Radicals and Roots

Index, Radical, Radicand

$\sqrt[3]{27}$ under radical

$\sqrt[3]{3 \cdot 3 \cdot 3} = 3$

$\sqrt[3]{27}$: 0 1 2 3

Real or Imaginary?

Even indexed radicals must have a positive number underneath as the radicand in order to be a Real number

Odd indexed radicals can have a positive or negative number underneath as the radicand and be a Real number

$\sqrt[4]{16}$ → $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}$ → 2

$\sqrt[2]{-25}$ → $-5 \cdot -5 = 25$ → Imaginary number → $5i$

$\sqrt[3]{27}$ → $\sqrt[3]{3 \cdot 3 \cdot 3}$ → 3

$\sqrt[5]{-32}$ → $\sqrt[5]{-2 \cdot -2 \cdot -2 \cdot -2 \cdot -2}$ → -2

Radicals as powers

*Radicals are fractional exponents!

Numerator -The exponent under the radical of the radicand

Denominator - the index of the radical

Rewrite the following radicals as fractional exponents and simplify.

$\sqrt[4]{16}$ → $16^{\frac{1}{4}}$ → 16 ^(1/4)

$\sqrt[2]{25x^6}$ → $25^{\frac{1}{2}} x^{\frac{6}{2}}$ → $5x^3$

$\sqrt[3]{27x^4}$ → $27^{\frac{1}{3}} x^{\frac{4}{3}}$ → $3x^{\frac{4}{3}}$

$\sqrt[5]{-32x^{10}}$ → $(-32)^{\frac{1}{5}} x^{\frac{10}{5}}$ → $-2x^2$

$-8^{2/3}$ → $-\sqrt[3]{8^2}$ → $-\sqrt[3]{64}$ → $\boxed{-4}$

$(-8)^{2/3}$ → $\sqrt[3]{(-8)^2}$ → $\sqrt[3]{64}$ → $\boxed{4}$

$31^{-2/5}$ → $\sqrt[5]{31^{-2}}$ or $(\sqrt[5]{31})^{-2}$

$\sqrt[5]{\frac{1}{31^2}}$ → $\frac{\sqrt[5]{1}}{\sqrt[5]{31^2}}$

$\frac{1}{(\sqrt[5]{31})^2}$ → $\frac{1}{(1.987)^2}$

take radical of top & bottom seperately

$\boxed{\frac{1}{\sqrt[5]{961}} \text{ or } \frac{1}{3.95} \text{ or } 0.253}$

$162^{-4/3}$

Rational Exponents

Exponents that are fractions

$x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$ Index

Denominator is Root Radical Index

Numerator is base's exponent

EXAMPLE 2 Understand Rational Exponents

Try It!

2. Explain what each fractional exponent means, then evaluate.

a. $25^{\frac{1}{2}}$ $= 5$ or -5

$\sqrt[2]{25}$ or $\sqrt{25}$

b. $32^{\frac{2}{5}}$

$\left(\sqrt[5]{32}\right)^2$

$(2)^2$

4

Rewrite these as radicals and evaluate.

$27^{1/3}$ $\quad$ $16^{1/4}$ $\quad$ $4^{3/2}$

$\sqrt[3]{27}$ $\quad$ $\sqrt[4]{16}$ $\quad$ $\left(\sqrt{4}\right)^3$

3 $\quad\quad\quad\quad$ 2^3

8

Nth Root

what number times itself n times, equals the base?

Find the real cube roots of −64.

$\sqrt[3]{-64} = -4$

$\sqrt[3]{-4 \cdot -4 \cdot -4}$

$\cancel{\sqrt[3]{(-4)^3}}$

-4

The nth root of a number is the number raised to the 1/n power

$\sqrt[3]{(-4)^3} = \left((-4)^3\right)^{\frac{1}{3}}$

$(-4)^{3 \cdot \frac{1}{3}} = \boxed{-4^1}$

EXAMPLE 1 Find All Real *n*th Roots

Try It!

