Your Name

Mrs. Theo

12/5/22

Notes

2-6 Quadratic Formula

Discriminant

Workbooks pg. 54

Quadratic Formula

When you can't factor or it would take a long time to find the factors, you can always find the x-intercept solutions using

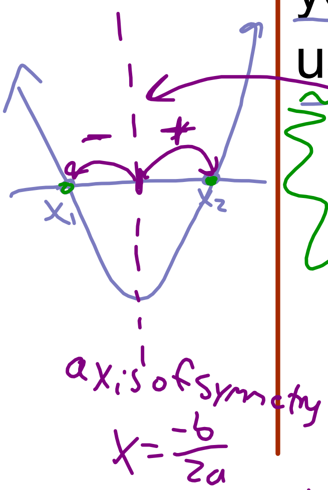
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\pm$ creates two solutions

x = __ and x = __

(x,0) and (x,0)

Why?



EXAMPLE 1 Solve Quadratic Equations

Workbooks pg. 54

Try It! $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

1. Solve using the Quadratic Formula.

a. $2x^2 + 6x + 3 = 0$

$a=2 \quad b=6 \quad c=3$

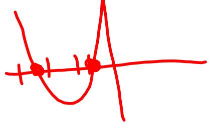
$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(3)}}{2(2)}$

$x = \frac{-6 \pm \sqrt{12}}{4}$

$x = \frac{-6 \pm 3.464}{4}$

$x = \frac{(-6+3.464)}{4}$ { $x = \frac{(-6-3.464)}{4}$

$x = (-0.634, 0)$ $\quad x = (-2.366, 0)$



b. $3x^2 - 2x + 7 = 0$

$a=3 \quad b=-2 \quad c=7$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(7)}}{2(3)}$

$x = \frac{2 \pm \sqrt{-80}}{6}$ $\quad \sqrt{80} \cdot \sqrt{-1}$ $\quad 8.944 \cdot i$

$x = \frac{2 + 8.944i}{6}$ $\quad x = \frac{2 - 8.944i}{6}$

$x = \frac{2}{6} + \frac{8.944i}{6}$ $\quad x = \frac{2}{6} - \frac{8.944i}{6}$

$x = 0.333 + 1.490i$ $\quad x = 0.333 - 1.490i$

complex conjugates

Discriminant | $b^2 - 4ac$ (The part under the root)

It Discriminates the type of solutions a Quadratic equations has

1 Rational Root | $b^2 - 4ac = 0$

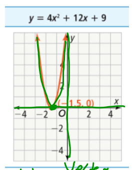
There is just the -b/2a part, no ±

ex. $x^2 - 6x + 9$

$a=1 \quad b=-6 \quad c=9$

$(-6)^2 - 4(1)(9)$

$36 - 36 = 0$ 1 solution



Vertex on x-axis

2 Rational Roots | $b^2 - 4ac$ = Positive and a perfect square

nice → Fractions decimals

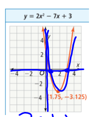
you will be ± a whole number

ex. $2x^2 - 6x + 4$

$a=2 \quad b=-6 \quad c=4$

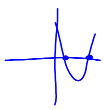
$(-6)^2 - 4(2)(4)$

$36 - 32 = 4$ Positive perfect square 2 solutions



2 Irrational Roots | $b^2 - 4ac$ = Positive but not a perfect square

ugly ↑ decimals



You will be left with a number under the radical

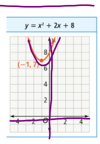
ex. $x^2 - 6x + 7$

$a=1 \quad b=-6 \quad c=7$

$(-6)^2 - 4(1)(7)$

$36 - 28 = 8$ Positive not P.S. so ugly 2 solutions

2 Complex Roots | $b^2 - 4ac < 0$



(its negative under √)

The solutions are complex conjugates with i

They come in pairs (Conjugate Zeros Theorem)

Graph will not show them.

ex. $x^2 + 16 = 0$ $\quad$ ex. $x^2 - 6x + 10 = 0$

$a=1 \quad b=0 \quad c=16$

$0^2 - 4(1)(16)$

$0 - 64$

-64

2 complex solutions

EXAMPLE 3 Identify the Number of Real-Number Solutions

Try It! b^2-4ac

3. Describe the nature of the solutions for each equation.

a. $16x^2 + 8x + 1 = 0$

$a=16 \quad b=8 \quad c=1$

b^2-4ac

$(8)^2-4(16)(1)$

$64-64$

0

1 Rational Root

b. $2x^2 - 5x + 6 = 0$

$a=2 \quad b=-5 \quad c=6$

$(-5)^2-4(2)(6)$

$25-48$

-23

2 Complex Roots

EXAMPLE 4 Interpret the Discriminant

Try It!

is it possible?

