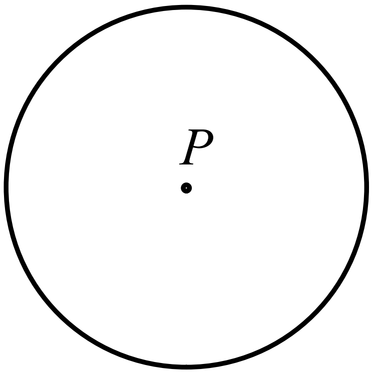
10.1 - Intro to Circles

Circle = Set of all points in a plane that are equidistant from a given point called the ***center***.



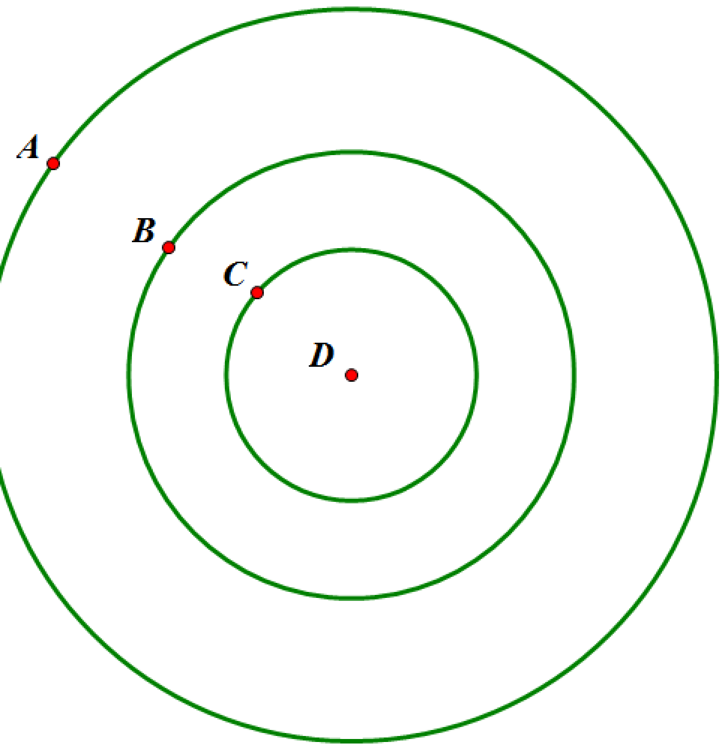
-Circles are labeled by the point at the center of the circle.

The circle to the left is called:

Circle P

$\odot P$

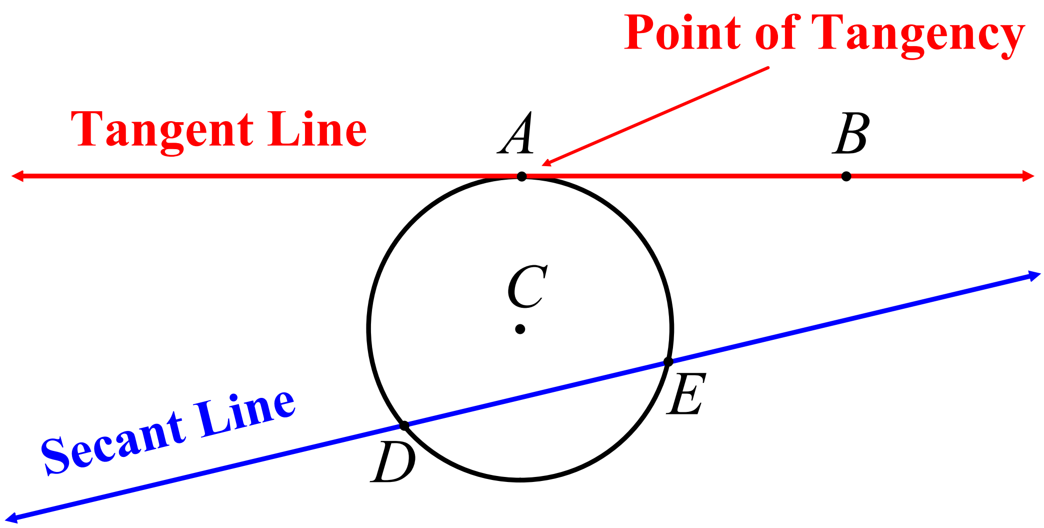
Concentric Circles: Circles that share a common center.



