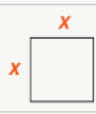



Explore and Reason pg. 31

The table represents $A(x)$, the area of a square as a function of side length x units, where x is a positive real number.

A. Consider the function where the areas in the table are doubled. Write the equation of a function that represents this.

Side Length (units)	x	1	2	3	4
Model					
Area (sq. units)	$A(x)$	1	4	9	16

$D(x) = 2 \quad 8 \quad 18 \quad 32$

$A(x) = x^2$
 $D(x) = 2x^2 = 2A(x)$

The table represents $A(x)$, the area of a square as a function of side length x units, where x is a positive real number.

C. Find the equation for a function whose x -values are the same as $A(x)$ but whose y -values are 2 units greater than each y -value in $A(x)$.

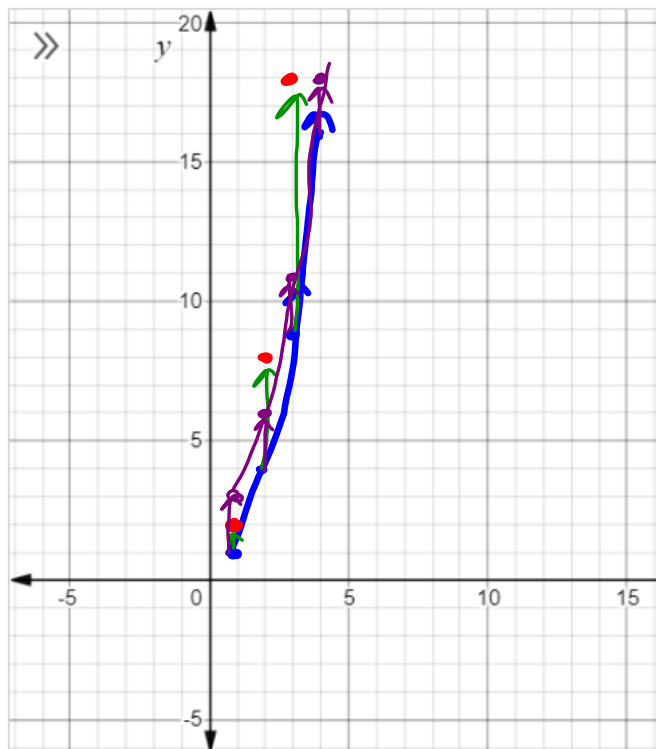
Side Length (units)	x	1	2	3
Area (sq. units)	$A(x)$	1	4	9
	$g(x)$	3	6	11

$A(x) + 2$
 $g(x) = x^2 + 2$

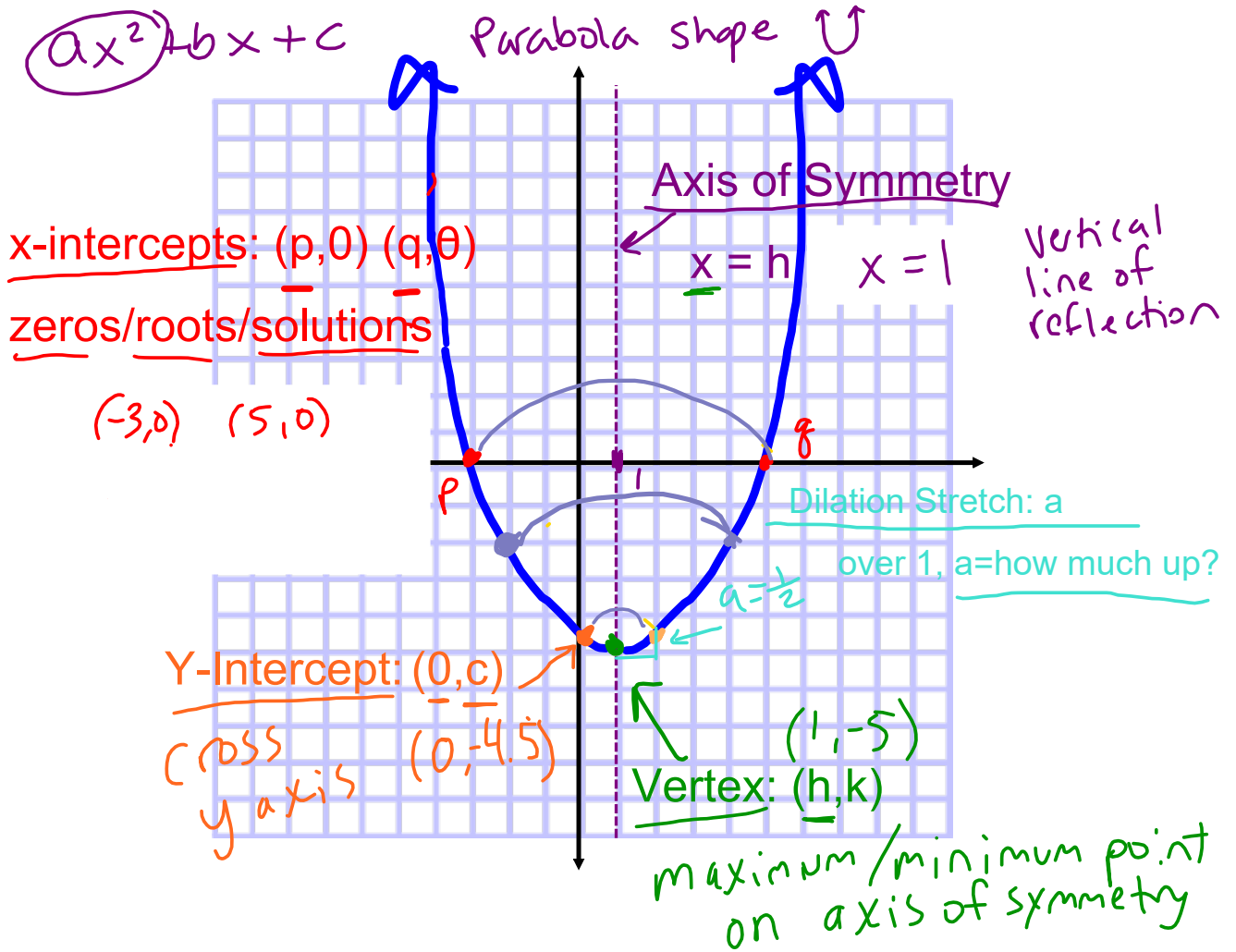
The table represents $A(x)$, the area of a square as a function of side length x units, where x is a positive real number.