4. According to the model of Rachel's serve, will the ball reach a height of 3 meters?

$h=-5t^2+5t+2$

$3=-5t^2+5t+2$

$-3 \qquad\qquad -3$

$0=-5t^2+5t-1$

$a=-5 \quad b=5 \quad c=-1$

b^2-4ac

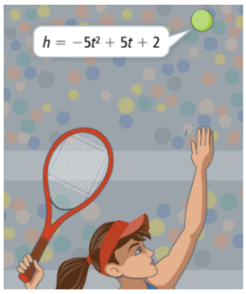
$(5)^2-4(-5)(-1)$

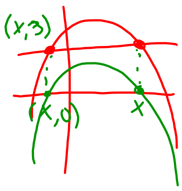
$25-20$

$5 \rightarrow$ 2 solutions

Yes the ball will reach 3 meters twice

Quadratic Formula only works when equation $=0$





7. At time t seconds, the height, h, of a ball thrown vertically upward is modeled by the equation $h = -5t^2 + 33t + 4$. About how long will it take for the ball to hit the ground? $h=0$

$0=-5t^2+33t+4$

$a=-5 \quad b=33 \quad c=4$

$x=\dfrac{-(33)\pm\sqrt{(33)^2-4(-5)(4)}}{2(-5)}$

$x=\dfrac{-33\pm\sqrt{1169}}{-10}$

$x=\dfrac{-33+34.19}{-10}$ and $x=\dfrac{-33-34.19}{-10}$

~~$x=-0.119$ seconds~~

$x=6.719$ seconds to hit the ground

when will the ball be at a height of 10 feet?

$h=10$

must $=0$ move 10 over, solve new equation

$10=-5t^2+33t+4$

$-10 \qquad\qquad -10$

$0=-5t^2+33t-6$

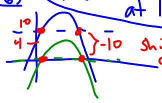
$a=-5 \quad b=33 \quad c=-6$

$x=\dfrac{-(33)\pm\sqrt{(33)^2-4(-5)(-6)}}{2(-5)}$

$x=\dfrac{-33\pm\sqrt{969}}{-10}$

$x=\dfrac{-33+31.13}{-10}$ and $x=\dfrac{-33-31.13}{-10}$

at $x=0.187$ seconds & $x=6.413$ seconds the ball will be at 10 ft



shifts critical values down onto x axis where $y=0$

when is the ball less than 5 feet?

$h=5$

$5=-5t^2+33t+4$

$-5 \qquad\qquad -5$

$0=-5t^2+33t-1$

$a=-5 \quad b=33 \quad c=-1$

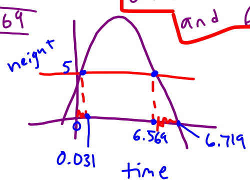
$x=\dfrac{-(33)\pm\sqrt{(33)^2-4(-5)(-1)}}{2(-5)}$

$x=\dfrac{-33\pm\sqrt{1069}}{-10}$

$x=\dfrac{-33+32.69}{-10}$ and $x=\dfrac{-33-32.69}{-10}$

$x=0.031$ & $x=6.569$

$0<x<0.031$ and $6.569<x<6.719$

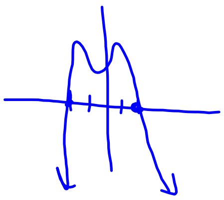


Quad Formula

Quad Form

Polynomials

$\underline{\ \ }^{2y} + \underline{\ \ }^{y} + \underline{\ \ }$



ex. $-2k^4 + 7k^2 = -9$

$+9 \qquad +9$

$a=-2 \quad b=7 \quad c=9$

$-2k^4 + 7(k^2) + 9 = 0$

Can use Quad Formula if highest degree is double middle exponent

$k^2 = \dfrac{-(7) \pm \sqrt{(7)^2 - 4(-2)(9)}}{2(-2)}$

$k^2 = \dfrac{-7 \pm \sqrt{49 + 72}}{-4}$

$k^2 = \dfrac{-7 \pm \sqrt{121}}{-4}$

$k^2 = \dfrac{-7 + 11}{-4}$ and $k^2 = \dfrac{-7 - 11}{-4}$

$k^2 = \dfrac{4}{-4} = -1 \qquad k^2 = \dfrac{-18}{-4}$

$k = \pm\sqrt{-1} \qquad k^2 = 4.5$

$k = \pm\sqrt{4.5}$

$k = i$ and $k = -i \qquad k = 2.12$ and $k = -2.12$

EXTEND 9-4

Algebra Lab

Applying Quadratic Equations

Many of the real-world problems you solved in Chapters 8 and 9 were physical problems involving the path of an object that is influenced by gravity. These paths, called **trajectories**, can be modeled by a quadratic function. The formula relating the height of the object $H(t)$ and time t is shown below.