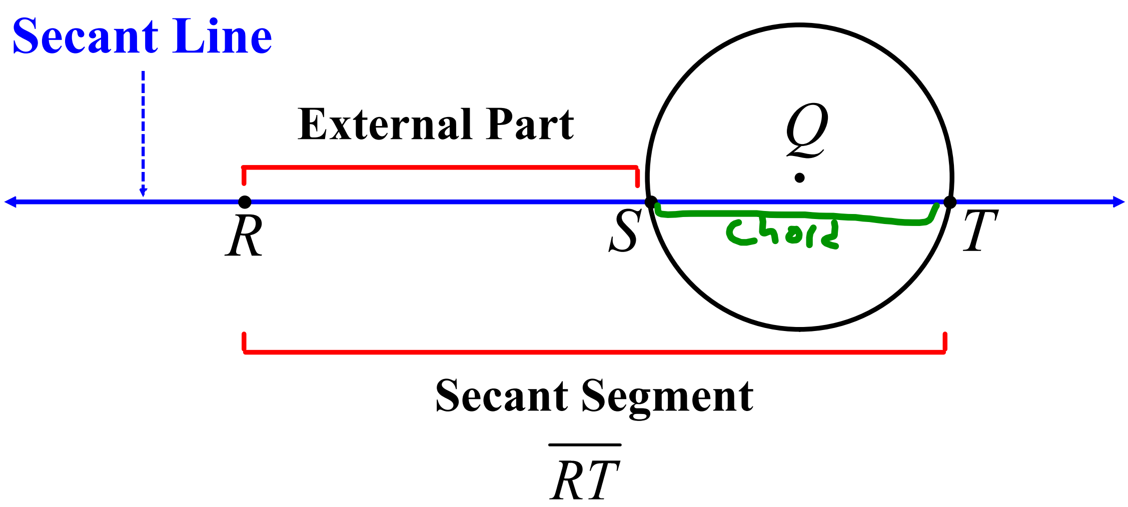
Secant Line = A line that intersects a circle at exactly two points.

Tangent Line = A line that intersects a circle at exactly one point.

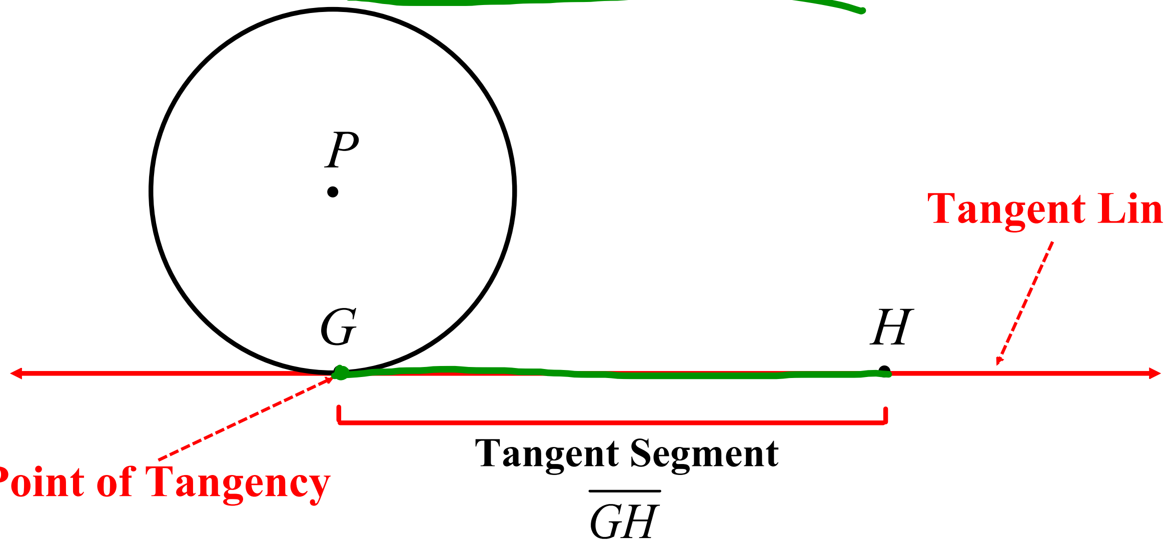
point of tangency **=** The point where a tangent line intersects a circle



Secant Segment = The part of a secant line connecting a point outside a circle with the farthest intersection point of the secant line and the circle.



