Perfect Squares and Perfect Cubes

Workbook pg. 114

Perfect Squares	
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$
$11^2 = 121$	$\sqrt{121} = 11$
$12^2 = 144$	$\sqrt{144} = 12$
$13^2 = 169$	$\sqrt{169} = 13$
$14^2 = 196$	$\sqrt{196} = 14$
$15^2 = 225$	$\sqrt{225} = 15$
$20^2 = 400$	$\sqrt{400} = 20$
$30^2 = 900$	$\sqrt{900} = 30$

$30 \cdot 30$

Perfect Cubes	
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 725$	$\sqrt[3]{725} = 9$
$10^3 = 1{,}000$	$\sqrt[3]{8} = 10$
$11^3 = 1{,}331$	$\sqrt[3]{1{,}000} = 11$
$12^3 = 1{,}728$	$\sqrt[3]{1{,}728} = 12$

Other Powers to know

$\overset{2\cdot2\cdot2\cdot2}{2^4} = 16$	$\sqrt[4]{16} = 2$
$3^4 = 81$	$\sqrt[4]{81} = 3$
$4^4 = 256$	$\sqrt[4]{256} = 4$
$2^5 = 32$	$\sqrt[5]{32} = 2$
$2^6 = 64$	$\sqrt[6]{64} = 2$

Nth Root

what number, times itself n times, equals the base?

Find the real cube roots of −64.

$\sqrt[3]{-64} = -4$

$\sqrt[3]{-4 \cdot -4 \cdot -4}$

$\cancel{\sqrt[3]{(-4)^3}}$

-4

The nth root of a number is the number raised to the 1/n power

$\sqrt[3]{(-4)^3} = ((-4)^3)^{\frac{1}{3}}$

$(-4)^{3 \cdot \frac{1}{3}} = \boxed{-4^1}$

EXAMPLE 1 Find All Real *n*th Roots

Try It!

1. Find the specified roots of each number.

a. real fourth roots of 81

Even Root

2 answers $\pm$

$\sqrt[4]{81} = \boxed{3}$

$3 \cdot 3 \cdot 3 \cdot 3 = 81$

$\sqrt[4]{81} = \boxed{-3}$

$\underbrace{-3 \cdot -3}_{9} \cdot \underbrace{-3 \cdot -3}_{9} = 81$

Note: can't take even roots of negative #s → Imaginary

b. real cube roots of 64

Odd root

1 answer

$\sqrt[3]{64}$

$4 \cdot 4 \cdot 4 = 64$

$\boxed{4}$

square root of 64

$\sqrt[2]{64} = \sqrt{8 \cdot 8}$

8

$\sqrt[2]{64} = \sqrt{-8 \cdot -8}$

-8

7th root of x^{14}

$(x^{14})^{\frac{1}{7}} = x^{14/7}$

$\sqrt[7]{x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2} = \boxed{x^2}$

$(4^2)^3 = 4^{2 \cdot 3} = 4^6$

$4^2 \cdot 4^2 \cdot 4^2$

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Rational Exponents

Exponents that are fractions

$$\boxed{x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a}$$

Index

Denominator is Root Radical Index

Numerator is base's exponent

EXAMPLE 2 Understand Rational Exponents

Try It!

2. Explain what each fractional exponent means, then evaluate.

a. $25^{\frac{1}{2}} = 5$ or -5

$\sqrt[2]{25}$ or $\sqrt{25}$

b. $32^{\frac{2}{5}}$

$\left(\sqrt[5]{32}\right)^2$

$(2)^2$

4

Rewrite these as radicals and evaluate.