$$H(t) = -\frac{1}{2}gt^2 + vt + h$$

Height of the object; Acceleration due to gravity; Initial velocity of the object; Initial height of the object

The acceleration due to gravity is 9.8 meters per second, per second. We express this by saying *9.8 meters per second squared*. Similarly, it is 32 feet per second squared.

If an object were projected downward, the initial velocity of the object is negativ

EXAMPLE 1

Juan kicks a football at a velocity of 25 meters per second. If the ball makes contact with his foot 0.5 meter off the ground, how long will the ball stay in the air?

We want to find the time t when $H(t)$ is 0. First substitute the known values into the motion formula. Since the known measures are written in terms of meters and meters per second, use 9.8 meters per second squared for the acceleration due to gravity.

$H(t) = -\frac{1}{2}gt^2 + vt + h$ Motion Formula

$0 = -\frac{1}{2}(9.8)t^2 + 25t + 0.5$ $H(t) = 0, g = 9.8, v = 25, h = 0.5$

$0 = -4.9t^2 + 25t + 0.5$ Simplify.

Use the Quadratic Formula to solve for t.

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \dfrac{-25 \pm \sqrt{25^2 - 4(-4.9)(0.5)}}{2(-4.9)}$ $a = -4.9, b = 25, c = 0.5$

$= \dfrac{-25 \pm \sqrt{634.8}}{-9.8}$ Simplify.

$t \approx -0.02$ or $t \approx 5.12$ Use a calculator.

Since time cannot be a negative value, discard the negative solution. The football will be in the air about 5 seconds.

EXAMPLE 2

Katharine is on a bridge 12 feet above a pond. She throws a handful of fish food straight down with a velocity of 8 feet per second. In how many second: will it reach the surface of the water?

Since the units given are in feet, use $g = 32\ ft/s^2$. Katharine throws the food down, so the initial velocity is negative. When the food hits the water, $H(t)$ will be 0 feet.

$H(t) = -\frac{1}{2}gt^2 + vt + h$ Motion Formula

$0 = -\frac{1}{2}(32)t^2 - 8t + 12$ $H(t) = 0, g = 32, v = -8, h = 12$

$0 = -16t^2 - 8t + 12$ Simplify.

$0 = -4t^2 - 2t + 3$ Divide each side by 4.

Use the Quadratic Formula to solve for t.

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \dfrac{2 \pm \sqrt{(-2)^2 - 4(-4)(3)}}{2(-4)}$ $a = -4, b = -2, c = 3$

$= \dfrac{2 \pm \sqrt{52}}{-8}$ Simplify.

$t \approx -1.15$ or $t \approx 0.65$ Use a calculator.

Discard the negative solution. The fish food will hit the water in 0.65 second.

Exercises

$v=10$

1. Darren swings at a golf ball on the ground with a velocity of 10 feet per second. How long was the ball in the air? about 0.625 s

$g=32$

$v=15$

2. Amalia hits a volleyball at a velocity of 15 meters per second. If the ball was hit from a height of 1.8 meters, determine the time it takes for the ball to land on the floor. Assume that the ball is not hit by another player. about 3.2 s

$g = 9.8$

3. Michael is repairing the roof on a shed. He accidentally dropped a box of nails from a height of 14 feet. How long did it take for the box to land on the ground? Since the box was dropped and not thrown, $v = 0$. about 0.94 s

4. Carmen threw a penny into a fountain. She threw it from a height of 1.2 meters and at a velocity of 6 meters per second. How long did it take for the penny to hit the surface of the water? about 0.17 s

EXTEND 9-4

Algebra Lab

Applying Quadratic Equations

Many of the real-world problems you solved in Chapters 8 and 9 were physical problems involving the path of an object that is influenced by gravity. These paths, called **trajectories**, can be modeled by a quadratic function. The formula relating the height of the object $H(t)$ and time t is shown below.

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The acceleration due to gravity is 9.8 meters per second, per second. We express this by saying *9.8 meters per second squared*. Similarly, it is 32 feet per second squared.

If an object were projected downward, the initial velocity of the object is negative.

