

Your Name

Mrs. Theo

11/28/22

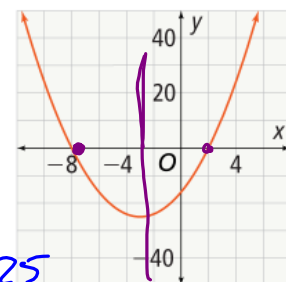
Notes

Lesson 2-3

INTERCEPT FORM QUADRATIC FUNCTIONS

CRITIQUE & EXPLAIN

Corey wrote an equation in factored form, $y = (x + 8)(x - 2)$, to represent a quadratic function. Kimberly wrote the equation $y = x^2 + 6x - 16$, and Joshua wrote the equation $y = (x + 3)^2 - 25$.



A. Reason Do all three equations represent the same function? If not, whose is different? Explain algebraically.

Enter your answer

$$y = (x+8)(x-2)$$

$$y = x^2 - 2x + 8x - 16$$

$$y = x^2 + 6x - 16 \checkmark$$

$$y = (x+3)^2 - 25$$

$$y = x^2 + 3x + 3x + 9 - 25$$

$$y = x^2 + 6x - 16 \checkmark$$

B. How else could you determine if all three equations represent the same function?

$$y = (x+3)^2 - 25$$

Vertex: $(-3, -25)$

$$a = 1 \checkmark$$

$$y = x^2 + 6x - 16$$

$$x = \frac{-6}{2(1)} = \frac{-6}{2} = -3$$

$$y = (-3)^2 + 6(-3) - 16$$

$$y = -25 \rightarrow V: (-3, -25) \checkmark$$

$$a = 1 \checkmark$$

$$y = (x+8)(x-2)$$

$$0 = (x+8)(x-2)$$

$$x+8=0 \quad x-2=0$$

$$-8 \quad -8 \quad +2 \quad +2$$

$$x = -8 \quad x = 2$$

$$V: (-3, -25)$$

C. What information can Corey's form help you find that is more difficult to find using Kimberly's or Joshua's form?

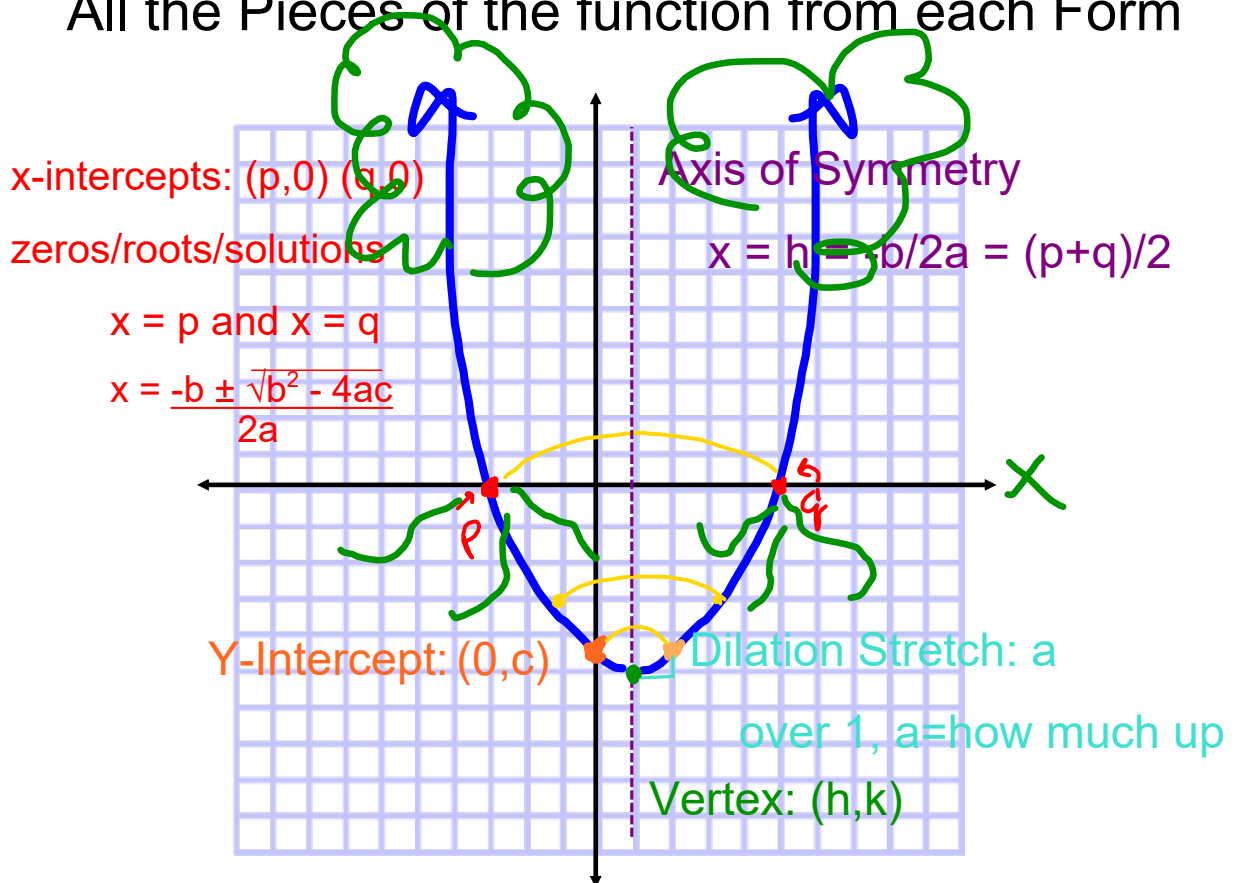
x intercepts / roots

$$y = (-3+8)(-3-2)$$

$$y = 5 \cdot -5 = -25$$

Vertex Form	Intercept Form	Standard Form