B. **Look for Relationships** Use the tool to graph the ordered pairs for both $A(x)$ and your new function, $D(x)$. How would you describe the differences in the locations of these points?

Side Length (units)	x	1	2	3
Area (sq. units)	$A(x)$	1	4	9
	$D(x)$	2	8	18



Vertex Form	Intercept Form	Standard Form
$f(x) = a(x-h)^2 + k$	$f(x) = a(x-p)(x-q)$	$f(x) = ax^2 + bx + c$
<p>Vertex: (h,k) h- horizontal shift k- vertical shift Dilation: a a is negative: Reflection $0 < a < 1$: shrink $a > 1$: stretch Axis of Symmetry: $x = h$</p>	<p>X intercepts/ Solutions/Roots/Zeros: $x = p$ and $x = q$ Dilation: a Axis of Symmetry: Half way between the roots $x = h = \frac{p + q}{2}$ To find Vertex: input the axis of symmetry x value, h, in the function, the y value will be the k</p>	<p>Axis of Symmetry: $x = h = \frac{-b}{2a}$ To find Vertex: input the axis of symmetry x value, h, in the function the y value will be the k Dilation: a Y Intercept: (0,c) X intercepts/Roots $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p>
Easily Translatable		



Vertex
Form
Quadratic
Function
Family

$$y = a \cdot (x - h)^2 + k$$

Dilation

Translation

$$V: (h, k)$$

is 'a' positive $a > 0$ No $\uparrow \uparrow$
 ex. $2, \frac{5}{3}, \frac{1}{3}$

is 'a' negative $a < 0$ $\downarrow \downarrow$
 ex. $-2, -\frac{5}{7}, -\frac{1}{3}$ **Reflected** Yes

Horizontal
Shift

Vertical
Shift

$x - \#$ ex. $x - 3$
 shift right $h = 3$

$+ \#$ ex. $+ 7$
 shift up $k = 7$

$x + \#$ ex. $x + 3$
 shift left $x - (-3)$
 $h = -3$

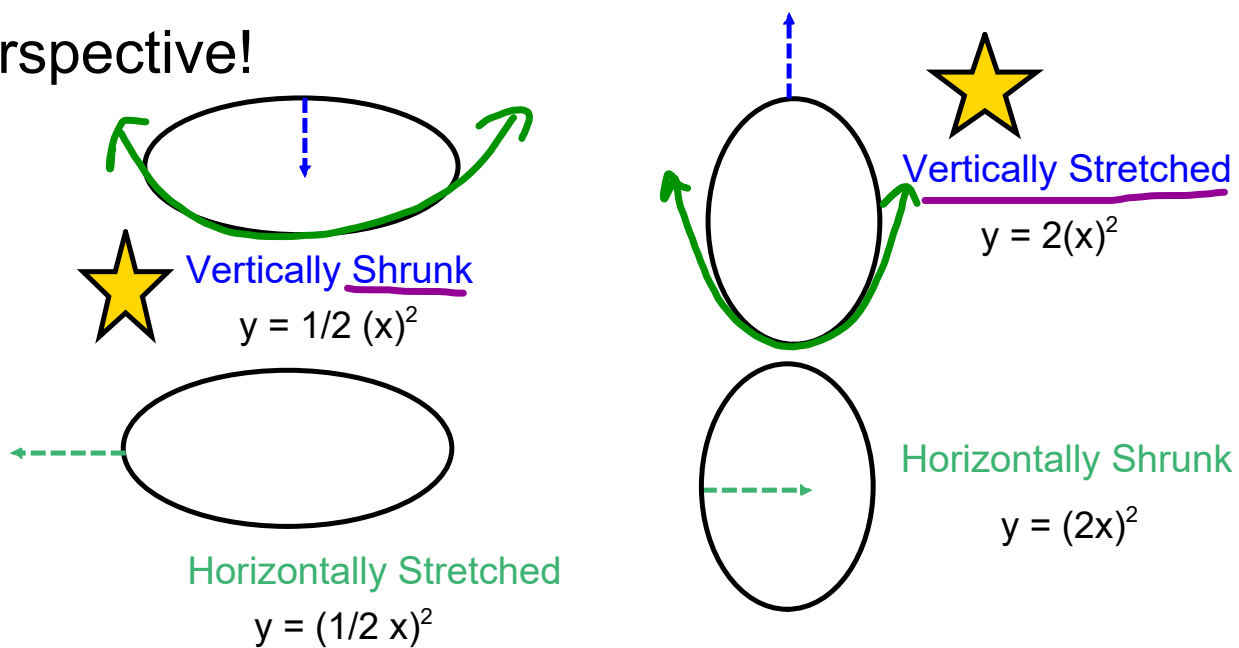
$- \#$ ex. $- 7$
 shift down $k = -7$

Stretch $|a| > 1$
 ex. $2, -2, \frac{5}{3}, \frac{5}{3}, 4.5$

Shrink $0 < |a| < 1$
 ex. $\frac{5}{7}, -\frac{1}{3}, 0.8, -0.35$

$\frac{1}{3}$

Perspective!



Wider

Skinnier

Which
Form is it
in?

Practice

$$0 = -7(3x - 1)(x - 2)$$

Intercept

$$f(x) = -(x - 3)^2 + 14$$

Vertex: (3, 14)

$$f(x) = (x + 7)(x + 9)$$

Intercept

$$f(x) = \frac{7}{4}x^2 - 6$$

Standard

Vertex: (0, -6)

$$\frac{7}{4}(x - 0)^2 - 6$$

$$f(x) = 4(x - 9)^2 + k$$

Vertex
(-9, 0) (9, 0)

$$f(x) = -75x^2 + 25x$$

Standard

$$f(x) = -\frac{1}{8}(x + 2)^2 - 5$$

Vertex: (-2, -5)