EXAMPLE 1 Gravity $9.8\ m/s^2$

Juan kicks a football at a velocity of 25 meters per second. If the ball makes contact with his foot 0.5 meter off the ground, how long will the ball stay in the air?

We want to find the time t when $H(t)$ is 0. First substitute the known values into the motion formula. Since the known measures are written in terms of meters and meters per second, use 9.8 meters per second squared for the acceleration due to gravity.

Meters

$H(t) = -\frac{1}{2}gt^2 + vt + h$ Motion Formula

$0 = -\frac{1}{2}(9.8)t^2 + 25t + 0.5$ $H(t) = 0, g = 9.8, v = 25, h = 0.5$

$0 = -4.9t^2 + 25t + 0.5$ Simplify.

upward velocity $V = +25$

Use the Quadratic Formula to solve for t.

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-25 \pm \sqrt{25^2 - 4(-4.9)(0.5)}}{2(-4.9)}$ $a = -4.9, b = 25, c = 0.5$

$= \frac{-25 \pm \sqrt{634.8}}{-9.8}$ Simplify.

$t \approx -0.02$ or $t \approx 5.12$ Use a calculator.

Since time cannot be a negative value, discard the negative solution. The football will be in the air about 5 seconds.

EXAMPLE 2 Gravity $32\ ft/s^2$

Katharine is on a bridge 12 feet above a pond. She throws a handful of fish food straight down with a velocity of 8 feet per second. In how many seconds will it reach the surface of the water?

Since the units given are in feet, use $g = 32\ ft/s^2$. Katharine throws the food down, so the initial velocity is negative. When the food hits the water, $H(t)$ will be 0 feet.

feet

$H(t) = -\frac{1}{2}gt^2 + vt + h$ Motion Formula

$0 = -\frac{1}{2}(32)t^2 - 8t + 12$ $H(t) = 0, g = 32, v = -8, h = 12$

$0 = -16t^2 - 8t + 12$ Simplify.

$0 = -4t^2 - 2t + 3$ Divide each side by 4.

downward velocity $V = -8$

Use the Quadratic Formula to solve for t.

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{2 \pm \sqrt{(-2)^2 - 4(-4)(3)}}{2(-4)}$ $a = -4, b = -2, c = 3$

$= \frac{2 \pm \sqrt{52}}{-8}$ Simplify.

$t \approx -1.15$ or $t \approx 0.65$ Use a calculator.

Discard the negative solution. The fish food will hit the water in 0.65 second.

EXERCISES

units must match

1. Darren swings at a golf ball on the ground with a velocity of 10 feet per second. How long was the ball in the air? about 0.625 s

use $32\ ft/s^2$

on ground $h = 0$

$H(t) = -\frac{1}{2}(32)t^2 + 10t + 0$

$H(t) = 0 = -16t^2 + 10t + 0$

$a = -16$, $b = 10$, $c = 0$

$t = \frac{-10 \pm \sqrt{(10)^2 - 4(-16)(0)}}{2(-16)}$

$t = \frac{-10 \pm \sqrt{100 + 0}}{-32} = \frac{-10 \pm 10}{-32}$

$t = \frac{-10+10}{-32}$ $t = \frac{-10-10}{-32}$

$t = 0$ $t = \frac{-20}{-32}$

$\boxed{t = 0.625 \text{ seconds}}$

2. Amalia hits a volleyball at a velocity of 15 meters per second. If the ball was hit from a height of 1.8 meters, determine the time it takes for the ball to land on the floor. Assume that the ball is not hit by another player. about 3.2 s

$V = 15$

$h(t) = -\frac{1}{2}(9.8)t^2 + 15t + 1.8$

$h(t) = -4.9t^2 + 15t + 1.8$

$t = \frac{-(15) \pm \sqrt{(15)^2 - 4(-4.9)(1.8)}}{2(-4.9)}$

$t = \frac{-15 \pm \sqrt{225 + 35.28}}{-9.8}$ $t = 3.177$ seconds

3. Michael is repairing the roof on a shed. He accidentally dropped a box of nails from a height of 14 feet. How long did it take for the box to land on the ground? Since the box was dropped and not thrown, $v = 0$. about 0.94 s