$f(x) = a(x-h)^2 + k$	$f(x) = a(x-p)(x-q)$	$f(x) = ax^2 + bx + c$
<p>Vertex: (h,k) h- horizontal shift k- vertical shift</p> <p>Dilation: a a is negative: Reflection $0 < a < 1$: shrink $a > 1$: stretch</p> <p>Axis of Symmetry: $x = h$</p> <p>Easily Translatable</p>	<p>X intercepts/ Solutions/Roots/Zeros: $x = p$ and $x = q$</p> <p>Dilation: a</p> <p>Axis of Symmetry: Half way between the roots $x = h = \frac{p+q}{2}$</p> <p>To find Vertex: input the axis of symmetry x value, h, in the function, the y value will be the k</p>	<p>Axis of Symmetry: $x = h = \frac{-b}{2a}$</p> <p>To find Vertex: input the axis of symmetry x value, h, in the function, the y value will be the k</p> <p>Dilation: a</p> <p>Y Intercept: (0,c)</p> <p>X intercepts/Roots $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p>

All the Pieces of the function from each Form



2-3 Intercept Form pg. 40

EXAMPLE 1 Factor a Quadratic Expression

Try It!

1. Factor the expression.

a. $x^2 - 9$ $\sqrt{x^2} = x$
 $(x+3)(x-3)$ $\sqrt{9} = 3$

b. $3x^2 - 7x + 2$

① $\begin{matrix} a & b & c \\ 3 & -7 & 2 \end{matrix}$

$(x - \frac{1}{3})(x - \frac{6}{3})$
 $(3x - 1)(x - 2)$

② $\begin{matrix} a \cdot c & & \\ 3 \cdot 2 & & \\ & 6 & \\ -1 & + & -6 \\ & -7 & \end{matrix}$

③ \div by a
 ④ simplify + move denominator up

2-3 Intercept Form pg. 40

EXAMPLE 1 Factor a Quadratic Expression

Try It!

