a Queen, AND an Ace?

$$P(5, Q, \text{And Ace}) = P(5) \cdot P(Q) \cdot P(\text{Ace})$$

$$\frac{4 \div 4}{52 \div 4} \cdot \frac{4}{52} \cdot \frac{4}{52} = \frac{64}{140{,}608}$$

$$\frac{1}{13} \cdot \frac{1}{13} \cdot \frac{1}{13} = \boxed{\frac{1}{2{,}197}}$$

Mixed Examples

a) Rolling two dice such that only one is a 5 or the sum of the dice is 8

P(only One 5) + P(sum is 8) P(5 and 3)

$$\left(\underset{\text{roll 5}}{\frac{1}{6}} \cdot \underset{\text{roll Not 5}}{\frac{5}{6}}\right) + \overset{5+3}{\left(\frac{1}{6} \cdot \frac{1}{6}\right)} + \overset{6+2}{\left(\frac{1}{6} \cdot \frac{1}{6}\right)} + \overset{4+4}{\left(\frac{1}{6} \cdot \frac{1}{6}\right)} - \left(\frac{1}{6} \cdot \frac{1}{6}\right)$$

$$\frac{5}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} - \frac{1}{36} = \boxed{\frac{7}{36}}$$

b) Given a standard 52-card deck, what is the probability of pulling a club or a Jack?

$$P(C \text{ or } J) = P(\text{Club}) + P(J) - P(J \text{ of Club})$$ add overlap

$$\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16 \div 4}{52 \div 4} = \boxed{\frac{4}{13}}$$

0:00:00

c) In order to receive his driver's license, Mark must pass both the driving portion and written portion of the license test. On the average, the probability of passing the driving portion on the first time is 7/10 and the probability of passing the written portion on the first attempt is 4/5. Overall, the probability of people passing both portions on the first try is 2/3.

What is the probability that Mark passes the first or the second test on the first try?

$$P(\text{Pass D or W}) = P(D) + P(W) - \overset{\text{and}}{P(D \cap W)}$$ add overlap

$$\frac{7}{10} + \frac{4}{5} - \frac{2}{3}$$

Common Denominator
LCM: 30

$$\frac{21}{30} + \frac{24}{30} - \frac{20}{30} = \frac{25}{30} = \boxed{\frac{5}{6}}$$

D M MD

$$\frac{13}{19} + \frac{14}{19} - \frac{5}{19} =$$

Replacement

When picking more than one thing, you are putting it back each time. This keeps the number of options the same.

multiply probabilities
keep option amount the same

Examples

a). 5 red apples and 3 green apples. How many ways is there to pick two green apples, if they are replaced?

Pick 1st green apple · Pick 2nd green apple

$$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$$

b). 5 red apples and 3 green apples. What is the probability of picking two red apples if you put it back each time?

Pick 1st red · Pick 2nd red

$$\frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$$

Examples

52 cards | 2 colors: Red 26, Black 26 | ♥ 13, ♣ 13 club, ♠ 13 spade, ♦ 13 | 2-10: 4 of each | Ace, Jack, Queen, King: 4 of each

a) Using a standard 52-card deck, what is the probability of drawing a heart with the first draw, replacing it, then drawing an ace?

draw 1st heart · draw 2nd ace

$$\frac{13}{52} \cdot \frac{4}{52} = \frac{52}{2704} = \frac{52}{52^2} = \boxed{\frac{1}{52}}$$

b) According to the U.S. Department of Transportation, airlines arrive on time 80% of the time. During their vacation, the Sorensen family flew to Seattle, Denver, Phoenix, and back to Chicago. What is the probability that they arrived on time for all of their flights?

$80\% \rightarrow 0.80$

$\rightarrow S \quad \rightarrow D \quad \rightarrow P \quad \rightarrow C$

$$0.8 \cdot 0.8 \cdot 0.8 \cdot 0.8 = 0.4096$$

$$\frac{80}{100} \quad \frac{8}{10} \quad \frac{4}{5} \quad \frac{4}{5}$$

$\boxed{40.96\%}$

c) Kobe Bryant makes 83.7% of the free throws that he shoots. What is the probability that if he shoots 2 free throws, he misses the first and makes the second?

Examples

dividing by total

a) Using a standard 52-card deck, what is the probability of drawing a heart with the first draw, replacing it, then drawing an ace?

1st draw ♡ 2nd Draw Ace

$$\frac{13}{52} \cdot \frac{4}{52} = \frac{52}{52^2} = \boxed{\frac{1}{52}}$$

b) According to the U.S. Department of Transportation, airlines arrive on time 80% of the time. During their vacation, the Sorensen family flew to Seattle, Denver, Phoenix, and back to Chicago. What is the probability that they arrived on time for all of their flights?

→S →D →P →C

80% → 0.80 Decimal

$0.80 \cdot 0.8 \cdot 0.8 \cdot 0.8 = 0.4096$ → 40.96%

$\frac{80}{100}$

c) Kobe Bryant makes 83.7% of the free throws that he shoots. What is the probability that if he shoots 2 free throws, he misses the first and makes the second?

Probability of Two Dependent Events Without Replacement

If two events, A and B, are dependent, then the probability of both events occurring is the product of each individual probability

Def: Not put back, not same amount of options

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)}$$

Without Replacement

When picking more than one thing, you are NOT putting it back each time. This decreases the number of options each time you have to pick.

Examples

no + dividing

ex. 5 red apples and 3 green apples. How many ways is there to pick two green apples, if not replaced?

Pick 1st green · Pick 2nd green apple

$3 \cdot 2 = 6$ ways

divide by total

ex. 5 red apples and 3 green apples. What is the probability that you pick two red apples to put in your basket to keep?

Pick 1st red · Pick 2nd red

$\frac{5}{\cancel{8}\,2} \cdot \frac{\cancel{4}\,1}{7}$ one less red, one less total $= \frac{20}{56}$

$\frac{5}{14}$