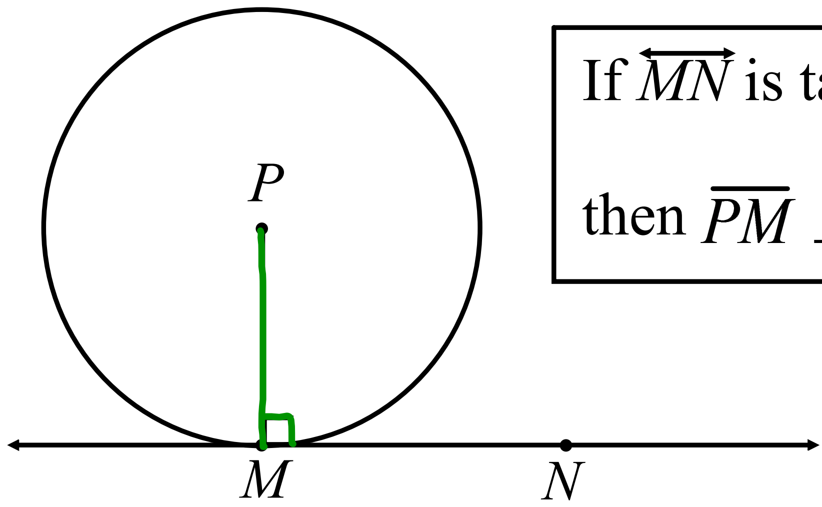
Tangent Segment = The part of a tangent line between the point of tangency and a point outside the circle.



Tangent Segment Theorem #1 = If a segment, ray, or line is tangent to a circle, then it is perpendicular to the radius connected to the point of tangency.

$\perp$ 90°

Note: The converse of this theorem is also true.

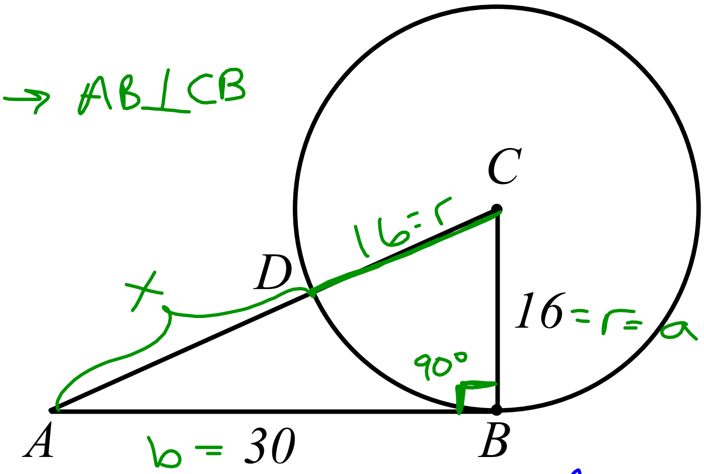


If $\overleftrightarrow{MN}$ is tangent to $\odot P$, then $\overline{PM} \perp \overleftrightarrow{MN}$.

Example #1

Given: $\overline{AB}$ is tangent to $\odot C$ $\rightarrow$ $AB \perp CB$

Find: Length of $\overline{AD}$ = 18



$a^2 + b^2 = c^2$

$16^2 + 30^2 = (x+16)^2$

$256 + 900 = (x+16)^2$

$\sqrt{1156} = \sqrt{(x+16)^2}$

$34 = x + 16$

$-16 \quad -16$

$18 = x$

or recognize Pythagorean Triple!