(2, 5)

$$f(x) = -5x^2 + 7x + 6$$

Standard

EXAMPLE 1 Transform a Quadratic Function

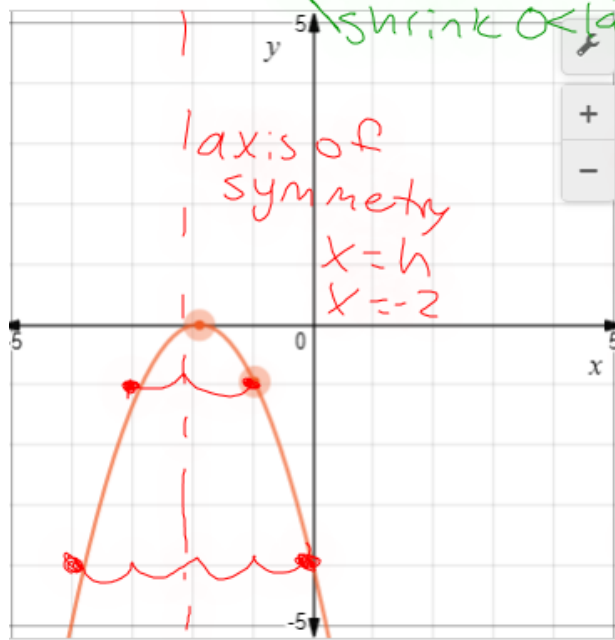
Try It!

1. Describe the transformations $f(x) = x^2$. Then graph the function.

a. $g(x) = -(x + 2)^2$

reflection
left + 2
 $V: (-2, 0)$

$a(x-h)^2 + k$
 • Reflection if a is neg
 • Stretch $|a| > 1$
 • Shrink $0 < |a| < 1$
 Vertex (h, k)
 right $x - \#$ up $+ \#$
 left $x + \#$ down $- \#$



Steps to Graph
 1) Graph Vertex
 2) go right 1 and move 'a'
 3) Graph y int $(0,)$
 Plug 0 in for x
 4) Reflect over line of symmetry

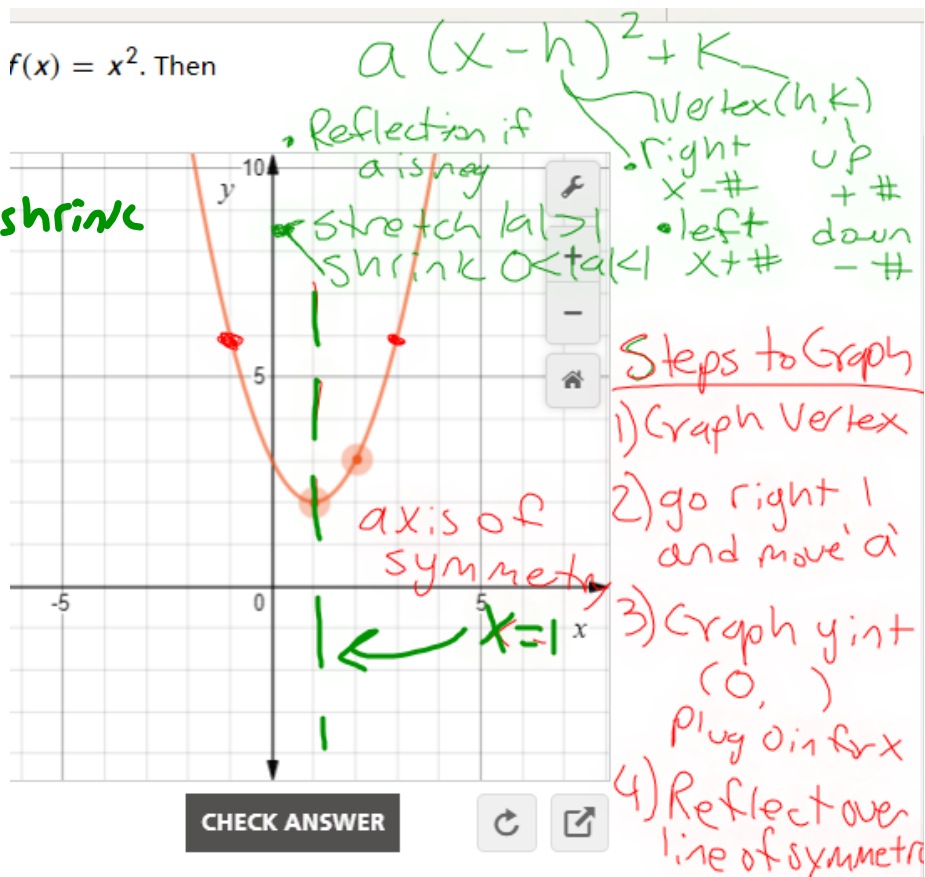
EXAMPLE 1 Transform a Quadratic Function

1. Describe the transformations of the parent function graph the function.

b. $g(x) = (x - 1)^2 + 2$

$f(x) = x^2$. Then

No Reflection
stretch or shrink
right 1
up 2
V: (1, 2)



2-1 Vertex Form pg. 32

Try It! 2-1 Vertex Form
 $a(x-h)^2+k$

2. Identify the vertex, axis of symmetry, minimum or maximum, domain, and range of the function $f(x) = -(x+4)^2 - 5$.

$-(x-(-4))^2 + (-5)$
 $a = -1$ $h = -4$ $k = -5$

Vertex
 (h, k)
 V: $(-4, -5)$

Axis of Symmetry

- Vertical line goes through the vertex
- all points are reflected around it

Always $x = h$
 A.o.S.: $x = -4$

minimum or maximum
 • if a is positive then right side is up
 Vertex is low
 Vertex is a min

• if a is negative then right side is down
 Vertex is high
 Vertex is a max
 Vertex is a max

Domain
 X values can put in left to right
 $-\infty < x < \infty$
 X is all #'s between $-\infty$ & ∞

Range
 y values you get out lowest to highest

★ Check out Vertex
 (h, k) is it a max or min?
 $-\infty < y \leq k$ $k \leq y < \infty$
 Range: $-\infty < y \leq -5$

Try It!

3. What is the equation of a parabola with a vertex of $(1, -4)$ and which passes through $(-2, -1)$?

$$y = a(x-h)^2 + k$$

$$y = \frac{1}{3}(x-1)^2 - 4$$

$$a =$$

$$h = 1$$

$$k = -4$$

1) Plug in Vertex $h+k$

2) Plug in extra point $x+y$
solve for a

$$(-1) = a((-2) - 1)^2 - 4$$

$$-1 = a(-3)^2 - 4$$

$$-1 = a \cdot (9) - 4$$

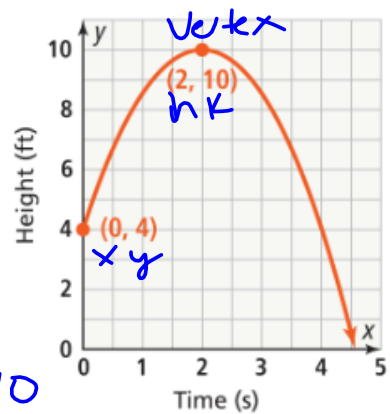
$$+4 \qquad +4$$

$$\frac{3}{9} = \frac{9a}{9}$$

$$\frac{1}{3} = a$$

Try It!