1. Find the specified roots of each number.

a. real fourth roots of 81 — Even Root — 2 answers $+$ $-$

$\sqrt[4]{81} = \boxed{3}$

$3 \cdot 3 \cdot 3 \cdot 3 = 81$

$\sqrt[4]{81} = \boxed{-3}$

$-3 \cdot -3 \cdot -3 \cdot -3 = 81$

$9 \quad 9$

Note: can't take even roots of negative #s → Imaginary

b. real cube roots of 64 — Odd root — 1 answer

$\sqrt[3]{64}$

$4 \cdot 4 \cdot 4 = 64$

$\boxed{4}$

square root of 64

$\sqrt[2]{64} = \sqrt{8 \cdot 8}$

8

$\sqrt{64} = \sqrt{-8 \cdot -8}$

-8

7th root of x^{14}

$(x^{14})^{\frac{1}{7}} = x^{14/7}$

$\sqrt[7]{x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2} = \boxed{x^2}$

$(4^2)^3 = 4^{2 \cdot 3} = 4^6$

$4^2 \cdot 4^2 \cdot 4^2$

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Radical and Rational Exponents Level 1 Name: ______________________

Perfect Squares		Perfect Cubes	
$2^2 = 4$	$\sqrt{4} = 2$	$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 9$	$\sqrt{9} = 3$	$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^2 = 16$	$\sqrt{16} = 4$	$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^2 = 25$	$\sqrt{25} = 5$	$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^2 = 36$	$\sqrt{36} = 6$	$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^2 = 49$	$\sqrt{49} = 7$	$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^2 = 64$	$\sqrt{64} = 8$	$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^2 = 81$	$\sqrt{81} = 9$	$9^3 = 725$	$\sqrt[3]{725} = 9$
$10^2 = 100$	$\sqrt{100} = 10$	$10^3 = 1{,}000$	$\sqrt[3]{1000} = 10$
$11^2 = 121$	$\sqrt{121} = 11$	$11^3 = 1{,}331$	$\sqrt[3]{1331} = 11$
$12^2 = 144$	$\sqrt{144} = 12$	$12^3 = 1{,}728$	$\sqrt[3]{1{,}728} = 12$
$13^2 = 169$	$\sqrt{169} = 13$		
$14^2 = 196$	$\sqrt{196} = 14$		
$15^2 = 225$	$\sqrt{225} = 15$		
$20^2 = 400$	$\sqrt{400} = 20$		
$30^2 = 900$	$\sqrt{900} = 30$		

Other Powers to know

$2^4 = 16$	$\sqrt[4]{16} = 2$
$3^4 = 81$	$\sqrt[4]{81} = 3$
$4^4 = 256$	$\sqrt[4]{256} = 4$
$2^5 = 32$	$\sqrt[5]{32} = 2$
$2^6 = 64$	$\sqrt[6]{64} = 2$

To solve and get the Real Roots for an equation with an exponent…

1. You need to get the variable with the exponent alone (undo addition/subtraction, undo multiplication/division)
2. Undo the exponent by taking the root with the same index as the exponent. (square root undoes square, cube root undoes cube, $\sqrt[4]{\ }$ undoes x^4)
3. Type that radical into your calculator using the blue 2nd button and the ^ to get $\sqrt[x]{\ }$ ($\sqrt[4]{16}$ would be 4 $\sqrt[x]{\ }$ 16)

	Explain what happened from the step before
$14x^5 + 7 = 100$ $-7 \quad -7$	
$14x^5 = 93$ $\overline{14} \quad \overline{14}$	Subtracted 7 on both sides
$x^5 = 6.64$	÷ by 14 on both sides
$\sqrt[5]{x^5} = \sqrt[5]{6.64}$	Undo exponent of 5 with 5th root on both
x = 1.46 and 4 imaginary non-real solutions.	Odd roots have 1 real and the rest imaginary

Radical and Rational Exponents Level 1 Name: ______________________

L1		
Find the real roots of the following questions and say how many imaginary.	$x^3 = -125$ $x^3 + 125 = 0$ $(x+5)(x^2 - 5x + 25)$ $x = -5 \quad x = \frac{+5 \pm \sqrt{(-5)^2 - 4(1)(25)}}{2(1)}$	$x^4 = 81$ $x^4 - 81 = 0$ $(x^2-9)(x^2+9)$ $(x+3)(x-3)(x+3i)(x-3i)$ $\sqrt[4]{x^4} = \sqrt[4]{81}$ $x = +3$ $x = -3$ and 2 imaginary and 3i and -3i
	$2x^3 = 16$	$x^2 + 7 = 71$