$H(t) = -\frac{1}{2}(32)t^2 + 0t + 14$

$H(t) = -16t^2 + 0t + 14$

$a = -16$, $b = 0$, $c = 14$

$t = \frac{-0 \pm \sqrt{(0)^2 - 4(-16)(14)}}{2(-16)}$

$t = \frac{\pm\sqrt{256}}{-32}$

$t = \frac{16}{-32}$ $t = -\frac{16}{-32}$

$t = \cancel{-0.5 \text{ seconds}}$ not negative $\boxed{t = 0.5 \text{ seconds}}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Carmen threw a penny into a fountain. She threw it from a height of 1.2 meters and at a velocity of 6 meters per second. How long did it take for the penny to hit the surface of the water? about 0.17 s

downward

$g = 9.8$

$h(t) = -\frac{1}{2}(9.8)t^2 - 6t + 1.2$

$h(t) = -4.9t^2 - 6t + 1.2 = 0$

$a = -4.9$, $b = -6$, $c = 1.2$

$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-4.9)(1.2)}}{2(-4.9)}$

$t = \frac{6 \pm \sqrt{36 + 23.52}}{-9.8}$

$t = \frac{6 \pm \sqrt{59.52}}{-9.8}$

$t = \frac{6 + 7.714}{-9.8}$ $t = \frac{6 - 7.714}{-9.8}$

$t = \cancel{-1.399}$ $t = 0.175$ seconds

1. ESSENTIAL QUESTION How can you use the Quadratic Formula to solve quadratic equations or to predict the nature of their solutions?

2. Vocabulary Why is the discriminant a useful tool to use when solving quadratic equations?

3. Error Analysis Rick claims that the equation $x^2 + 5x + 9 = 0$ has no solution. Jenny claims that there are two solutions. Explain how Rick could be correct, and explain how Jenny could be correct.

4. Use Appropriate Tools What methods can you use to solve quadratic equations?

DO YOU KNOW HOW?

5. Describe the number and type of solutions of the equation $2x^2 + 7x + 11 = 0$.

6. Use the Quadratic Formula to solve the equation $x^2 + 6x - 10 = 0$.

7. At time t seconds, the height, h, of a ball thrown vertically upward is modeled by the equation $h = -5t^2 + 33t + 4$. About how long will it take for the ball to hit the ground?

8. Use the Quadratic Formula to solve the equation $x^2 - 8x + 16 = 0$. Is this the only way to solve this equation? Explain.

1. ESSENTIAL QUESTION How can you use the Quadratic Formula to solve quadratic equations or to predict the nature of their solutions?

CORRECT ANSWER

The Quadratic Formula provides the solutions to a quadratic equation in the form $ax^2 + bx + c = 0$. The discriminant in the formula, $b^2 - 4ac$, lets you predict the number and type of solutions. If $b^2 - 4ac > 0$, there will be 2 real solutions. If $b^2 - 4ac = 0$, the equation has 1 real solution. If $b^2 - 4ac < 0$, then the equation has 2 nonreal solutions.

2. Vocabulary Why is the discriminant a useful tool to use when solving quadratic equations?

CORRECT ANSWER

The discriminant, $b^2 - 4ac$, tells the number and type of solutions of a quadratic equation in standard form, $ax^2 + bx + c = 0$.

3. Error Analysis Rick claims that the equation $x^2 + 5x + 9 = 0$ has no solution. Jenny claims that there are two solutions. Explain how Rick could be correct, and explain how Jenny could be correct.

CORRECT ANSWER

Rick may be looking at the graph for real solutions. The discriminant for this equation is $5^2 - 4(1)(9) = 25 - 36 = -11$. There are no real-number solutions for this equation, but there are 2 nonreal solutions.

4. Use Appropriate Tools What methods can you use to solve quadratic equations?

CORRECT ANSWER

Methods include factoring, completing the square, factor by grouping, and the Quadratic Formula. Calculators can help with graphs and estimating square roots.

DO YOU KNOW HOW?

5. Describe the number and type of solutions of the equation $2x^2 + 7x + 11 = 0$.

CORRECT ANSWER

The equation has 2 non-real solutions.

6. Use the Quadratic Formula to solve the equation $x^2 + 6x - 10 = 0$.

CORRECT ANSWER

The solutions are $-3 \pm \sqrt{19}$.

7. At time *t* seconds, the height, *h*, of a ball thrown vertically upward is modeled by the equation $h = -5t^2 + 33t + 4$. About how long will it take for the ball to hit the ground?

CORRECT ANSWER

It will take the ball about 6.7 seconds to hit the ground.

8. Use the Quadratic Formula to solve the equation $x^2 - 8x + 16 = 0$. Is this the only way to solve this equation? Explain.

CORRECT ANSWER

The solution is $x = 4$. Another way to solve this equation is to recognize and factor the perfect square trinomial.