4. The graph shows the height of the flying disk with respect to time. What is the equation of the function? Write the equation in vertex form. Then write the equation in the form $y = ax^2 + bx + c$.



Enter your answer

$$y = -1.5(x - 2)^2 + 10$$

$$a =$$

$$h = 2$$

$$k = 10$$

CHECK ANSWER

$$4 = a(0 - 2)^2 + 10$$

$$4 = a(-2)^2 + 10$$

$$4 = a \cdot 4 + 10$$

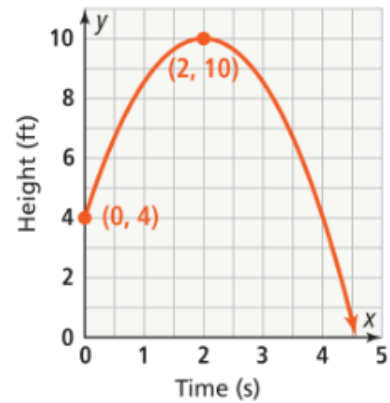
$$-10 \qquad -10$$

$$\frac{-6}{4} = \frac{4a}{4}$$

$$-1.5 = a$$

Try It!

4. The graph shows the height of the flying disk with respect to time. What is the equation of the function? Write the equation in vertex form. Then write the equation in the form $y = ax^2 + bx + c$.



Enter your answer

$$y = -\frac{3}{2}(x-2)^2 + 10$$

CHECK ANSWER

4)

$$y = a(x-h)^2 + k$$

$$y = -\frac{3}{2}(x-2)^2 + 10$$

plug in
(0, 4)
x y

$$4 = a(0-2)^2 + 10$$

$$4 = a \cdot 4 + 10$$

$$-6 = 4a$$

$$-\frac{3}{2} = a = -1.5$$

$$y = -\frac{3}{2}(x-2)^2 + 10 + 10$$

$$y = -\frac{3}{2}(x-2)(x-2) + 10$$

$$y = -\frac{3}{2}(x^2 - 2x - 2x + 4)$$

$$y = -\frac{3}{2}(x^2 - 4x + 4) + 10$$

$$y = -\frac{3}{2}x^2 + 6x - 6 + 10$$

$$y = -\frac{3}{2}x^2 + 6x + 4$$

a b c

Standard Form

Vertex Form
Features to graph:

$a(x-h)^2+k$

1. $f(x) = 1(x - 2)^2 + 5$

$a=1$ $h=2$ $k=5$

Vertex: $(h,k) \rightarrow (2,5)$

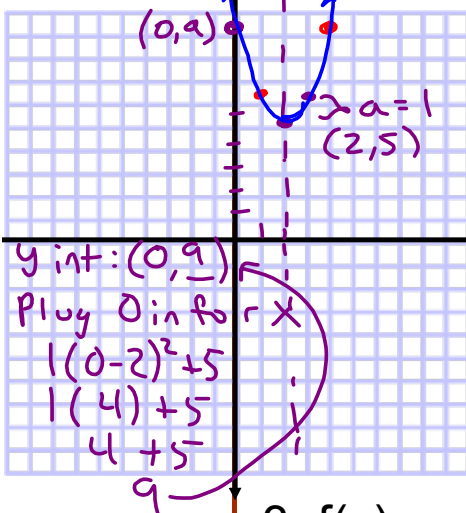
Reflection: a is positive } No Reflection

Dilation: $|a|=|1|=1$

So No Dilation

Steps to graph

- 1) Graph the Vertex
- 2) from Vertex, move over 1 and up 'a'
- 3) Graph y int
- 4) Draw A.O.S through Vertex
- 5) Reflect all points



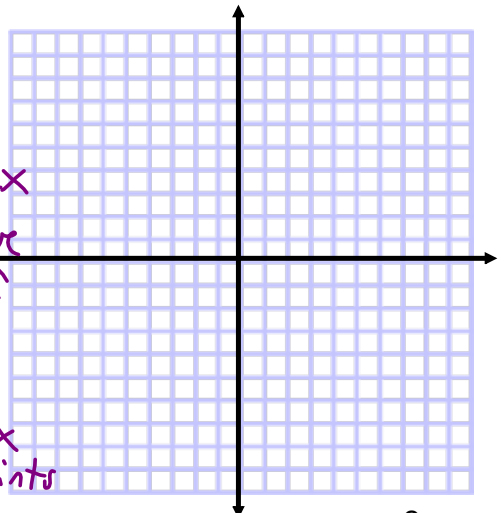
2. $f(x) = 2(x + 4)^2 - 7$

$a=$ $h=$ $k=$

Vertex:

Reflection:

Dilation:



3. $f(x) = -1(x + 3)^2 + 6$

$a=-1$ $h=-3$ $k=6$

Vertex: $(-3, 6)$

Reflection: Yes Reflection a is negative }

Dilation: $|-1|=1$
No stretch or shrink

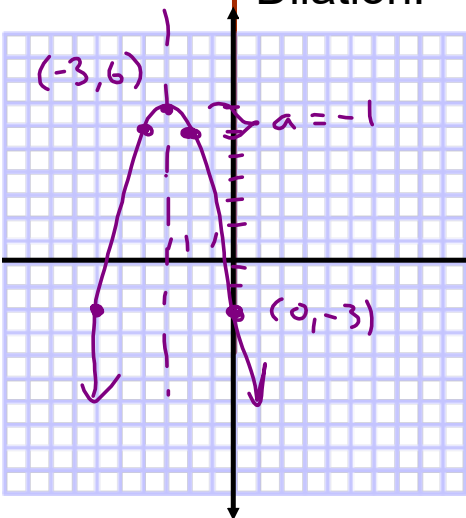
y int $(0, -3)$

$f(0) = -1(0+3)^2 + 6$

$f(0) = -1(9) + 6$

$f(0) = -9 + 6$

$f(0) = -3$



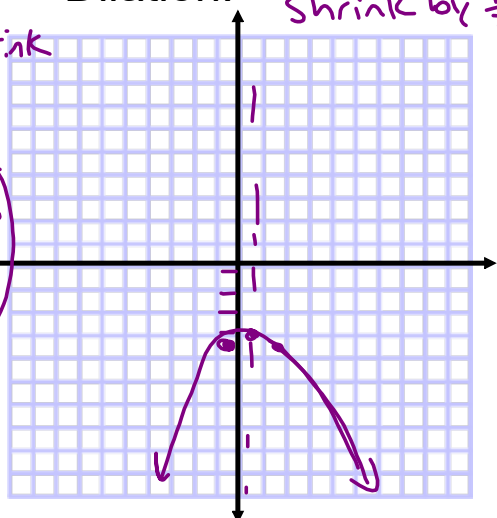
4. $f(x) = -0.5(x - 1)^2 - 4$

$a=-0.5$ $h=1$ $k=-4$

Vertex: $(1, -4)$

Reflection: Yes a is negative

Dilation: $|-0.5|=0.5 < 1$
Shrink by $\frac{1}{2}$



Vertex Form
Features to graph:

$a(x-h)^2+k$

1. $f(x) = 1(x - 2)^2 + 5$

$a = 1$ $h = 2$ $k = 5$

Vertex: $(h, k) \rightarrow (2, 5)$

Reflection: No Reflection
'a is positive'

Dilation: $|a| = 1$ No Dilation

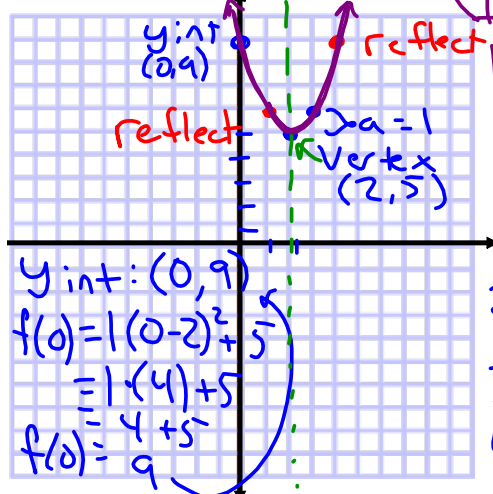
2. $f(x) = 2(x + 4)^2 - 7$

$a = 2$ $h = -4$ $k = -7$

Vertex: $(-4, -7)$

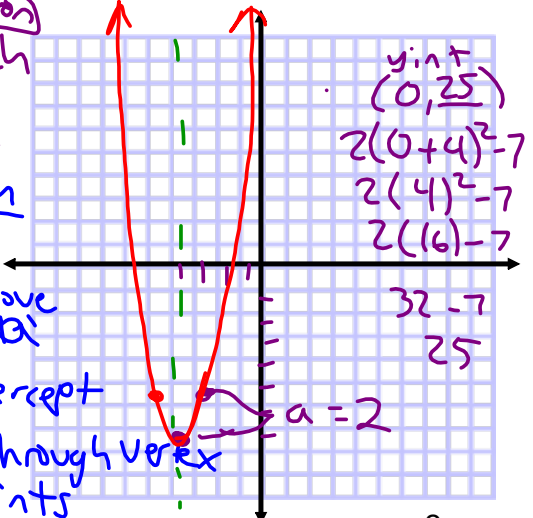
Reflection: No

Dilation: Stretch by 2



$|a| > 1$ Stretch
 $|a| < 1$ Shrink

- Steps to graph
- 1) graph Vertex
 - 2) from vertex move over 1 and up 'a'
 - 3) graph y intercept
 - 4) draw A.o.S. through vertex
 - 5) reflect points



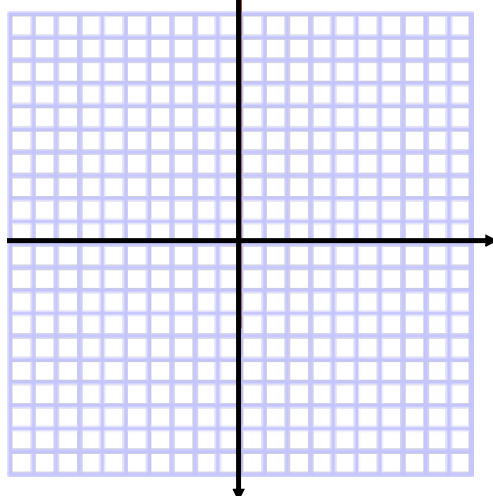
3. $f(x) = -1(x + 3)^2 + 6$

$a = -1$ $h = -3$ $k = 6$

Vertex: $(-3, 6)$

Reflection:

Dilation:



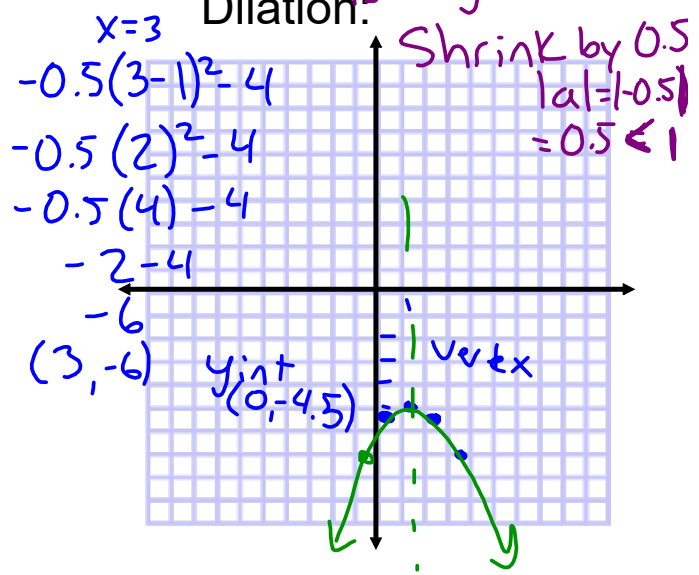
4. $f(x) = -0.5(x - 1)^2 - 4$

$a = -0.5$ $h = 1$ $k = -4$

Vertex: $(1, -4)$

Reflection: Yes Reflect
'a is negative'