17.2 | 34 | 16 | 8.2 | 30 | 15.2

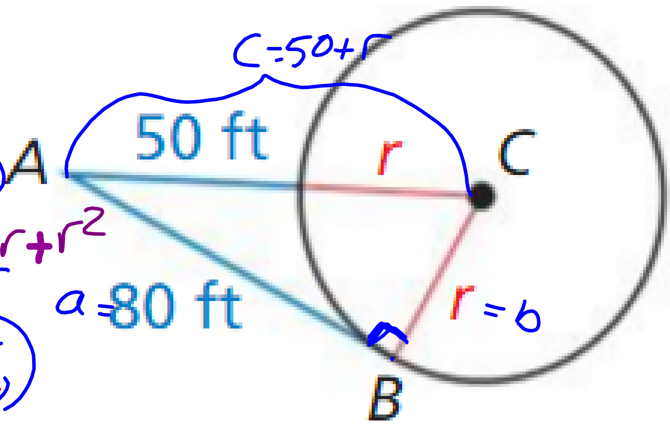
8-15-17

$\overline{AD} = 34 - 16$

$\overline{AD} = 18$

In the diagram, B is a point of tangency of circle C.

Find the radius of the circle.



$a^2 + b^2 = c^2$

$80^2 + r^2 = (50+r)^2$

$6400 + r^2 = (50+r)(50+r)$

$6400 + r^2 = 2500 + 50r + 50r + r^2$

$6400 + r^2 = 2500 + 100r + r^2$

$-r^2 \quad -r^2$

$6400 = 2500 + 100r$

$-2500 \quad -2500$

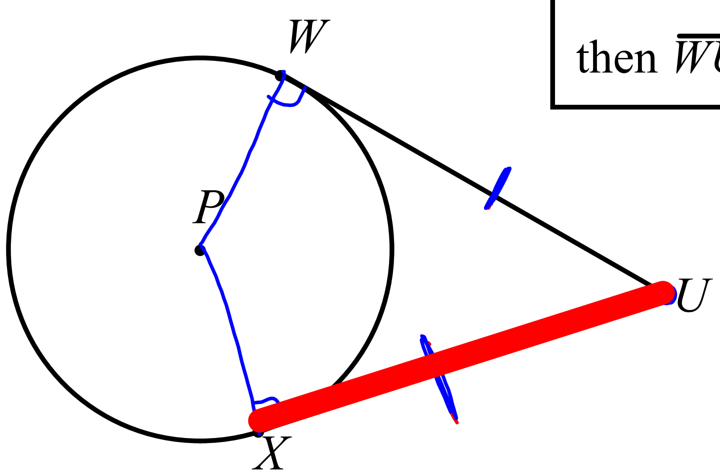
$\frac{3900}{100} = \frac{100r}{100}$

$39 = r$

Tangent Segment Theorem #2 = If two segments starting from the same external point are tangent to the same circle, then they are congruent.

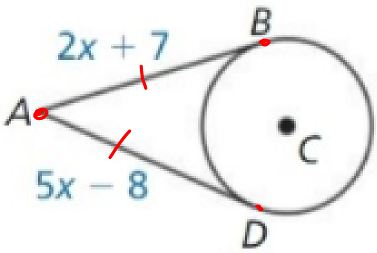
If $\overline{WU}$ and $\overline{XU}$ are tangent to $\odot P$

then $\overline{WU} \cong \overline{XU}$.



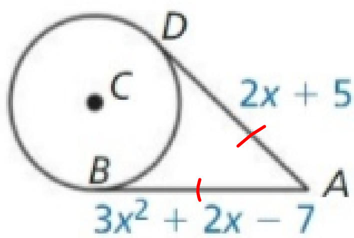
In Exercises 29–32, points *B* and *D* are points of tangency. Find the value(s) of *x*. *(See Example 5.)*

29.



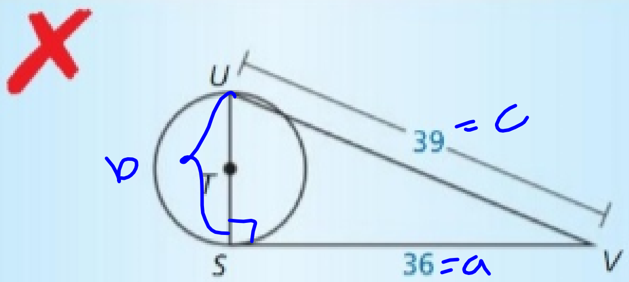
$2x+7=5x-8$
$-2x \quad -2x$
$7=3x-8$
$+8 \quad +8$
$\frac{15}{3}=\frac{3x}{3}$
$5=x$

32.