L1			
Rewrite in radical form and simplify the following.	Ex. $64^{\frac{2}{3}}$ $(\sqrt[3]{64})^2$ Since $4 \cdot 4 \cdot 4 = 64$ The inside becomes $(4)^2$ Now you can apply the exponent 16	$125^{\frac{2}{3}}$	$49^{\frac{1}{2}}$
		$32^{\frac{3}{5}}$	$144^{\frac{3}{2}}$ $(\sqrt[2]{144})^3$ $(12)^3 = 1{,}728$

L1			
Rewrite using rational exponents (exponents with fractions 😊). denominator is index	Ex. $\sqrt[4]{256^3}$ $(256)^{\frac{3}{4}}$	$\sqrt[3]{6}$	$\sqrt[5]{31}$
		$\sqrt[3]{50^2}$ $50^{\frac{2}{3}}$	$(\sqrt{70})^5$

L1			
Simplify the radical.	Ex. $\sqrt{16x^6}$ $\sqrt[2]{2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$ Index 2 = need groups of 2 $\sqrt{2^2 \cdot 2^2 \cdot x^2 \cdot x^2 \cdot x^2}$ Square root undoes the squares $2 \cdot 2 \cdot x \cdot x \cdot x = \boxed{4x^3}$	$\sqrt{196x^3}$ $(14^2 \cdot x^2 \cdot x)^{\frac{1}{2}}$ $14^{\frac{2}{2}} \cdot x^{2/2} \cdot x^{\frac{1}{2}}$ $\boxed{14x\sqrt{x}}$ $\sqrt{2 \cdot 2 \cdot 7 \cdot 7 \cdot x \cdot x \cdot x}$ $\sqrt{2^2 \cdot 7^2 \cdot x^2 \cdot x}$ $2 \cdot 7 \cdot x\sqrt{x}$ $14x\sqrt{x}$	$\sqrt{81x^5}$
	Or $\sqrt{4^2 \cdot x^2 \cdot x^2 \cdot x^2}$ $(4^2 \cdot x^6)^{\frac{1}{2}}$ $4^{\frac{2}{2}} \cdot x^{\frac{6}{2}}$ $4x^3$	$\sqrt{25x^{10}}$	$\sqrt{400x^4}$

$-32^{3/5}$

$-(32)^{\frac{3}{5}}$

$-\left(\sqrt[5]{32}\right)^3$

$-(2)^3$

-8

$27^{-2/3} = \frac{1}{9}$

$\frac{1}{27^{2/3}}$

$\frac{1}{\left(\sqrt[3]{27}\right)^2}$

$\frac{1}{3^2}$

$\frac{1}{9}$

EXAMPLE 3 Evaluate Expressions With Rational Exponents

Try It!

3. What is the value of each expression? Round to the nearest hundredth if necessary.

a. $-(16^{\frac{3}{4}})$

$-(16^{\frac{3}{4}})$

$-\left(\sqrt[4]{16}\right)^3$

$-(2)^3$

-8

b. $\sqrt[5]{3.5^4}$

$3.5^{\frac{4}{5}}$

type in calculator

3.5^(4/5)

2.72

B. The ~~Fujita scale rating~~, F, of a tornado is represented by $F = \sqrt[3]{\left(\frac{W}{14.1}\right)^2} - 2$, where W is the estimated wind speed of ~~the tornado in miles per hour~~. What is the Fujita scale rating of a tornado with estimated wind speeds of 100 mph?

$F = \sqrt[3]{\left(\frac{100}{14.1}\right)^2} - 2$

$F = \left(\frac{100}{14.1}\right)^{\frac{2}{3}} - 2$

(100/14.1)^(2/3) −2

F=1.69

F1 tornado

Simplify Radical expressions

1. Turn into a fractional exponent
2. Apply distribution multiplication of exponents
3. Simplify fraction exponents
4. Any leftover fractions can be changed back to radical form

IMPORTANT: Even root requires Absolute Value Bars.