$27^{1/3}$

$\sqrt[3]{27}$

3

$16^{1/4}$

$\sqrt[4]{16}$

$4^{3/2}$

$\left(\sqrt{4}\right)^3$

2^3

8

$-8^{2/3}$

$-\sqrt[3]{8^2}$

$-\sqrt[3]{64}$

$\boxed{-4}$

$(-8)^{2/3}$

$\sqrt[3]{(-8)^2}$

$\sqrt[3]{64}$

$\boxed{4}$

$31^{-2/5}$

$\sqrt[5]{31^{-2}}$ or $\left(\sqrt[5]{31}\right)^{-2}$

$\sqrt[5]{\frac{1}{31^2}}$ $\qquad$ $\frac{1}{\left(\sqrt[5]{31}\right)^2}$

$\frac{\sqrt[5]{1}}{\sqrt[5]{31^2}}$ $\qquad$ $\frac{1}{(1.987)^2}$

take radical of top & bottom seperately

$\boxed{\frac{1}{\sqrt[5]{961}} \text{ or } \frac{1}{3.95} \text{ or } 0.253}$

$162^{-4/3}$

$-32^{3/5}$

$-(32)^{\frac{3}{5}}$

$-(\sqrt[5]{32})^3$

$-(2)^3$

-8

$27^{-2/3} = \frac{1}{9}$

$\frac{1}{27^{2/3}}$

$\frac{1}{(\sqrt[3]{27})^2}$

$\frac{1}{3^2}$

$\frac{1}{9}$

EXAMPLE 3 Evaluate Expressions With Rational Exponents

Try It!

3. What is the value of each expression? Round to the nearest hundredth if necessary.

a. $-(16^{\frac{3}{4}})$

$-(16^{\frac{3}{4}})$

$-(\sqrt[4]{16})^3$

$-(2)^3$

-8

b. $\sqrt[5]{3.5^4}$

$3.5^{\frac{4}{5}}$

type in calculator

3.5^(4/5)

2.72

B. The Fujita scale rating, *F*, of a tornado is represented by $F = \sqrt[3]{\left(\frac{W}{14.1}\right)^2} - 2$, where *W* is the estimated wind speed of the tornado in miles per hour. What is the Fujita scale rating of a tornado with estimated wind speeds of 100 mph?

$F = \sqrt[3]{\left(\frac{100}{14.1}\right)^2} - 2$

$F = \left(\frac{100}{14.1}\right)^{\frac{2}{3}} - 2$

(100/14.1)^(2/3) − 2

$F = 1.69$

F1 tornado

Simplify Radical expressions

1. Turn into a fractional exponent
2. Apply distribution multiplication of exponents
3. Simplify fraction exponents
4. Any leftover fractions can be changed back to radical form

IMPORTANT: Even root requires Absolute Value Bars.

These bars remain on any variable with an ODD exponent, to ensure its positivity.

EXAMPLE 4 Simplify *n*th Roots

Try It!

4. Simplify each expression.

a. $\sqrt[3]{-8a^3b^9}$

$(-8a^3b^9)^{\frac{1}{3}}$

$(-8)^{1\cdot\frac{1}{3}}\ a^{\frac{3}{1}\cdot\frac{1}{3}}\ b^{\frac{9}{1}\cdot\frac{1}{3}}$

$(-8)^{\frac{1}{3}}\ a^{\frac{3}{3}}\ b^{\frac{9}{3}}$

$\boxed{-2a^1b^3}$

b. $\sqrt[4]{256x^{12}y^{24}}$

$(256^1x^{12}y^{24})^{\frac{1}{4}}$

$256^{1\cdot\frac{1}{4}}\ x^{12\cdot\frac{1}{4}}\ y^{24\cdot\frac{1}{4}}$

$256^{\frac{1}{4}}\ x^{\frac{12}{4}}\ y^{\frac{24}{4}}$

$\boxed{4|x^3|y^6}$

Odd exponent needs Abs. Value

need groups of 3

$\sqrt[3]{54x^4}$

54
2 · 27
2 · 3 · 9
2 · 3 · 3 · 3

$(54x^4)^{\frac{1}{3}}$

$(2\cdot\underbrace{3\cdot3\cdot3}\cdot\underbrace{xxx}x)^{1/3}$

$(2\cdot3^3\cdot x^3\cdot x)^{1/3}$

$2^{1/3}\cdot3^{3/3}\cdot x^{3/3}\cdot x^{1/3}$

$3^1\cdot x^1(2\cdot x)^{1/3}$

$3x\sqrt[3]{2x}$

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These bars remain on any variable with an ODD exponent, to ensure its positivity.

EXAMPLE 4 Simplify *n*th Roots

Try It!