1. Factor the expression.

a. $x^2 - 9$

$(x+3)(x-3)$

Difference of squares
 $\sqrt{x^2} = x$
 $\sqrt{9} = 3$
 Square root each term
 (+)(-)

b. $3x^2 - 7x + 2$

① $\begin{matrix} a & b & c \\ 3 & -7 & 2 \end{matrix}$

$3x^2 - 1x - 6x + 2$
 $x(3x-1) - 2(3x-1)$
 $(3x-1)(x-2)$

② split middle term
 ③ Factor by grouping

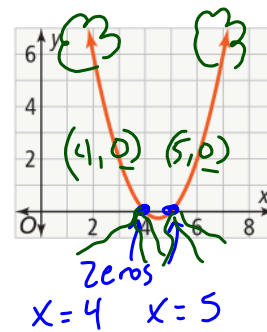
② $\begin{matrix} a \cdot c & & \\ 3 \cdot 2 & & \\ & 6 & \\ -1 & + & -6 \\ & -7 & \end{matrix}$

EXAMPLE 2 Relate Factors to Zeros of a Function

ESS

Try It!

2. The graph shows the function $y = x^2 - 9x + 20$. Identify the zeros of the function. How do the zeros relate to the factors of $x^2 - 9x + 20$?



Zeros - x-intercepts
 - where function crosses x-axis
 y is 0
 - roots, solutions

$$y = x^2 - 9x + 20$$

a	b	c	
		20	-9
-4	-5	-9	b
+4	+5		

$$0 = (x-4)(x-5)$$

$$x-4=0 \quad x-5=0$$

$$+4 \quad +4 \quad +5 \quad +5$$

$$x=4 \quad x=5$$

Zeros are opposite signs than factors

EXAMPLE 3 Solve Quadratic Equations by Factoring

Try It!

3. Solve the equation by factoring.

a. $x^2 + 8x = 20$

$$\frac{-20 - 20}{(x-2)(x+10)} = 0$$

-20	-20	
-2	10	a.c
+2	+10	-4
+2	-10	+1
-2	-10	-3
+2	+10	b

$$x-2=0 \quad x+10=0$$

$$+2 \quad +2 \quad -10 \quad -10$$

$$x=2 \quad x=-10$$

$$(2, 0) \quad (-10, 0)$$

b. $2x^2 = 3x + 2$

$$2x^2 - 3x - 2 = 0$$

$$2x^2 - 4x + x - 2 = 0$$

$$2x(x-2) + 1(x-2) = 0$$

$$(x-2)(2x+1) = 0$$

$$x-2=0 \quad 2x+1=0$$

$$+2 \quad +2 \quad 2x \quad -1 \quad -1$$

$$\frac{2x}{2} = -\frac{1}{2}$$

$$x=2 \quad x=-\frac{1}{2}$$

Intercept Form
Features to graph:
 $a(x-p)(x-q)$

Example 1:
 $f(x) = (x-3)(x+1)$
 X-Intercepts: $x-3=0 \rightarrow x=3$, $x+1=0 \rightarrow x=-1$
 Stretch: None ($a=1$)
 Reflection/Opens: up or down (a is positive)
 Axis of Symmetry: $x = \frac{p+q}{2} = \frac{3+(-1)}{2} = 1$
 Vertex: $x=1$
 Graph:

Example 2:
 $f(x) = -1/4(2x-5)(x-1)$
 X-Intercepts: $2x-5=0 \rightarrow x=2.5$, $x-1=0 \rightarrow x=1$
 Stretch: Shrink by $\frac{1}{4}$
 Reflection/Opens: up or down (a is negative)
 Axis of Symmetry: $x = \frac{2.5+1}{2} = 1.75$
 Vertex: $(1.75, 0.281)$
 Graph:

Example 3:
 $f(x) = 1/2(x+4)(x-2)$
 X-Intercepts: $x+4=0 \rightarrow x=-4$, $x-2=0 \rightarrow x=2$
 Stretch: Shrink by $\frac{1}{2}$
 Reflection/Opens: up or down (a is positive)
 Axis of Symmetry: $x = \frac{-4+2}{2} = -1$
 Vertex:

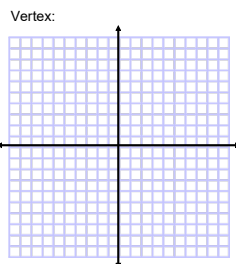
Example 4:
 $0 = -3x(x+7)$
 X-Intercepts: $x=0$, $x=-7$
 Stretch: Stretch by 3
 Reflection/Opens: up or down (a is negative)
 Axis of Symmetry: $x = \frac{0+(-7)}{2} = -3.5$
 Vertex:

Handwritten notes:
 To Find Vertex: Plug in A.o.S. as x value to get y value.
 $f(1) = (1-3)(1+1) = (-2)(2) = -4$
 $f(1) = -4$
 $(1, -4)$
 1) Plot x-int, 2) Plot A.o.S, 3) Plot Vertex, 4) use 'a' value or y-int and reflect.
 0 = $-3x(x+7) = -3x^2 - 21x$

Homework: Intercept Form Features to graph:

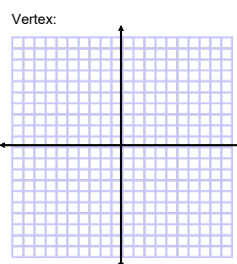
1. $f(x) = (x-5)(x+1)$

X-Intercepts:
 Stretch:
 Reflection/Opens: up or down
 Axis of Symmetry:



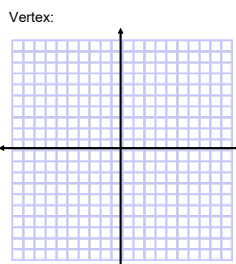
2. $f(x) = 1/4(x+6)(x-4)$

X-Intercepts:
 Stretch:
 Reflection/Opens: up or down
 Axis of Symmetry:



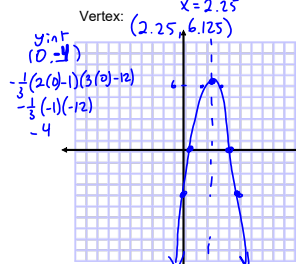
3. $y = 2x(x-5)$

X-Intercepts:
 Stretch:
 Reflection/Opens: up or down
 Axis of Symmetry:



4. $y = -1/3(2x-1)(3x-12)$

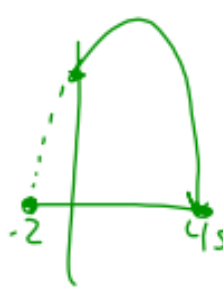
X-Intercepts: $2x-1=0 \rightarrow x=0.5$, $3x-12=0 \rightarrow x=4$
 Stretch: Shrink by $\frac{1}{3}$
 Reflection/Opens: up or down
 Axis of Symmetry: $x = \frac{0.5+4}{2} = 2.25$
 Vertex: $(2.25, 6.125)$



Try It!

4. A baseball is thrown from the upper deck of a stadium, 128 ft above the ground. The function $h(x) = -16x^2 + 32x + 128$ gives the height of the ball x seconds after it is thrown. How long will it take the ball to reach the ground?

$h(x) = 0$



$$0 = -16x^2 + 32x + 128$$

$$0 = -16(x^2 - 2x - 8)$$

$$0 = -16(x - 4)(x + 2)$$

$-16 \neq 0$ $x - 4 = 0$ $x + 2 = 0$

$x = 4$ seconds

~~$x = -2$ seconds~~
can't have negative time

Identify the interval(s) on which the function $y = x^2 - 2x - 3$ is positive.