These bars remain on any variable with an ODD exponent, to ensure its positivity.

EXAMPLE 4 Simplify *n*th Roots

Try It!

4. Simplify each expression.

a. $\sqrt[3]{-8a^3b^9}$

$(-8a^3b^9)^{\frac{1}{3}}$

$(-8)^{\frac{1}{3}}\ a^{\frac{3}{1}\cdot\frac{1}{3}}\ b^{\frac{9}{1}\cdot\frac{1}{3}}$

$(-8)^{\frac{1}{3}}\ a^{\frac{3}{3}}\ b^{\frac{9}{3}}$

$\boxed{-2a^1b^3}$

b. $\sqrt[4]{256x^{12}y^{24}}$

$(256^1x^{12}y^{24})^{\frac{1}{4}}$

$256^{1\cdot\frac{1}{4}}\ x^{12\cdot\frac{1}{4}}\ y^{24\cdot\frac{1}{4}}$

$\left|256^{\frac{1}{4}}\ x^{\frac{12}{4}}\ y^{\frac{24}{4}}\right|$

$4|x^3|y^6$

odd exp. needs Abs. Value

even expo. no abs. val.

need groups of 3

$\sqrt[3]{54x^4}$

$(54x^4)^{\frac{1}{3}}$

$(2\cdot 3\cdot 3\cdot 3\cdot xxxx)^{1/3}$

$(2\cdot 3^3\cdot x^3\cdot x)^{1/3}$

$2^{1/3}\cdot 3^{3/3}\cdot x^{3/3}\cdot x^{1/3}$

$3^1\cdot x^1(2\cdot x)^{1/3}$

$3^1x^1\sqrt[3]{2x}$

54

2 · 27

2 · 3 · 9

2 · 3 · 3 · 3

Radical and Rational Exponents Level 2 Name: ____________________

L2 Find the real roots of the following questions and say how many imaginary.	$3x^3 = 648$	$15x^2 - 10 = 230$
	$50 - x^5 = 293$	$50 + 2x^4 = 562$

L2 Rewrite in radical form and simplify the following.	Ex. $\boxed{-64^{\frac{2}{3}}}$ $-(\sqrt[3]{64})^2$ $-(4)^2$ -16 The negative is not raised to the power.	$-216^{\frac{2}{3}}$	$-36^{\frac{1}{2}}$ $-(\sqrt{36})$ $-(6)$ -6	$(-36)^{\frac{1}{2}}$ $\sqrt{-36} = \sqrt{-1} \cdot \sqrt{36}$ imaginary $i \cdot 6$ $6i$
	Ex. $\boxed{(-64)^{\frac{2}{3}}}$ $(\sqrt[3]{-64})^2$ $(-4)^2$ 16 The negative is raised to the power. Note: $-4 \cdot -4 = +16$ -4 · -4 · -4 = - 64	$(-27)^{\frac{4}{3}}$	$(-32)^{\frac{3}{5}}$	

Rewrite using positive rational exponents (exponents with fractions 😊) AND SIMPLIFY if possible.

L2	Ex. $\sqrt[4]{512^{-3}}$ $(512)^{-\frac{3}{4}} = \frac{1}{512^{\frac{3}{4}}}$	$\sqrt{90^8}$	$\sqrt[7]{100^{-2}}$ $100^{-\frac{2}{7}} = \frac{1}{100^{2/7}}$	$(\sqrt[3]{10})^6$

L2 Simplify the radical. **Note:** Odd roots don't need absolute value bars, even roots do!	Method 1 $\sqrt{108x^4y^3}$ $\sqrt[2]{2 \cdot 3 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y}$ Index 2 = need groups of 2 $\sqrt{2^2 \cdot 3^2 \cdot x^2 \cdot x^2 \cdot y^2 \cdot 3y}$ $2 \cdot 3 \cdot \lvert x\rvert \cdot \lvert x\rvert \cdot \lvert y\rvert \cdot \sqrt{3y}$ $6x^2\lvert y\rvert\sqrt{3y}$	$\sqrt{80c^9d^4}$	$\sqrt{128x^{12}y^7}$
	Method 2 $\sqrt[3]{1024x^{13}}$ $\sqrt[3]{8 \cdot 8 \cdot 8 \cdot 2 \cdot x^3 \cdot x^3 \cdot x^3 \cdot x^3 \cdot x}$ Index 3 = need groups of 3 $\sqrt[3]{8^3 \cdot x^{12} \cdot 2x}$ $8^{\frac{3}{3}}x^{\frac{12}{3}}\sqrt[3]{2x}$ $8x^4\sqrt[3]{2x}$	$\sqrt[3]{81x^5y^8}$	$\sqrt[3]{343a^6d^9}$