4. Simplify each expression.

a. $\sqrt[3]{-8a^3b^9}$

$(-8^1 a^3 b^9)^{\frac{1}{3}}$

$(-8)^{1/3} \cdot a^{\frac{3}{1}\cdot\frac{1}{3}} \cdot b^{\frac{9}{1}\cdot\frac{1}{3}}$

$-2 \quad a^{\frac{3}{3}} \quad b^{\frac{9}{3}}$

$\boxed{-2ab^3}$

b. $\sqrt[4]{256x^{12}y^{24}}$

$(256x^{12}y^{24})^{\frac{1}{4}}$

$256^{\frac{1}{4}} \; x^{\frac{12}{1}\cdot\frac{1}{4}} \; y^{24\cdot\frac{1}{4}}$

$|256^{1/4} \; x^{\frac{12}{4}} \; y^{\frac{24}{4}}|$

$\boxed{4|x^3|y^6}$

odd exp. needs Abs. Value

even expo. no need abs. val.

$\sqrt[3]{54x^4}$

$\sqrt[3]{27\cdot 2\cdot x^3\cdot x}$

$(27^1 \cdot 2^1 \cdot x^3 \cdot x^1)^{\frac{1}{3}}$

$27^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} \cdot x^{\frac{3}{3}} \cdot x^{\frac{1}{3}}$

$3x \cdot 2^{1/3} \cdot x^{1/3}$

$\boxed{3x\sqrt[3]{2x}}$

Exponent tells you # of solutions

- when you take an odd root
  - only 1 answer
- when you take an even root
  - 2 answers + and −
- fill in rest with imaginary, which come in pairs

EXAMPLE 5 Use *n*th Roots to Solve Equations

Try It!

5. a. Find the real solutions to the equation $5x^3 = 320$.

$$\frac{5x^3}{5} = \frac{320}{5}$$

$$x^3 = 64$$

$$\cancel{\sqrt[3]{x^3}} = \sqrt[3]{64}$$

$$\boxed{x = 4 \text{ and 2 imaginary solutions}}$$

b. Find the real solutions to the equation $2p^4 = 162$.

$$\frac{2p^4}{2} = \frac{162}{2}$$

$$p^4 = 81$$

$$\cancel{\sqrt[4]{p^4}} = \sqrt[4]{81}$$

$3 \cdot 3 \cdot 3 \cdot 3 = 81$

and

$-3 \cdot -3 \cdot -3 \cdot -3 = 81$

$$\boxed{p = \pm 3 \text{ and 2 imaginary solutions}}$$

Radicals and Roots

Index, Radical, Radicand

$\sqrt[3]{27}$ under radical

$\sqrt[3]{3 \cdot 3 \cdot 3} = 3$

$\sqrt[3]{27}$

0 1 2 3

Real or Imaginary?

Even indexed radicals must have a positive number underneath as the radicand in order to be a Real number

Odd indexed radicals can have a positive or negative number underneath as the radicand and be a Real number

$\sqrt[4]{16}$ $\sqrt[2]{-25}$ $\sqrt[3]{27}$ $\sqrt[5]{-32}$

$\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}$ $-5 \cdot -5 = 25$ Imaginary number $\sqrt[3]{3 \cdot 3 \cdot 3}$ $\sqrt[5]{-2 \cdot -2 \cdot -2 \cdot -2 \cdot -2}$

2 $5i$ 3 -2

Radicals as powers

*Radicals are fractional exponents!

Numerator - The exponent under the radical of the radicand

Denominator - the index of the radical

Rewrite the following radicals as fractional exponents and simplify.

$\sqrt[4]{16^1}$ $\sqrt[2]{25^1x^6}$ $\sqrt[3]{27x^4}$ $\sqrt[5]{-32^1x^{10}}$

$16^{\frac{1}{4}}$ $25^{\frac{1}{2}}x^{\frac{6}{2}}$ $27^{\frac{1}{3}}x^{\frac{4}{3}}$ $(-32)^{\frac{1}{5}}x^{\frac{10}{5}}$

16 ^(1/4) $5x^3$ $3x^{\frac{4}{3}}$ $-2x^2$