Dilation: Shrink by 0.5
 $|a| = |-0.5| = 0.5 < 1$



Vertex
Form
Features
to graph:

Homework

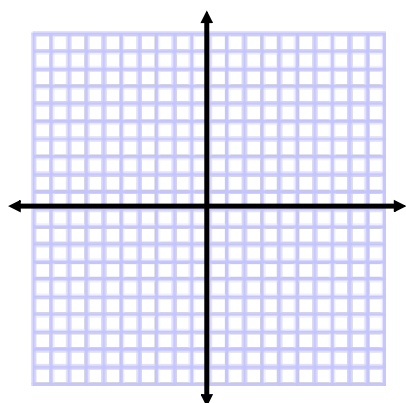
5. $f(x) = -3(x - 5)^2$

$a =$ $h =$ $k =$

Vertex:

Reflection:

Dilation:



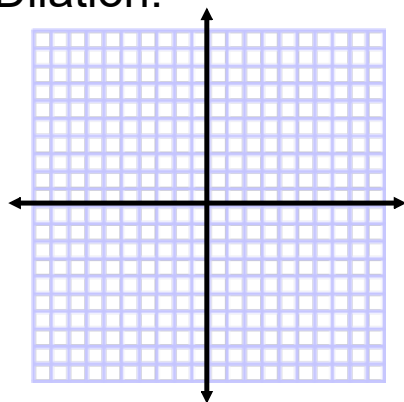
7. $f(x) = -0.5(x + 6)^2 + 3$

$a =$ $h =$ $k =$

Vertex:

Reflection:

Dilation:



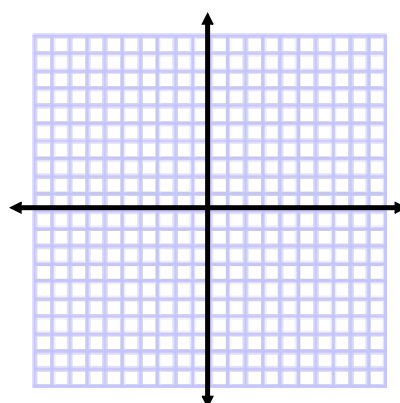
6. $f(x) = 5/3x^2 + 5$

$a =$ $h =$ $k =$

Vertex:

Reflection:

Dilation:



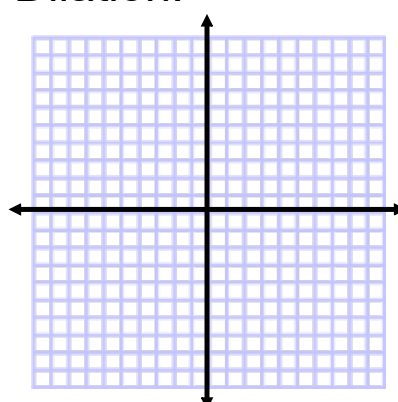
8. $f(x) = 1/4(x - 2)^2 - 6$

$a =$ $h =$ $k =$

Vertex:

Reflection:

Dilation:



Vertex Form Features to graph:

1. $f(x) = 1(x - 2)^2 + 5$

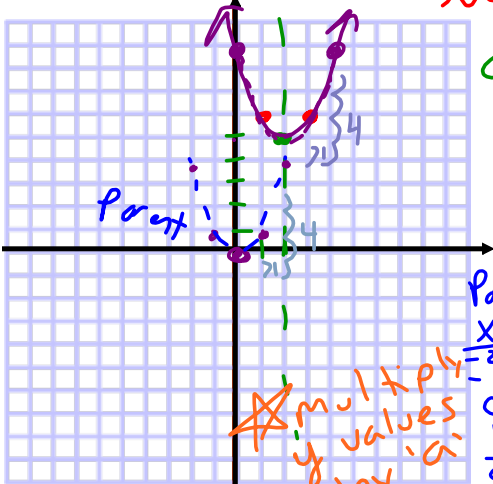
$a = 1$ $h = 2$ $k = 5$

Vertex: $(2, 5)$

Reflection: None 'a' is positive

Dilation: None $a = 1$

axis of symmetry $x = h = 2$



Parent x	New y = x^2 * a
-2	4 * 1 = 4
-1	1 * 1 = 1
0	0 * 1 = 0
1	1 * 1 = 1
2	4 * 1 = 4

* multiply y values by 'a'

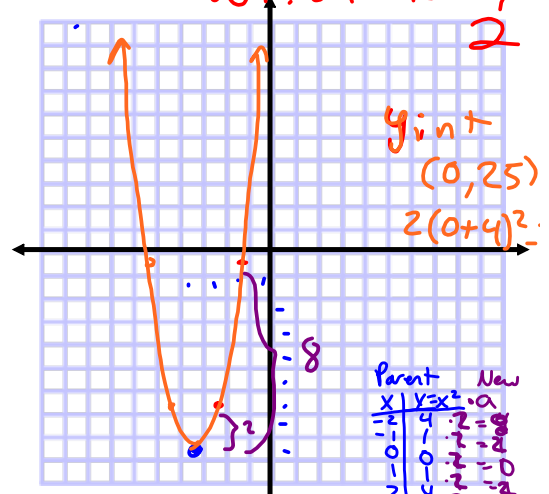
2. $f(x) = 2(x + 4)^2 - 7$

$a = 2$ $h = -4$ $k = -7$

Vertex: $(-4, -7)$

Reflection: None 'a' = 2 which is positive

Dilation: Vert. stretch by 2



Parent x	New y = x^2 * a
-2	4 * 2 = 8
-1	1 * 2 = 2
0	0 * 2 = 0
1	1 * 2 = 2
2	4 * 2 = 8