Radicals Level 3 and 4 Name: Date:

L3 Rewrite in radical form and simplify the following.	$16^{\frac{-3}{2}}$	$49^{\frac{3}{4}} \cdot 49^{\frac{-1}{4}}$	First, distribute exponents Then, turn into radical w/ exponent to simplify	$\left(\frac{3}{32^{\frac{2}{5}}}\right)^{\frac{1}{2}}$	$2a^{\frac{1}{3}}\left(ab^{\frac{1}{2}}\right)^{\frac{2}{3}}$

L3 Rewrite using rational and positive exponents.	$\left(\sqrt[3]{50}\right)^{-1}$	$\left(\sqrt[5]{5^3}\right)^2$	$\sqrt[3]{\frac{x^4y^2}{125x}}$

L3 Simplify the radical expression.	$\sqrt[3]{1024x^{13}}$ Index 3 = need groups of 3 $\sqrt[3]{8 \cdot 8 \cdot 8 \cdot 2 \cdot x^3 \cdot x^3 \cdot x^3 \cdot x^3 \cdot x}$ $\sqrt[3]{8^3 \cdot x^{12} \cdot 2x}$ $8^{\frac{3}{3}}x^{\frac{12}{3}}\sqrt[3]{2x}$ $8x^4\sqrt[3]{2x}$	$\sqrt{200x^3y^6}$	$\sqrt[3]{250v^2w^{11}x^6}$
L4	$\sqrt[5]{64w^{11}x^{25}}$	$\sqrt[6]{64w^{11}x^{25}y^{-12}}$	$\sqrt[5]{\frac{7}{16x^3}}$

Solving Radical and Exponent Equations Level 3 and 4

To solve and get the solutions to an equation with a radical… You need to get the radical alone first, then you undo the radical with the exponent matching the index on both sides. This might cause you to have to distribute binomials

$3^2 = 9$ but $(x+3)^2 = x^2 + 6x + 9$ (not $x^2 + 9$)

ex. 1 $3\sqrt{x+4} - 6 = 12$

Treat this equation like a normal 2-step equation… add 6 then divide by 3.

$$3\sqrt{x+4} - 6 = 12$$
$$+6 \quad +6$$
$$\frac{3\sqrt{x+4}}{3} = \frac{18}{3}$$
$$\sqrt{x+4} = 6$$

Now we need to get rid of the square root… to do this you will SQUARE both sides!

$$(\sqrt{x+4})^2 = (6)^2$$

The square root will cancel and you're left w/ a one-step equation!

$$x + 4 = 36$$
$$-4 \quad -4$$
$$x = 32$$

try a few on the back!

Radicals Level 3 and 4 Name: Date:

For this question instead of squaring both sides at the end you'll need to CUBE both sides!

1.) $2\sqrt{x+15}+5=11$

2.) $-10\sqrt[3]{x}+18=48$

SHOW YOUR WORK!

SHOW YOUR WORK!

$x=-6$

$x=-27$

When your equation has a rational exponent then try solving like this ex. $(x-6)^{\frac{1}{2}}=5$

$\left((x-6)^{\frac{1}{2}}\right)^2=(5)^2$

$x-6=25$

$\boxed{x=31}$

Try one here

5.) $(x-5)^{\frac{1}{3}}=6$

$(x+2)^{\frac{1}{4}}=3$

$\left((x+2)^{\frac{1}{4}}\right)^4=(3)^4$

$x+2=81$

$\boxed{x=79}$

3. $\sqrt{x-7}+4=9$ L3

4. $-5\sqrt{x}+19=34$ L3

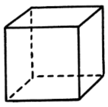
6. $3(x+1)^{\frac{1}{2}}-5=4$ L3

Level 3 and 4 Solving Equations

7. Jeanne's bank account earns interest annually. The equation below shows her starting balance of \$400 and her balance at the end of five years, \$535.29. At what rate r did Jeanne earn interest?