$2x+5=3x^2+2x-7$
$-2x \quad -2x$
$5=3x^2-7$
$+7 \quad +7$
$\frac{12}{3}=\frac{3x^2}{3}$
$\sqrt{4}=\sqrt{x^2}$
$\pm 2=x$
$x=2,\ x=-2$

34. ERROR ANALYSIS Describe and correct the error in finding the radius of $\odot T$.



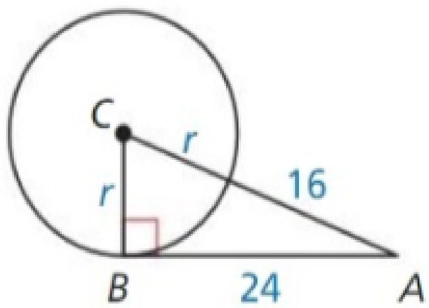
$39^2 - 36^2 = 15^2$
So, the radius is 15.

the diameter is 15
the radius is then 7.5

$d = 2r$
$\frac{15}{2} = \frac{2r}{2}$
$7.5 = r$

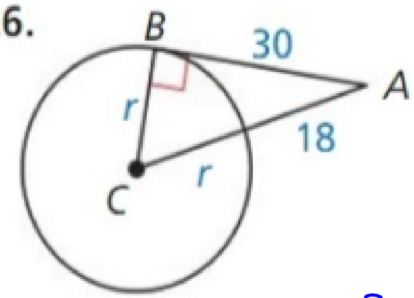
In Exercises 23–26, point B is a point of tangency. Find the radius r of $\odot C$. *(See Example 4.)*

23.



$r^2 + 24^2 = (r+16)^2$
$r^2 + 576 = r^2 + 32r + 256$
$-r^2 \qquad -r^2$
$576 = 32r + 256$
$-256 \qquad -256$
$\frac{320}{32} = \frac{32r}{32}$
$10 = r$

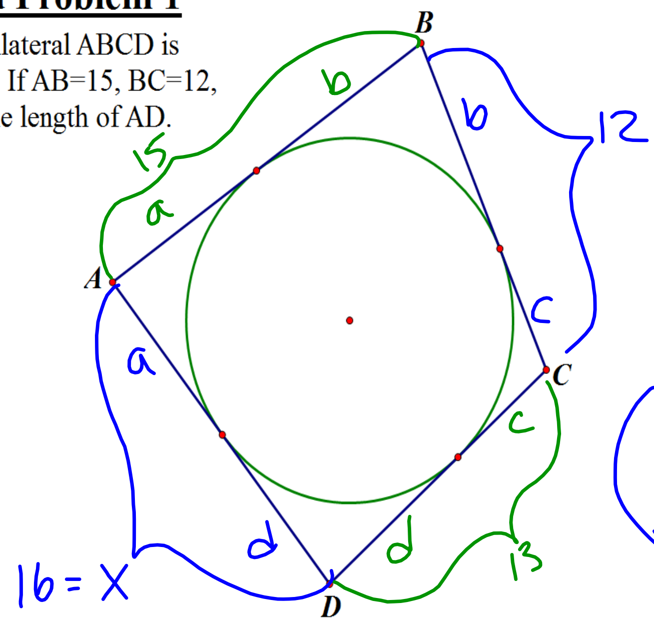
26.



$r^2 + 30^2 = (r+18)^2$
$r^2 + 900 = r^2 + 36r + 324$
$900 = 36r + 324$
$576 = 36r$
$r = 16$

Walk-Around Problem 1

Each side of quadrilateral ABCD is tangent to circle E. If AB=15, BC=12, and CD=13, find the length of AD.



$a+b+c+d$

$15 + 13 = 28$

$b+c + a+d$

$12 + X$

but

$a+b+c+d = 28$

so

$12 + x = 28$ too!

$-12 \quad\quad -12$

$x = 16$