The y-values of a quadratic function can only turn from positive to negative or from negative to positive when the graph crosses the x-axis. Find the zeros of the function to identify these points.

$0 = x^2 - 2x - 3$ Set expression equal to 0.
 $0 = (x - 3)(x + 1)$ Factor.
 $x - 3 = 0$ or $x + 1 = 0$ Zero Product Property.
 $x = 3$ or $x = -1$ Solve.

The zeros of the function are $x = 3$ and $x = -1$.

Two zeros create three intervals. Choose an x-value to test in each interval. Substitute the x-value into the original expression to determine if the corresponding y-value is positive or negative.

$x < -1$	$-1 < x < 3$	$x > 3$
Choose $x = -3$. $(-3)^2 - 2(-3) - 3$ $= 9 + 6 - 3$ $= 12$	Choose $x = 1$. $(1)^2 - 2(1) - 3$ $= 1 - 2 - 3$ $= -4$	Choose $x = 6$. $(6)^2 - 2(6) - 3$ $= 36 - 12 - 3$ $= 21$
Positive	Negative	Positive

Try It!

5. Identify the interval(s) on which the function $y = x^2 - 4x - 21$ is negative.

1) Find x intercepts
Factor

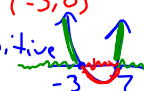
2) Visualize graph
using 'a'

Positive Interval
 $-\infty < x < -3$ \cup $7 < x < \infty$

Negative interval?
 $x \in (-3, 7)$
or $-3 < x < 7$

$a = 1$

$y = (x - 7)(x + 3)$ ~~-21~~
 $x - 7 = 0$ $x + 3 = 0$
 $x = 7$ $x = -3$
 $(-3, 0)$



EXAMPLE 6 Write the Equation of a Parabola in Factored Form

Write an equation of a parabola with x-intercepts at $(-2, 0)$ and $(-1, 0)$ and which passes through the point $(-3, 20)$.

$$y = a(x-p)(x-q)$$

$$\begin{array}{l} x = -2 \\ +2 + 2 \\ x + 2 = 0 \end{array} \qquad \begin{array}{l} x = -1 \\ +1 + 1 \\ x + 1 = 0 \end{array}$$

$$\begin{aligned} y &= a(x+2)(x+1) \\ 20 &= a(-3+2)(-3+1) \\ 20 &= a(-1)(-2) \\ \frac{20}{2} &= \frac{2a}{2} \rightarrow a = 10 \end{aligned}$$

$$y = 10(x+2)(x+1)$$

6. Write an equation of a parabola with x-intercepts at $(3, 0)$ and $(-3, 0)$ and which passes through the point $(1, 2)$.

$$x = 3 \qquad x = -3$$

$$\begin{aligned} y &= a(x-3)(x+3) \\ 2 &= a(1-3)(1+3) \\ 2 &= a(-2)(4) \\ \frac{2}{-8} &= \frac{-8a}{-8} \\ -\frac{1}{4} &= a \end{aligned}$$

$$y = -\frac{1}{4}(x-3)(x+3)$$

Intercept Form
Features to graph:

$$f(x) = (x-3)(x+1) = 0$$

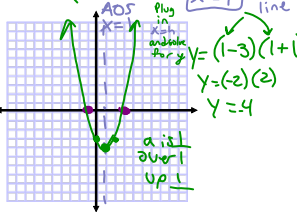
X-Intercepts: $x-3=0$ or $x+1=0$
 $x=3$ or $x=-1$
 $(3,0)$ $(-1,0)$

Stretch: $a=1$ No stretch

Reflection/Opens: up or down a is positive

Axis of Symmetry: $x = \frac{3+(-1)}{2} = 1$

Vertex: $(1, -4)$



$$f(x) = -\frac{1}{4}(2x-5)(x-1)$$

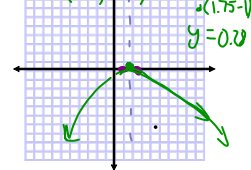
X-Intercepts: $2x-5=0$ or $x-1=0$
 $2x=5$ or $x=1$
 $x=2.5$ or $x=1$
 $(2.5, 0)$ $(1, 0)$