$535.29 = 400(1+r)^5$

8. One cube has an edge length 5 cm shorter than the edge length of the second cube. The volume of the smaller cube is 216 cm^3. What is the volume of the larger cube?



$volume = (side)^3$

Radicals Level 3 and 4 Name: Date:

L4 Rewrite the Product

$8^{1/6}$

$(2^3)^{1/6}$

$2^{3/6}$

$\sqrt[6]{8x}\cdot\sqrt[3]{2x}$

$(8x)^{1/6}(2x)^{1/3}$

$8^{1/6}\cdot x^{1/6}\cdot 2^{1/3}\cdot x^{1/3}$

$2^{3/6+\frac{2}{6}}$ $x^{1/6+2/6}$

$2^{5/6}\cdot x^{3/6} \rightarrow \sqrt[6]{2^5\cdot x^3}$

$\sqrt[5]{16}\cdot\sqrt[5]{8} = 2\sqrt[5]{4}$

$2^4\cdot 2^3$

$\sqrt[5]{16\cdot 8}$

$\sqrt[5]{128}$

$\sqrt[5]{2^7}=\sqrt[5]{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2}=\sqrt[5]{2^5\cdot 2^2}$

group of 5

$2\sqrt[5]{2^2}$

$\sqrt[4]{x^8}\cdot\sqrt{x^3}$

$x^{\frac{8}{4}}\cdot x^{\frac{3}{2}}$

To add exponents, you need a common denominator

$x^{\frac{4}{2}}\cdot x^{\frac{3}{2}}$

$x^{\frac{4}{2}+\frac{3}{2}}$

$x^{\frac{7}{2}} = |x^3|\sqrt{x}$

$\sqrt[2]{x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x} = x\cdot x\cdot x\cdot\sqrt{x}$

$\sqrt[4]{125yx^6}\cdot\sqrt{5xy^2}$

Explain the error in the work below:

$\sqrt{-3}\cdot\sqrt{-12}=\sqrt{(-3)(-12)}=\sqrt{36}=6$

L4 Add and Subtract Radical Expressions

Must be same radical with same number underneath to be like terms and combined. So you must simplify each one first.

$\sqrt{20}-\sqrt[3]{16}+\sqrt[3]{250}-\sqrt{5}$

$\sqrt{4\cdot5}-\sqrt[3]{8\cdot2}+\sqrt[3]{125\cdot2}$

$\sqrt{2^2\cdot5}-\sqrt{5}-\sqrt[3]{2^3\cdot2}+\sqrt[3]{5^3\cdot2}$

$2\sqrt{5}-\sqrt{5}-2\sqrt[3]{2}+5\sqrt[3]{2}$

$(2-1)\sqrt{5}+(-2+5)\sqrt[3]{2}$

$\sqrt{5}+3\sqrt[3]{2}$

$\sqrt{63}-\sqrt{700}-\sqrt{112}$

$\sqrt{20}-\sqrt{600}-\sqrt{125}$

$\sqrt[3]{2{,}000}+\sqrt{2}-\sqrt[3]{128}$

$\sqrt{75}-\sqrt[3]{81}+\sqrt{3}+\sqrt[3]{192}$

L4 Multiply Binomial Radical Expressions

Numbers outside of radical can be multiplied together. If the radical's index is the same, then the numbers underneath (radicands) can be multiplied together

Ex. $\sqrt[3]{7}(2-\sqrt[3]{49})$

$\sqrt[3]{7}(2)-\sqrt[3]{7}(\sqrt[3]{49})$

$\sqrt[3]{7}(2)-\sqrt[3]{343}$

$2\sqrt[3]{7}-7$

$\sqrt{6}\,(5+\sqrt{3})$

$\sqrt{5}\,(6+\sqrt{2})$

$(x-\sqrt{10})(x+\sqrt{10})$

$(3x-\sqrt{5})(4x+2\sqrt{10})$

$12x^2+6x\sqrt{10}-4x\sqrt{5}-2\sqrt{50}$

$2\sqrt{25\cdot2}$

$2\sqrt{5^2\cdot2}$

$2\cdot5\sqrt{2}$

$\boxed{12x^2+6x\sqrt{10}-4x\sqrt{5}-10\sqrt{2}}$

Exponent tells you # of solutions

- when you take an odd root – only 1 answer
- when you take an even root – 2 answers ± and –
- fill in rest with imaginary, which come in pairs

EXAMPLE 5 Use *n*th Roots to Solve Equations

Try It!