3. $f(x) = -1(x + 3)^2 + 6$

$a = -1$ $h = -3$ $k = 6$

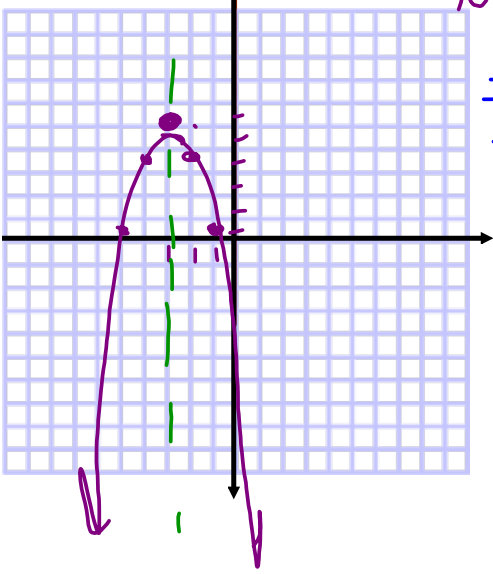
Vertex: $(-3, 6)$

Reflection: Yes 'a' = -1 'a' is negative

Dilation: No $|a| = 1$

x	y = x^2 * a
-2	4 * -1 = -4
-1	1 * -1 = -1
0	0 * -1 = 0
1	1 * -1 = -1
2	4 * -1 = -4

axis of symmetry $x = h = -3$



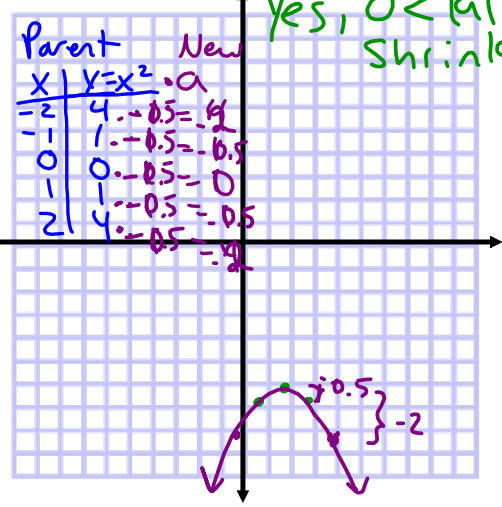
4. $f(x) = -0.5(x - 1)^2 - 4$

$a = -0.5$ $h = 1$ $k = -4$

Vertex: $(1, -4)$

Reflection: Yes 'a' = -0.5 'a' is negative

Dilation: Yes, $0 < |a| < 1$ Shrink



Parent x	New y = x^2 * a
-2	4 * -0.5 = -2
-1	1 * -0.5 = -0.5
0	0 * -0.5 = 0
1	1 * -0.5 = -0.5
2	4 * -0.5 = -2

Vertex
Form
Features
to graph:

Homework
KEY

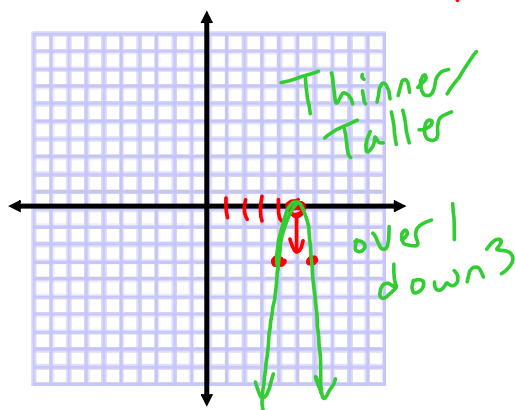
5. $f(x) = -3(x - 5)^2$

$a = -3$ $h = 5$ $k = 0$

Vertex: $(5, 0)$

Reflection: Yes

Dilation: Stretch by 3



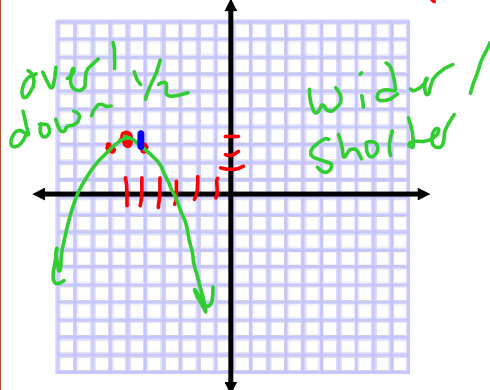
7. $f(x) = -0.5(x + 6)^2 + 3$

$a = -0.5$ $h = -6$ $k = 3$

Vertex: $(-6, 3)$

Reflection: Yes

Dilation: Shrink by $\frac{1}{2}$



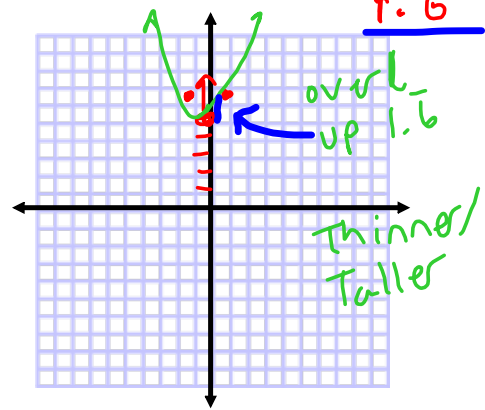
6. $f(x) = \frac{5}{3}x^2 + 5$

$a = \frac{5}{3}$ $h = 0$ $k = 5$

Vertex: $(0, 5)$

Reflection: No

Dilation: Stretch by 1.6



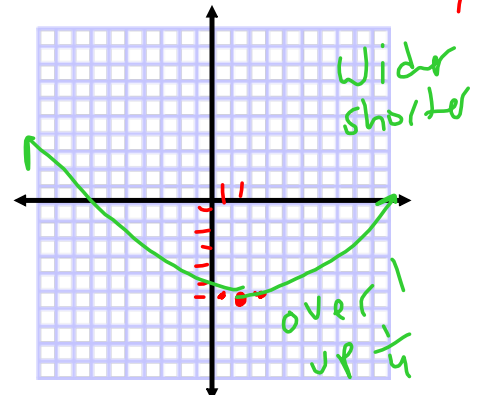
8. $f(x) = \frac{1}{4}(x - 2)^2 - 6$

$a = \frac{1}{4}$ $h = 2$ $k = -6$

Vertex: $(2, -6)$

Reflection: No

Dilation: Shrink by $\frac{1}{4}$



Summary of Transformation Options

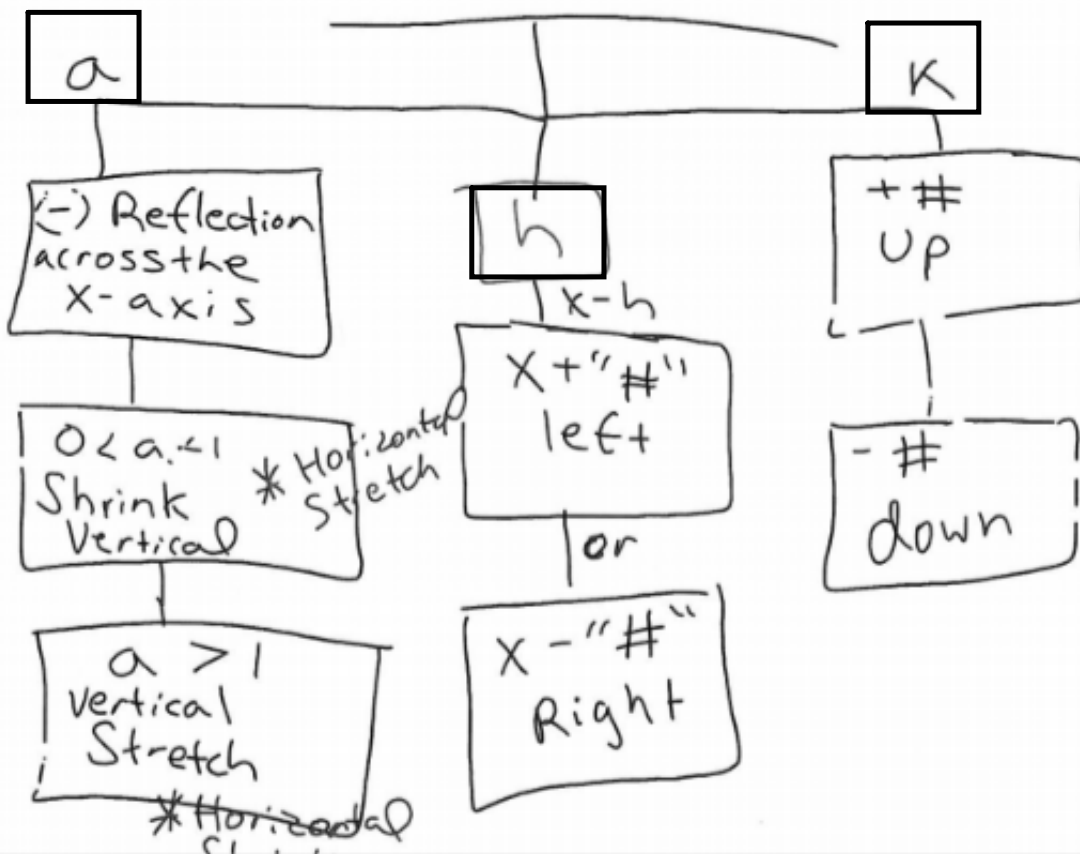
$a(x-h)^2 + k$

$a\sqrt{x-h} + k$

$a(x-h)^3 + k$

$ab^{x-h} + k$

$a f(x-h) + k$



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How does the equation of a quadratic function in vertex form highlight key features of the function's graph?

2. **Error Analysis** Given the function $g(x) = (x + 3)^2$, Martin says the graph should be translated right 3 units from the parent graph $f(x) = x^2$. Explain his error. **MP3**

3. **Vocabulary** What shape does a quadratic function have when graphed?

4. **Communicate Precisely** How are the graphs of $f(x) = x^2$ and $g(x) = -(x + 2)^2 - 4$ related? **MP6**

Do You KNOW HOW?

Describe the transformation of the parent function $f(x) = x^2$.

5. $g(x) = -(x + 5)^2 + 2$
 $a <$ reflected $a < 0$
 $|a| = 1$ No dilation
 $h = -5$ shift left 5
 $k = 2$ shift up 2

6. $h(x) = (x + 2)^2 - 7$

Write the equation of each parabola in vertex form.

7. Vertex: $(-3, 7)$; Point: $(-2, -5)$

$f(x) = a(x - h)^2 + k$

$f(x) = ?(x - 3)^2 + 7$

$-5 = a(-2 + 3)^2 + 7$

$-5 = a(1)^2 + 7$

$-5 = a + 7$

$-7 = a$

$-12 = a$

$f(x) = -12(x + 3)^2 + 7$

1st plug in vertex x
 2nd plug in other point to solve for a

9. Vertex: $(-4, 6)$; Point: $(-2, -2)$

10. Vertex: $(7, 4)$; Point: $(5, 16)$

Stretch or shrink?
 Reflect or no?
 left or right?
 UP or down?

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How does the equation of a quadratic function in vertex form highlight key features of the function's graph?

It is easy to see vertex and dilation factor and if its reflected and how

2. **Error Analysis** Given the function $g(x) = (x + 3)^2$, Martin says the graph should be translated right 3 units from the parent graph $f(x) = x^2$. Explain his error. **MP3**

He saw +3 and so thought right 3 but the formula is $x -$ so if its $x + 3$ its actually $x - (-3)$ and it went left 3

3. **Vocabulary** What shape does a quadratic function have when graphed?

a U shape or parabola shape

4. **Communicate Precisely** How are the graphs of $f(x) = x^2$ and $g(x) = -(x + 2)^2 - 4$ related? **MP6**

They are both quadratic $g(x)$ is reflected vertically, shifted left 2 and down 4 from $f(x)$

Do You KNOW HOW?

Describe the transformation of the parent function $f(x) = x^2$.

5. $g(x) = -(x + 5)^2 + 2$

Vertical reflection
 $a = -1$
 horiz. shift left 5 $h = -5$
 Vertical shift up 2 $k = 2$

6. $h(x) = (x + 2)^2 - 7$

horiz. shift left 2 $h = -2$
 Vert shift down 7 $k = -7$

Write the equation of each parabola in vertex form.

7. Vertex $(-3, 7)$; Point $(-2, -5)$

$f(x) = a(x - h)^2 + k$

$f(x) = a(x - (-3))^2 + 7$

$-5 = a(-2 + 3)^2 + 7$

$-5 = a(1)^2 + 7$

8. Vertex $(1, 3)$; Point $(2, 5)$

$f(x) = -12(x + 3)^2 + 7$

$f(x) = 2(x - 1)^2 + 3$

9. Vertex $(-4, 6)$; Point $(-2, -2)$

10. Vertex $(7, 4)$; Point $(5, 16)$

plug in vertex
 Plug in point for x and y
 $-5 = a + 7$
 $-7 = -7$ solve for a
 $-12 = a$

$5 = a(2 - 1)^2 + 3$

$5 = a(1)^2 + 3$

$5 = a + 3$

$2 = a$