Stretch: $a = -\frac{1}{4}$ shrink by $\frac{1}{4}$

Reflection/Opens: up or down a is negative

Axis of Symmetry: $x = \frac{2.5+1}{2} = 1.75$

Vertex: $(1.75, 0.28)$



$$f(x) = \frac{1}{2}(x+4)(x-2)$$

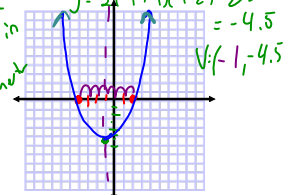
X-Intercepts: $x+4=0$ or $x-2=0$
 $x=-4$ or $x=2$
 $(-4, 0)$ $(2, 0)$

Stretch: $\frac{1}{2} < 1$ shrink by $\frac{1}{2}$

Reflection/Opens: up or down

Axis of Symmetry: $x = -1$

Vertex: $y = \frac{1}{2}(-1+4)(-1-2) = \frac{1}{2} \cdot 3 \cdot -3 = -4.5$
Vertex: $(-1, -4.5)$



Need y value so plug in Axis of Symmetry

$$0 = -3x(x+7)$$

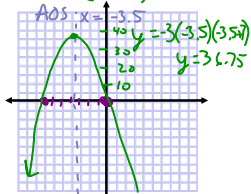
X-Intercepts: $-3x(x+7)=0$
 $x=0$ or $x+7=0$
 $x=0$ or $x=-7$
 $(0, 0)$ $(-7, 0)$

Stretch: stretch by 3

Reflection/Opens: up or down a is negative

Axis of Symmetry: $x = \frac{0+(-7)}{2} = -3.5$

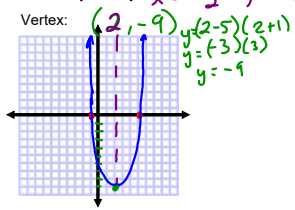
Vertex: $(-3.5, 36.75)$



Homework **Key**: Intercept Form Features to graph:

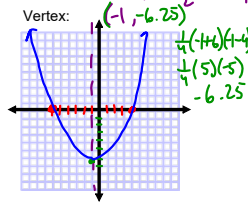
1. $f(x) = (x - 5)(x + 1)$

X-Intercepts: $x = 5, x = -1$
 Stretch: **No stretch or shrink**
 Reflection/Opens: **up** or down
 Axis of Symmetry: $x = \frac{5+(-1)}{2}, x = 2$



2. $f(x) = 1/4(x + 6)(x - 4)$

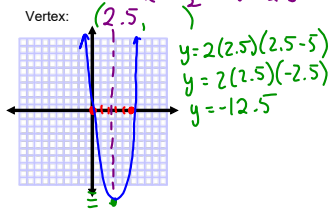
X-Intercepts: $x = -6, x = 4$
 Stretch: **shrink by 1/4**
 Reflection/Opens: **up** or down
 Axis of Symmetry: $x = \frac{-6+4}{2} \Rightarrow x = -1$



$\frac{2x=0}{2} \Rightarrow x-5=0$

3. $y = 2x(x - 5)$

X-Intercepts: $x = 0, x = 5$
 Stretch: **stretch by 2**
 Reflection/Opens: **up** or down
 Axis of Symmetry: $x = \frac{0+5}{2} \Rightarrow x = 2.5$



4. $y = -1/3(2x - 1)(3x - 12)$

X-Intercepts: $2x-1=0 \Rightarrow x=1/2, 3x-12=0 \Rightarrow x=4$
 Stretch: **shrink by 1/3**
 Reflection/Opens: **down**
 Axis of Symmetry: $x = \frac{0.5+4}{2} = 2.25$