5. a. Find the real solutions to the equation $5x^3 = 320$.

$$\frac{5x^3}{5} = \frac{320}{5}$$

$$x^3 = 64$$

$$\sqrt[3]{x^3} = \sqrt[3]{64}$$

$$\boxed{x = 4 \text{ and } 2 \text{ imaginary solutions}}$$

b. Find the real solutions to the equation $2p^4 = 162$.

$$\frac{2p^4}{2} = \frac{162}{2}$$

$$p^4 = 81$$

$$\sqrt[4]{p^4} = \sqrt[4]{81}$$

$3 \cdot 3 \cdot 3 \cdot 3 = 81$
and
$-3 \cdot -3 \cdot -3 \cdot -3 = 81$

$$\boxed{p = \pm 3 \text{ and } 2 \text{ imaginary solutions}}$$

Radicals Level 3 and 4 Name: Date:

L4 Rationalize a Binomial Denominator To divide radical expressions, you just rationalize them. Multiply by what it will take for the radical to become a whole number. Either the conjugate like the example below or the rest of the factors needed (ex. if you have $\sqrt[4]{5}$ you would multiply by $\sqrt[4]{5^3}$ so that it would become $\sqrt[4]{5^4} = 5$)	$\frac{-4x}{1-\sqrt{x}}$	$\frac{\sqrt{7}}{\sqrt{5}+3}$
$\frac{1}{2+\sqrt{5}}$ $\quad \frac{1}{2+\sqrt{5}} \cdot \frac{2-\sqrt{5}}{2-\sqrt{5}}$ $4 - 2\sqrt{5} + 2\sqrt{5} - \sqrt{25}$ $\frac{2-\sqrt{5}}{4-5}$ $\frac{2-\sqrt{5}}{-1}$ $\sqrt{5}-2$	$\frac{5-\sqrt{2}}{2-\sqrt{3}}$	$\frac{12}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}}$

L4 Solve the Equations Only one radical allowed on each side, raise it to the exponent to undo it. Expand and move to one side to see if you can factor. If not, isolate the radical expression, undo leftover radical, factor and solve. **Check for extraneous solutions!**	$x = \sqrt{7x+8}$	$(x+2)^2 = (\sqrt{x+2})^2$ $(x+2)(x+2) = x+2$ $x^2+4x+4 = x+2$ $-x \ -2 \quad -x \ -2$ $x^2+3x+2 = 0$ $(\quad x)(\quad) = 0$
$\sqrt{x+9} - \sqrt{2x} = 3$ $\sqrt{x+9} = \sqrt{2x} + 3$ $(\sqrt{x+9})^2 = (\sqrt{2x}+3)^2$ $\quad (\sqrt{2x}+3)(\sqrt{2x}+3)$ $x+9 = 2x + 6\sqrt{2x} + 9$ $\quad \sqrt{4x^2} + 3\sqrt{2x} + 3\sqrt{2x} + 9$ $-9 \quad -9$ $0 = x + 6\sqrt{2x}$ $\quad -x \quad -x$ $-x = 6\sqrt{2x}$ $(-x)^2 = (6\sqrt{2x})^2$ $\quad 36\sqrt{4x^2}$ $\quad 36 \cdot 2x$ $\quad 72x$ $x^2 - 72x = 0$ $x(x-72) = 0$ $x = 0$ or $x = 72$ ✓ extraneous	$\sqrt{x+4} - \sqrt{3x} = 2$	$\sqrt{15-x} - \sqrt{6x} = -3$

Level 4 Application Continued

B. The design shows the boards needed for bracing the back of some set scenery. Will 75 ft of wood be enough for all of the bracing?